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Lorentzian Quintessential Inflation

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אוניברסיטת בן-גוריון בנגב
Ben-Gurion University of the Negev

MG16 5-10 JULY 2021
SIXTEENTH MARCEL GROSSMANN MEETING
ON RECENT DEVELOPMENTS IN THEORETICAL AND EXPERIMENTAL GENERAL RELATIVITY, ASTROPHYSICS AND RELATIVISTIC FIEL THEORIES

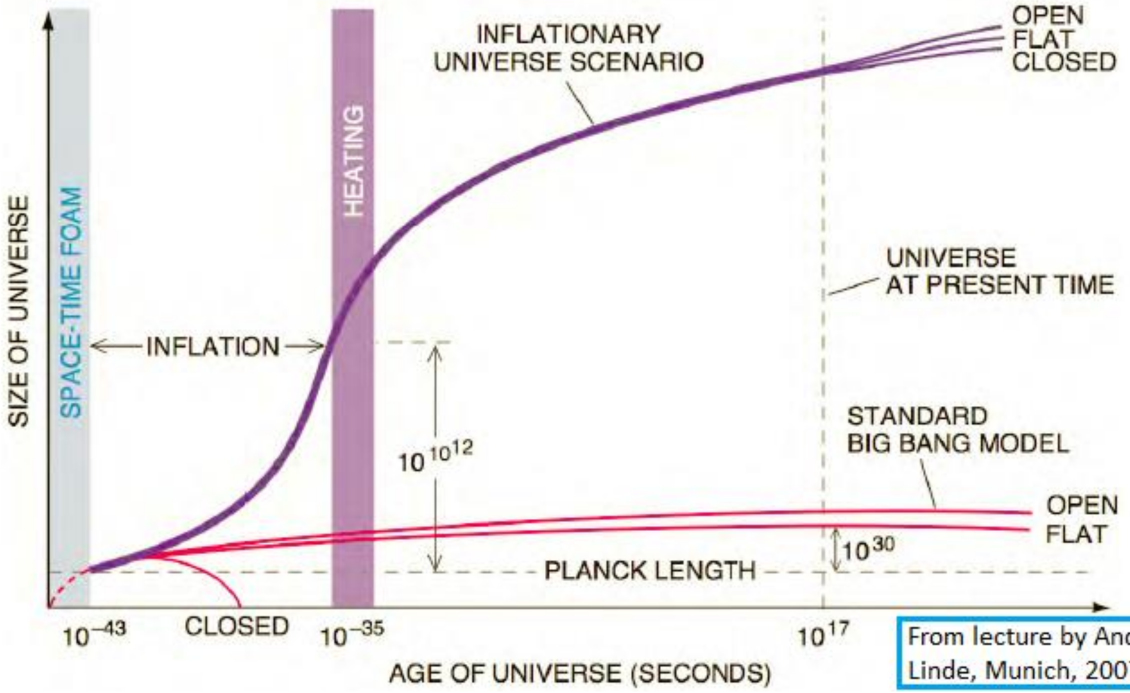
1905.03731 Eur.Phys.J. C80 (2020) no.5, 480

2004.00339 awarded 2nd prize in Gravity Research Foundations
International Journal of Modern Physics D 2042002

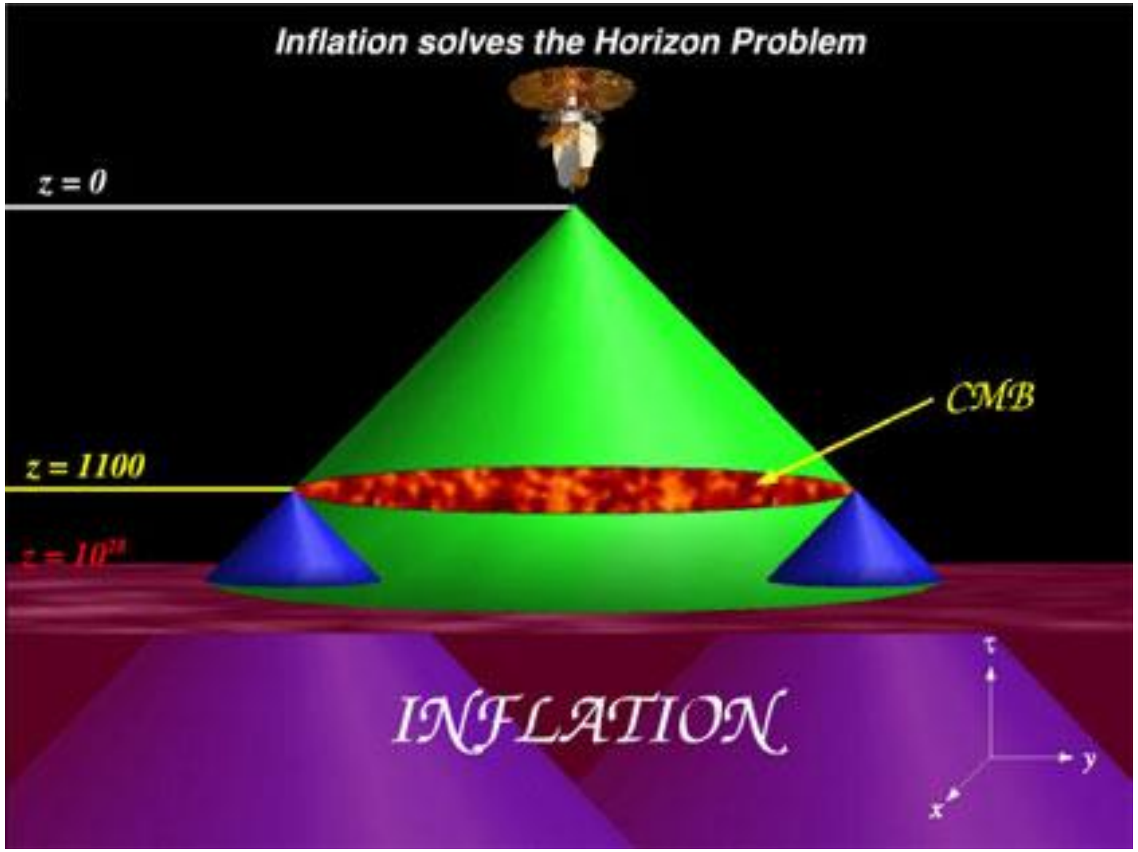
2006.04129 Eur.Phys.J. C80 (2020) no.6, 577

Problems with the early universe theory

- The Flatness Problem: $\mathrm{k} = 0$.
- The Horizon Problem



From lecture by Andrei Linde, Munich, 2007

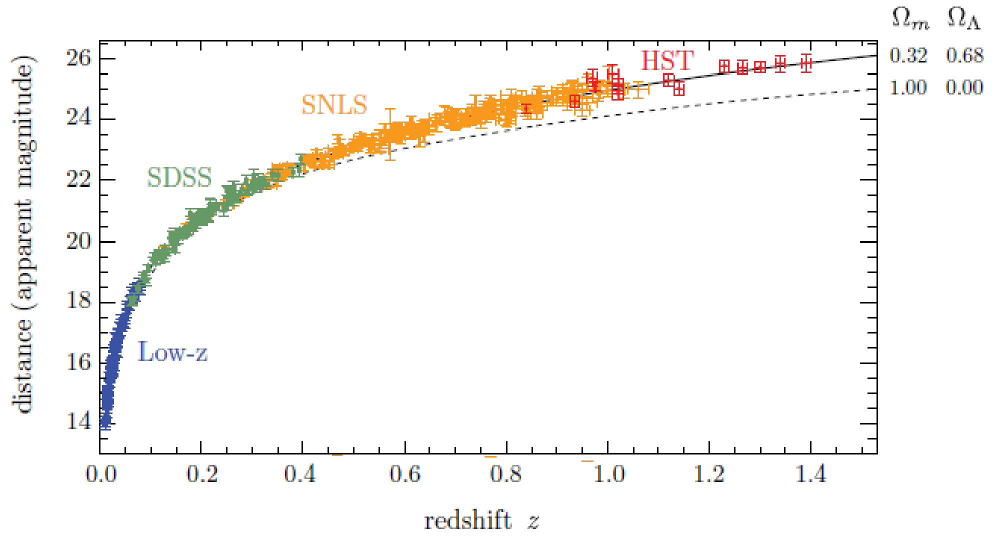


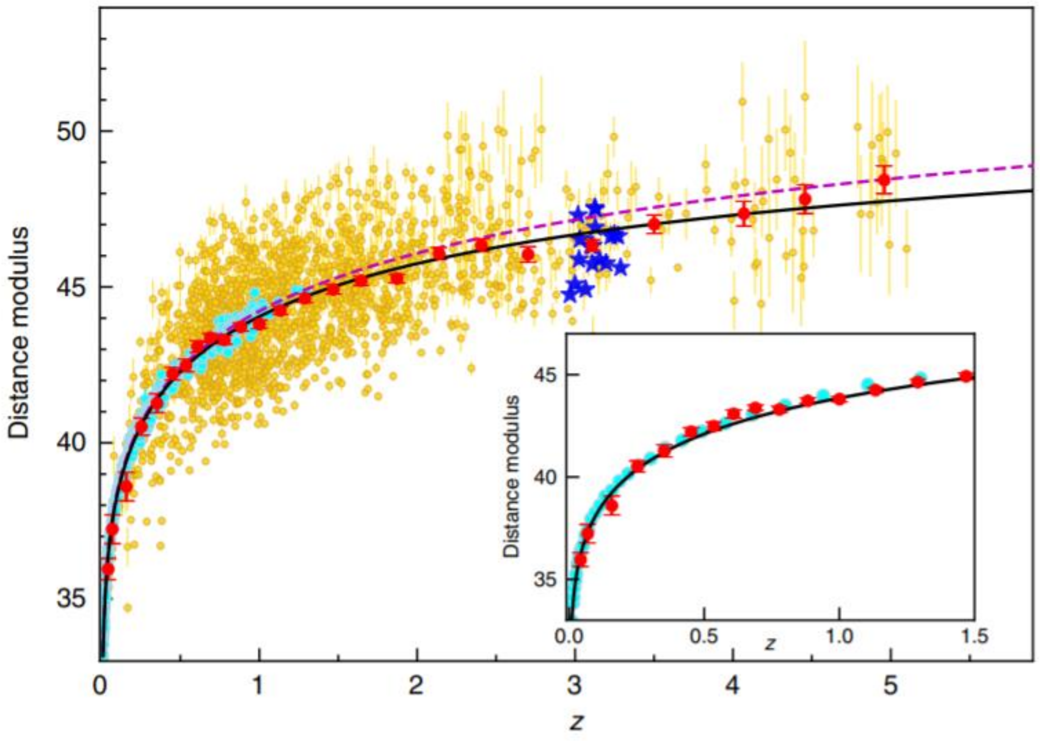
Signs for Dark Energy

- “Pantheon” - Type Ia Super Nova

Riess 2018,2019

- Quasars
- Cosmic Chronometers
- Baryon Acoustic Oscillations
- Perturbations (CMB, DES, DESI,LSST)





G. Risaliti and E. Lusso Nature Ast. 2019

A. D. Linde, Phys. Lett. 108B, 389 (1982)

A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982)

Inflaton field

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)$$

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$-3H^2 - 2\dot{H} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- We can address the “dark energy” equation of state, by the slow roll approximation:

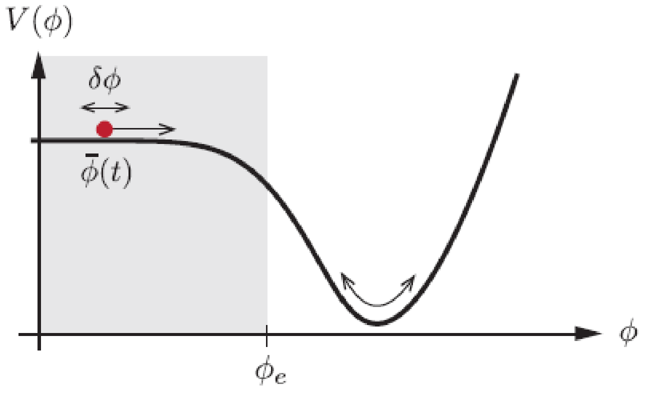
$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$$

- The Slow Roll parameters and the number of e-folds read:

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = \frac{V''}{V}, \qquad N = \int_{\phi_i}^{\phi_f} \frac{V}{V'}\,d\phi$$

The end of inflation $\epsilon = 1,\ N = 60$

Quantum Fluctuations



- Perturbed inflaton field: $\phi(\,\eta, x) = \phi(\eta) + \frac{f(\eta,x)}{a(\eta)}$
- Mukhanov-Sasaki equation for the FT of the modes at early times:
$$\ddot{f}_k + k^2 f_k = 0, \qquad k\,\eta \gg 1$$
- The Power Spectrum: $\langle |f|^2 \rangle = \Delta_f^2(k,\eta) = \frac{k^3}{2\pi^2}|f_k|^2$
- The curvature power spectrum: $\Delta_\mathrm{R}^2 = \frac{H^2}{8\pi\epsilon} \sim A_s k^{n_s - 1}$, the ***spectral index*** reads*:* $n_s = 1 - 6\epsilon + 2\eta$
- Tensor perturbations read: $\Delta_h^2 = \frac{2H^2}{\pi^2} \sim A_t k^{n_t - 1}$ with $n_t \approx -2\epsilon$. The ***scalar to tensor ratio***: $\mathrm{r} = \frac{A_t}{A_s} = 16\epsilon$

Starobinsky, A. A. (December 1979). ***Journal of Experimental and Theoretical Physics Letters***

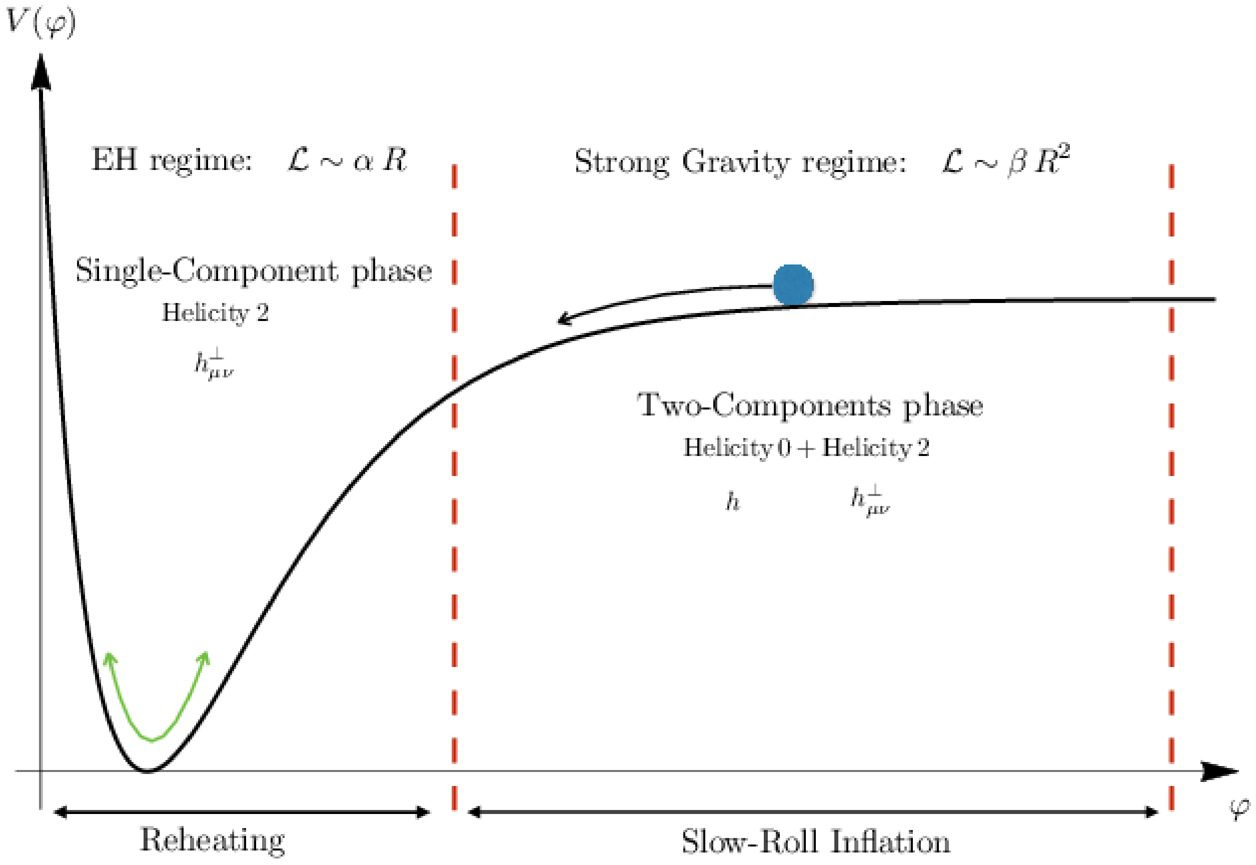
Starobinsky inflation

- Quantum corrections to GR are important for the early universe.

$$\mathcal{L} = \frac{1}{2}R + \frac{R^2}{12M^2}$$

In the Einstein Frame:

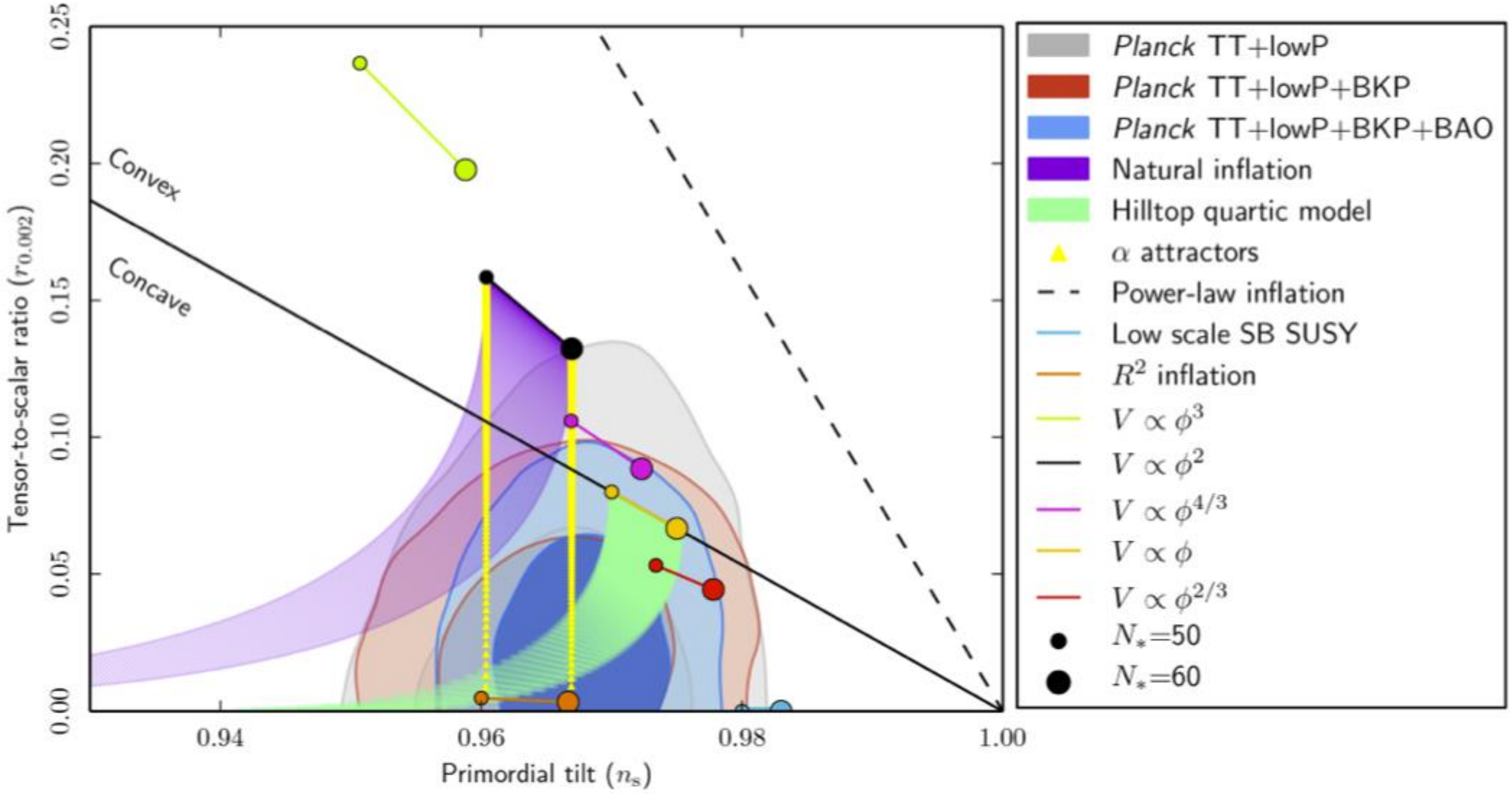
$$V(\phi) = \Lambda^4\left(1 - e^{-\sqrt{2/3}\phi/M_p}\right)^2$$



Predictions of Inflation

For Starobinsky Inflation

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12}{N^2}$$



$$0.95 < n_s < 0.97, \quad r < 0.064$$

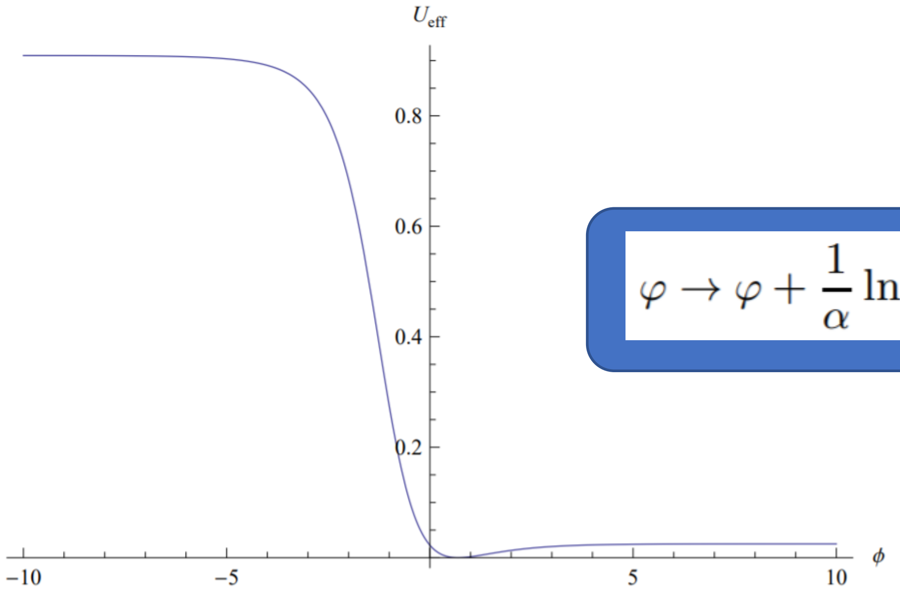
Y. Akrami et al. [Planck Collaboration], arXiv:1807.06211

Quintessential Inflation

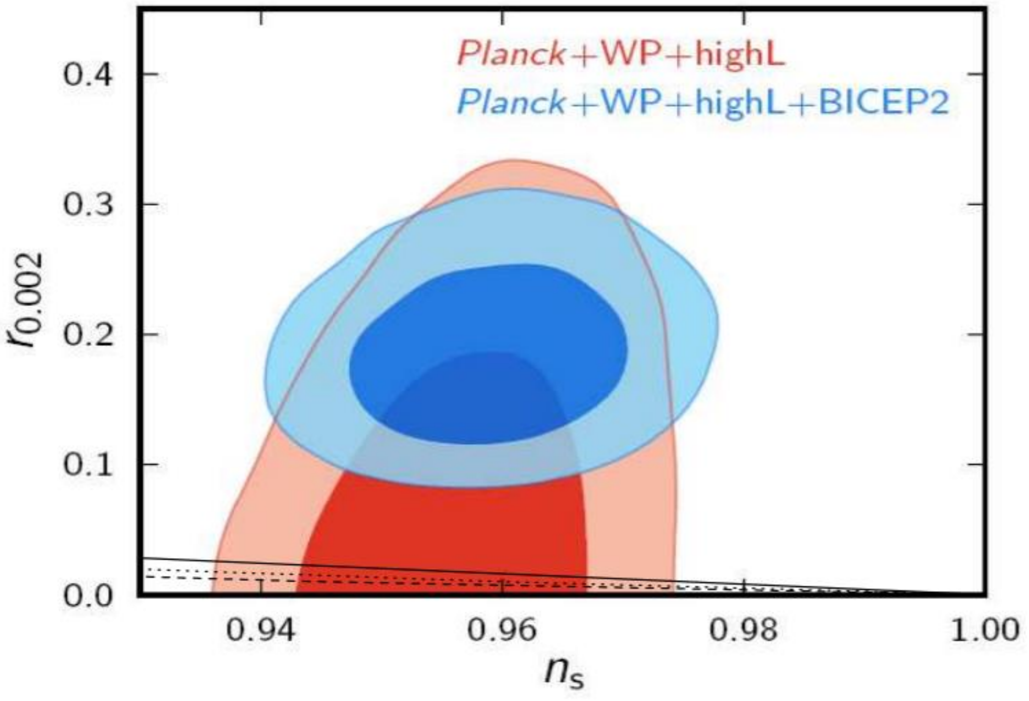
- Quintessential inflation P.J.E. Peebles, A. Vilenkin Phys.Rev. D063505 (1999)59

- Guendelman et al. 2015

$$U_{\rm eff}(\varphi) = \frac{\left(f_1 e^{-\alpha\varphi} - M_1\right)^2}{4\chi_2\left[f_2 e^{-2\alpha\varphi} + M_2 + \epsilon(f_1 e^{-\alpha\varphi} - M_1)^2\right]},$$



$$\varphi \to \varphi + \frac{1}{\alpha}\ln\lambda$$



Physics Letters B 517 (2001) 243–249

Higher order corrections to primordial spectra from cosmological inflation

Dominik J. Schwarz[a], César A. Terrero-Escalante[b], Alberto A. García[b]

Slow-roll condition

- For a general homogenous solution:

$$\epsilon_{n+1} = \left|\frac{d\epsilon_n}{dN}\right| \ll 1, \qquad \epsilon_0 = \frac{1}{H}, \qquad N = \ln a$$

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2},$$

$$\epsilon_2 \equiv \frac{\ddot{H}}{H\dot{H}} - \frac{2\dot{H}}{H^2},$$

$$\epsilon_3 \equiv \left(\ddot{H}H - 2\dot{H}^2\right)^{-1} \cdot \left[\frac{H\dot{H}\dddot{H} - \ddot{H}(\dot{H}^2 + H\ddot{H})}{H\dot{H}} - \frac{2\dot{H}}{H^2}(H\ddot{H} - 2\dot{H}^2)\right]$$

The Scale Factor Potential Approach to Inflation
D. Benisty, E. I. Guendelman, E, N. Saridakis
Eur.Phys.J. C80 (2020) no.5, 480

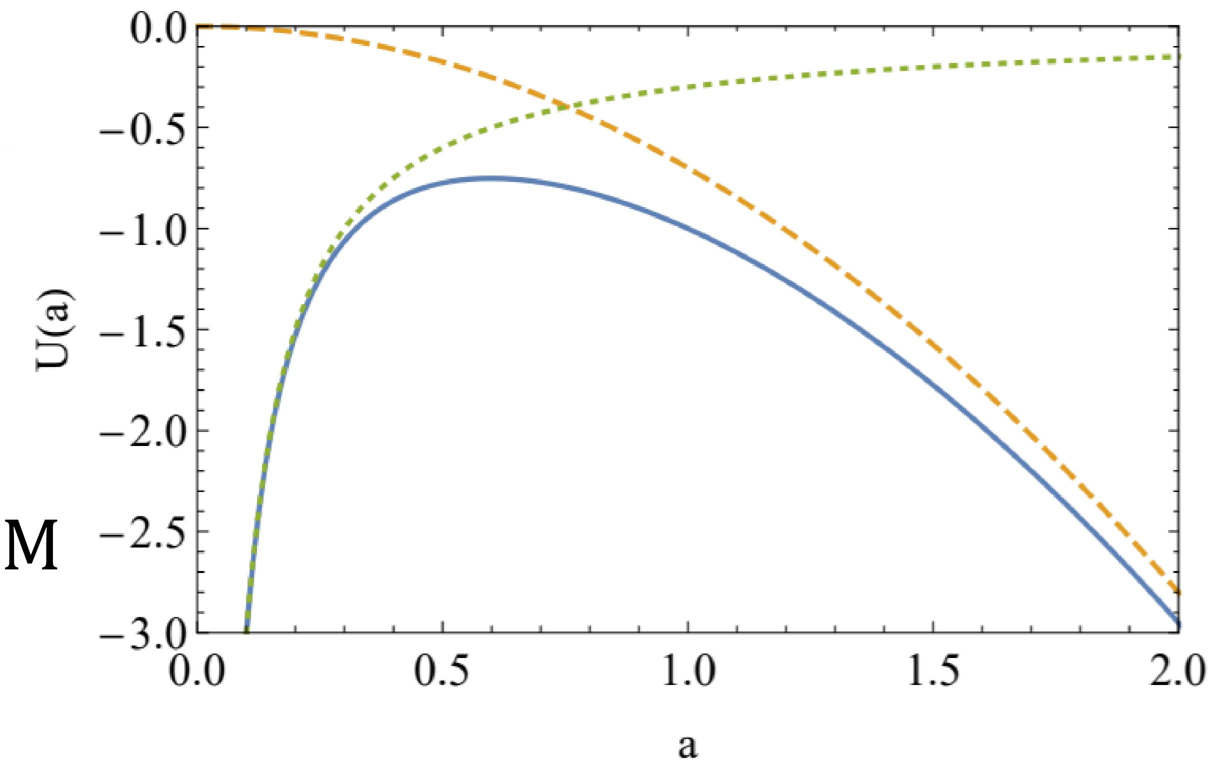
The Scale Factor Potential

- The potential reads:

$$\dot{a}^2 + U(a) = 0$$

- For ΛCDM:

$$U(a) = -a^2 H_0^2 \left[\Omega_\Lambda^{(0)} + \frac{\Omega_K^{(0)}}{a^2} + \frac{\Omega_m^{(0)}}{a^3} + \frac{\Omega_r^{(0)}}{a^4} \right]$$



Dashed yellow - Λ,

Dashed green – Cold DM, blue - ΛCDM

$V(\phi)$ is not $U(a)$

- The relations between those potentials read:

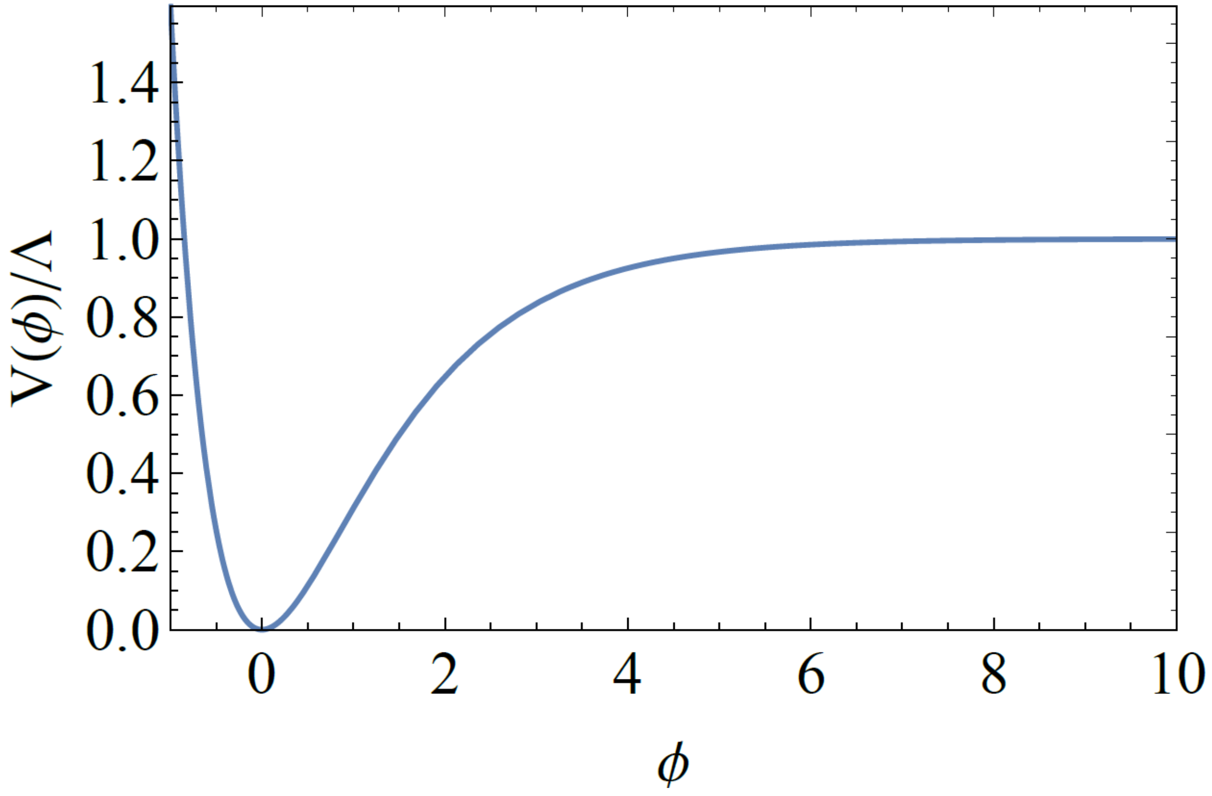
$$\phi(a) = -\int_{a_i}^{a_f} \frac{\sqrt{2U(a) - aU'(a)}}{a\sqrt{U(a)}}\, da,$$

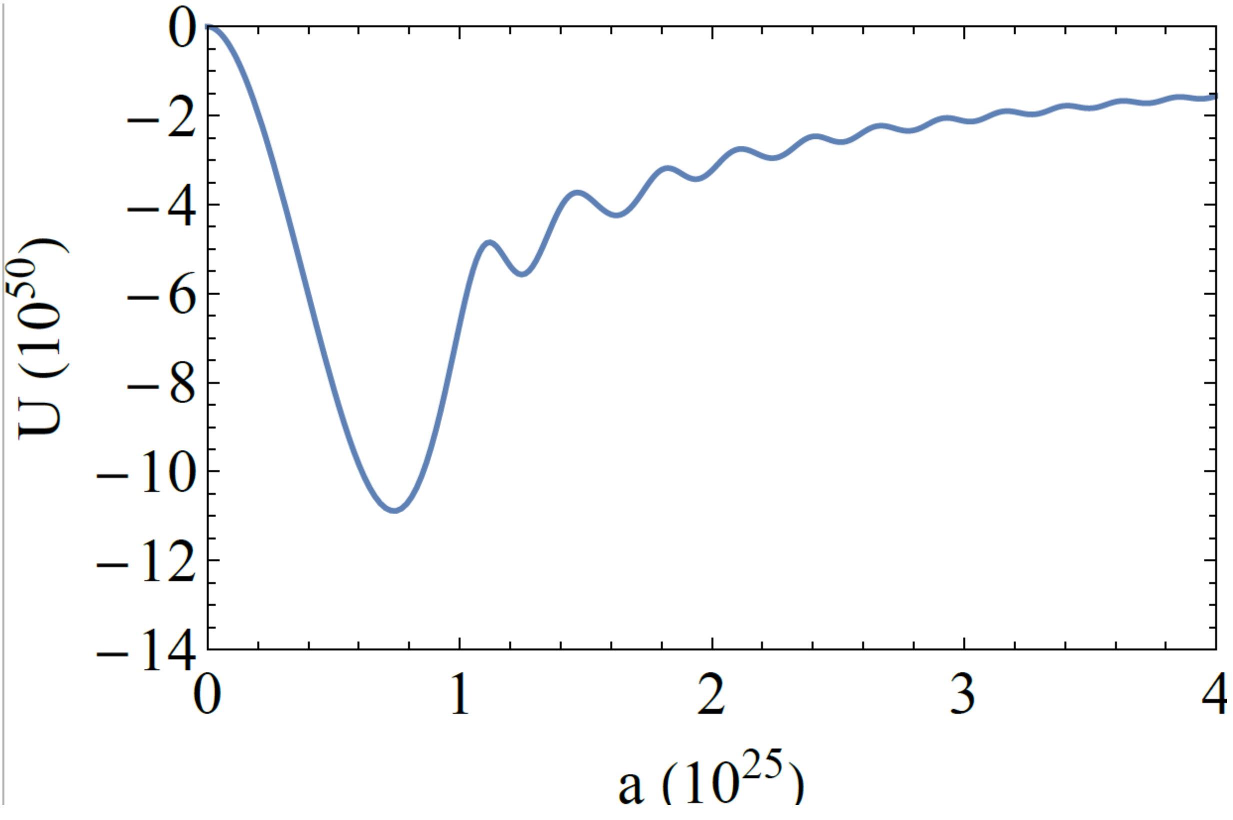
$$V(\phi(a)) = -\frac{a\,U'(a) + 4U(a)}{2a^2}.$$

- The $U(a)$ continues to the right hand side, where the $V(\phi)$ can go forward and back.

Scale factor approach to inflation

The end of inflation corresponds to a minimal point. $U'(a_f) = 0$





Lorentzian Slow Roll

10.1142/S021827182042002X **2004.00339**

David Benisty, Eduardo I. Guendelman

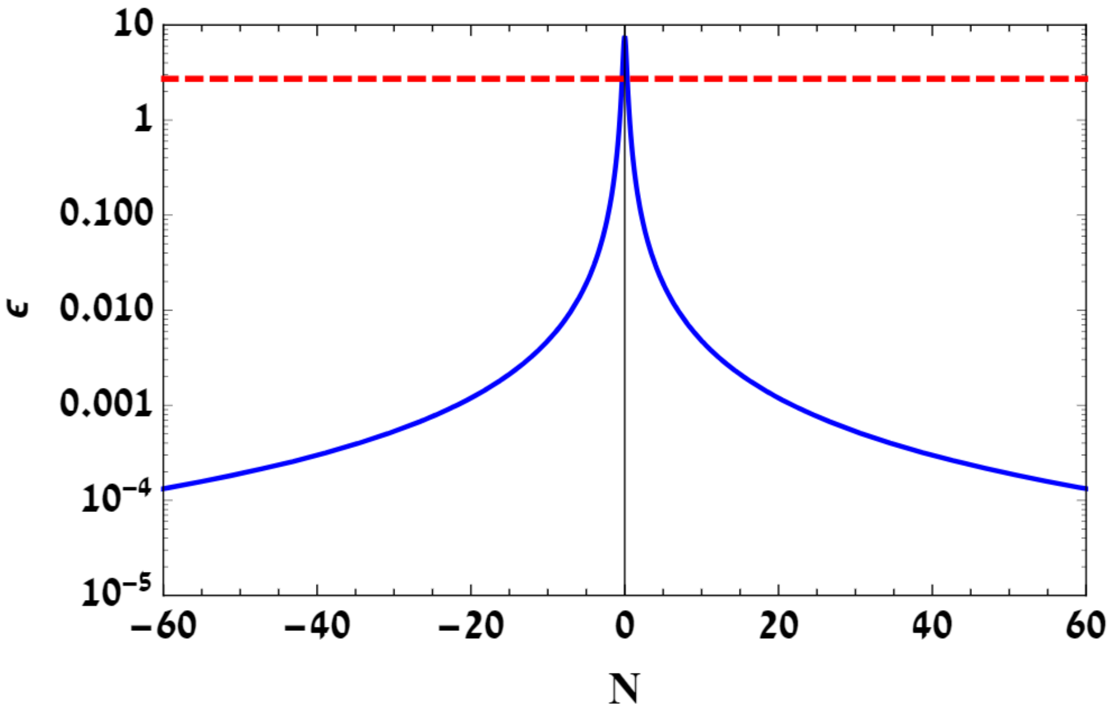
This essay awarded **second prize** in the 2020 Essay Competition of the **Gravity Research Foundation**

- We avoid the singularity with a small modification: $\epsilon(N) = \frac{2\xi}{\pi} \frac{\Gamma}{4N^2 + \Gamma^2}$.

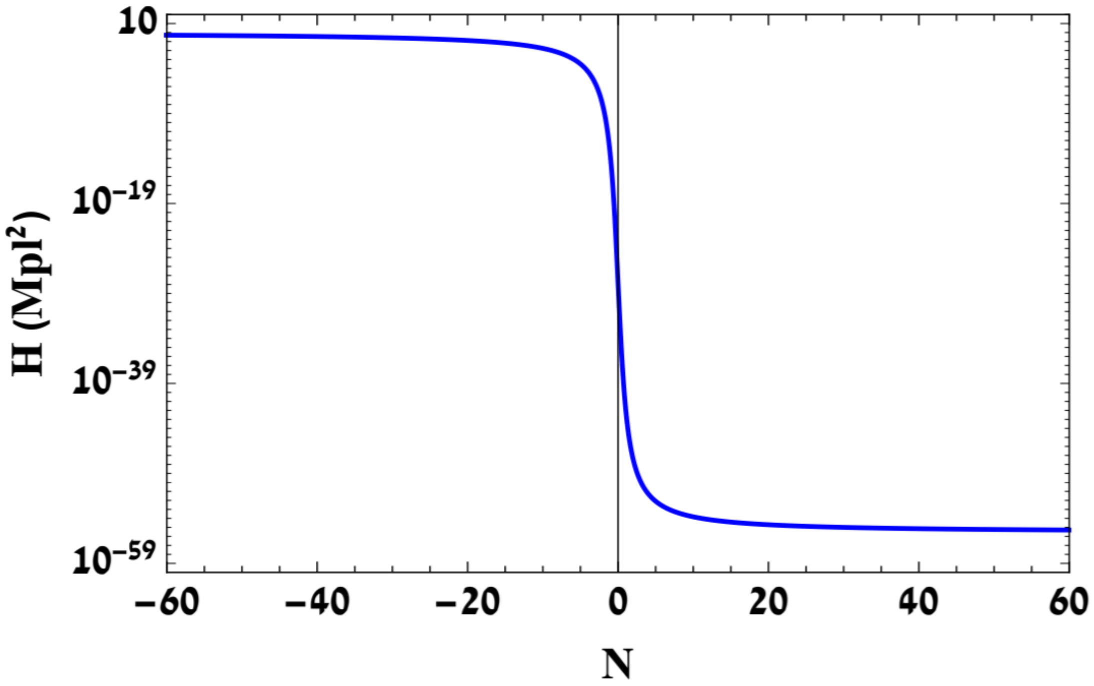
- $0 < \epsilon < 3$ yields $\Gamma < 2\xi/3\pi$

$$\epsilon_2 = -\frac{8N}{\Gamma^2 + 4N^2},$$

Etc.



The vacuum energy



- The vacuum energy term has an integration constant.

$$H = \sqrt{\frac{\Lambda_0}{3}} \exp[-\frac{\xi}{\pi} \tan^{-1}\left(\frac{2N}{\Gamma}\right)].$$

$$\xi \approx 129, \quad \Lambda_0 = 1.7 \cdot 10^{-32} Mpl^4.$$

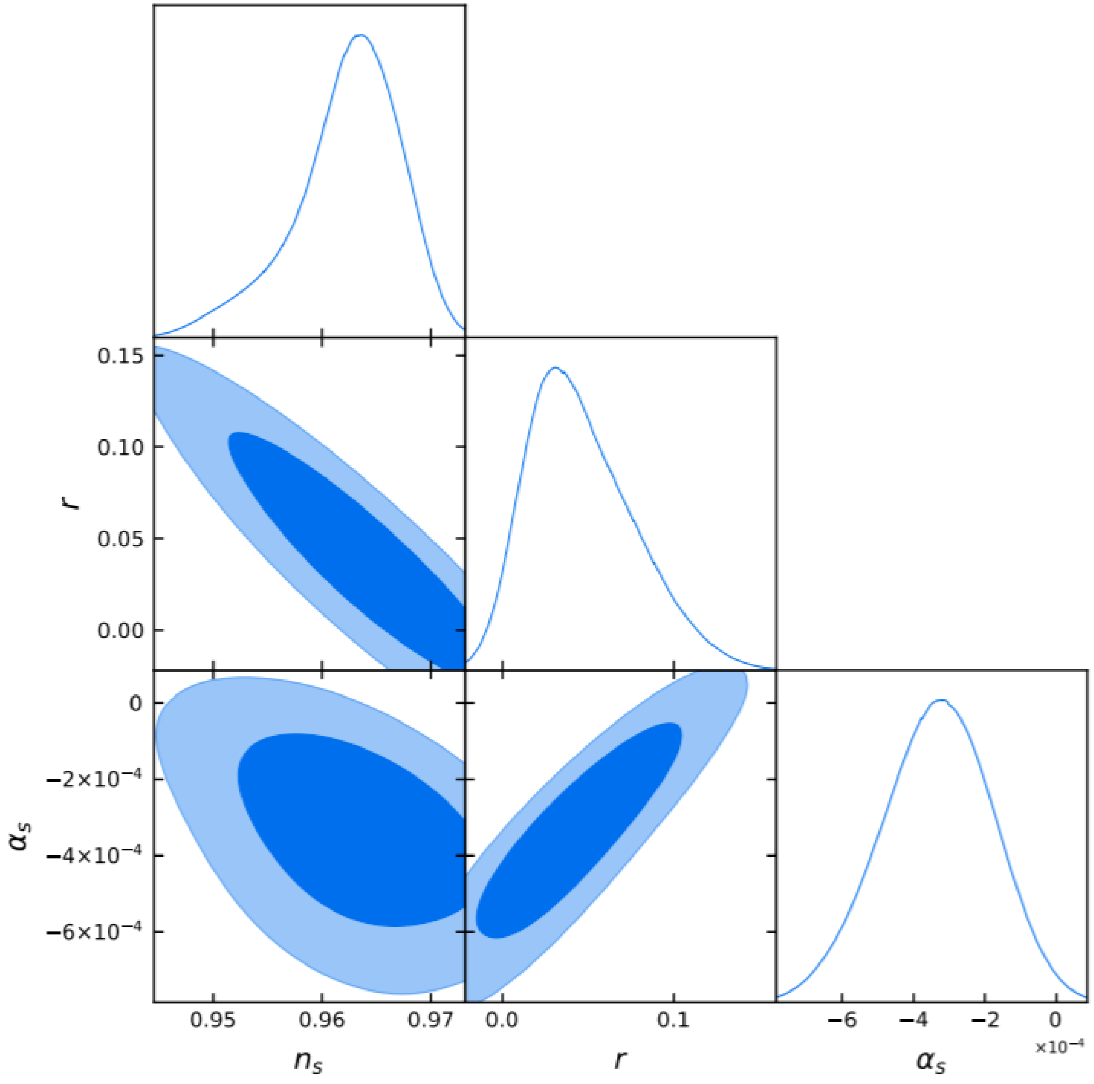
- With cosmological see-saw mechanism: $H_{\pm} = \sqrt{\frac{\Lambda_0}{3}} \exp^{\mp \xi/2}$

The observables

$$N_f = \pm\sqrt{\frac{\Gamma}{4\pi}(2\xi - \pi\Gamma)}$$

$$r = \frac{32\Gamma\xi}{\pi\Gamma^2 + 4\pi N_i^2},$$

$$n_s = \frac{\pi\left(\Gamma^2 + 4N_i(N_i+2)\right) - 4\Gamma\xi}{\pi\left(\Gamma^2 + 4N_i^2\right)}.$$



- A uniform prior

N ∈ [50; 70], ξ ∈ [100; 200], Γ ∈ [0; 1]

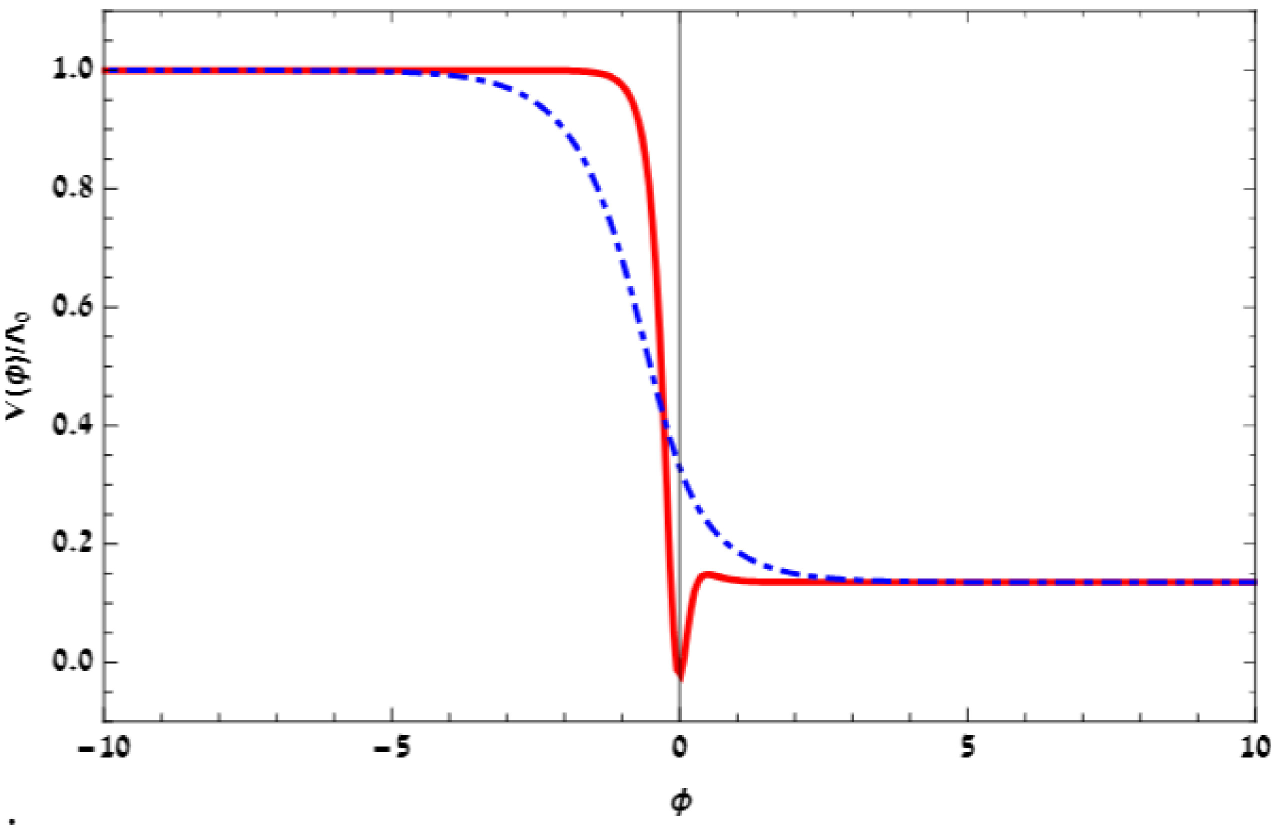
yields: $r = 0.045^{+0.065}_{-0.053}$, $n_s = 0.9624^{+0.0087}_{-0.011}$,

The potential

$$\phi = \int_0^N \sqrt{-2\frac{H'}{H}}\,dN, \quad V(\phi) = HH' + 3H^2.$$

$$N = \frac{\Gamma}{2}\sinh\left(\sqrt{\frac{\pi}{\xi\Gamma}}\phi\right),$$

$$V(N) = \Lambda_0 e^{-\frac{2\xi}{\pi}\tan^{-1}\left(\frac{2N}{\Gamma}\right)}\left(1 - \frac{2\Gamma\xi}{3\pi\Gamma^2 + 12\pi N^2}\right).$$



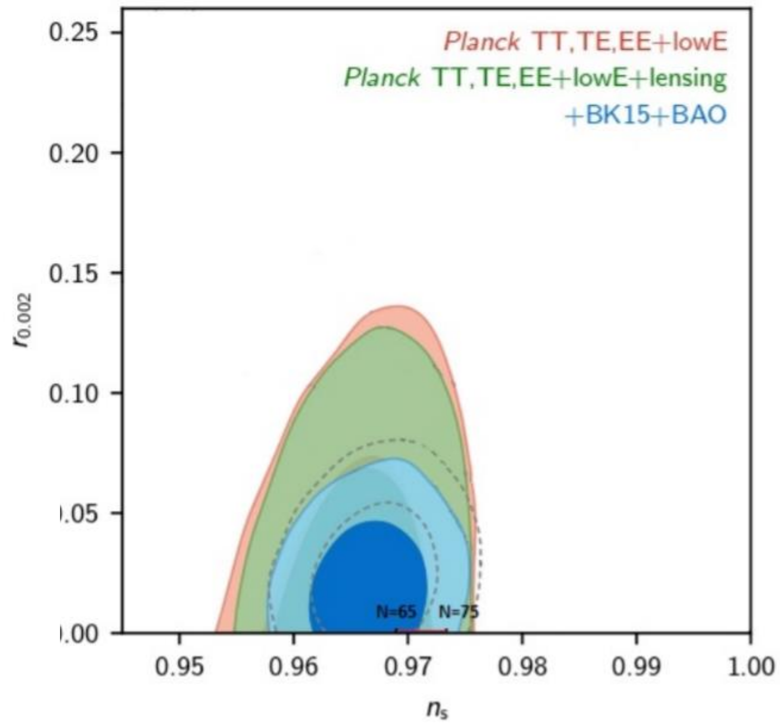
$$V(\phi) = \Lambda_0 e^{-\frac{2\xi}{\pi}\tan^{-1}(\sinh x)}\left(1 - \frac{2\xi}{3\pi\Gamma}\mathrm{sech}^2 x\right). \quad x \equiv \sqrt{\pi/\Gamma\xi}\,\phi.$$

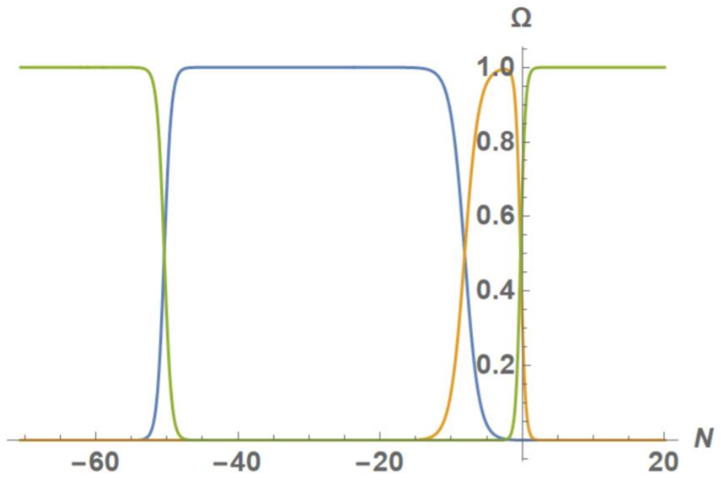
CSSM: Simpler potentials

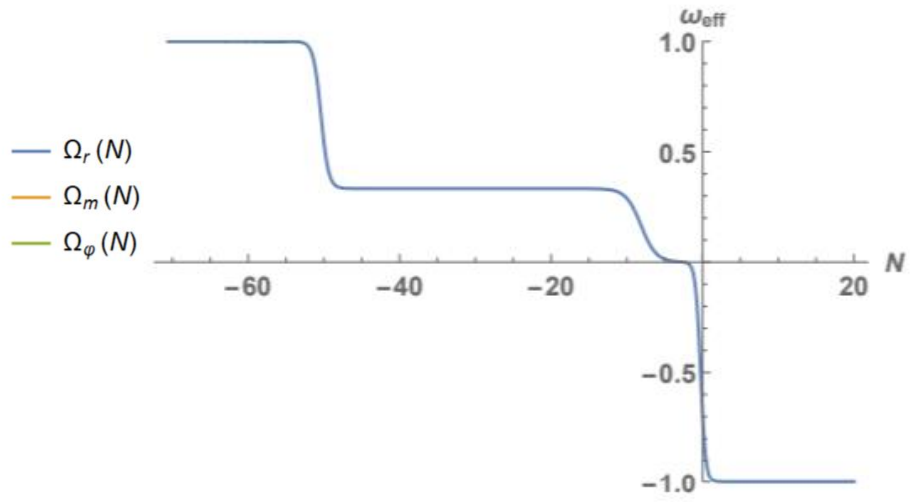
L. Aresté Saló, D. Benisty,
E. I. Guendelman, J. d. Haro
accepted in JCAP 2102.09514

- $$V(\varphi) = \lambda M_{pl}^4 \exp\left[-\frac{2\xi}{\pi}\arctan\left(\sinh\left(\gamma\varphi/M_{pl}\right)\right)\right],$$

- The See Saw Mechanism is applied.







Eur.Phys.J. C80 (2020) no.6, 577

David Benisty, Eduardo I. Guendelman

Reheating

- No oscillations in the potential.
- We introduce a new field with ϕ dependent mass:

$$m_\sigma^2 = \mu^2 \exp[-2\,\alpha\,\phi]$$

$\dot{\phi}_0$ is the velocity of the field

Where $\dot{m}_\sigma \sim m_\sigma^2$

- After the analysis, the total density reads:

$$\rho_\sigma = \left[\frac{\alpha\dot{\phi}_0}{\sqrt{2\pi\log\left[\alpha\dot{\phi}_0/\mu\right]}}\right]^4 \frac{1}{a^4}$$

- The particle creation does not produce exactly a thermal spectrum yet.

Data Fit

- CMB distant priors (chen 2019)
- Hubble Diagram of: Type IA supernova, Quasars, Gamma ray Bursts
- Cosmic Chronometers
- Baryon Acoustic Oscillations collation (Benisty & Staicova A&A 647, A38 (2021))

z	Parameter	Value	Error	Year	Survey	Ref.
0.106	r_d/D_V	0.336	0.015	2011	6dFGS	Beutler et al. (2011)
0.15	$D_V(r_{d,fidd}/r_d)$	664	25.0	2014	SDSS DR7	Ross et al. (2015)
0.275	r_d/D_V	0.1390	0.0037	2009	SDSS-DR7+2dFGRS	Percival et al. (2010)
0.32	$D_V(r_{d,fidd}/r_d)$	1264	25	2016	SDSS-DR11 LOWZ	Tojeiro et al. (2014)
0.44	r_d/D_V	0.0870	0.0042	2012	WiggleZ	Blake et al. (2012)
0.54	D_A/r_d	9.212	0.41	2012	SDSS-III DR8	Seo et al. (2012)
0.57	D_V/r_d	13.67	0.22	2012	SDSSIII/DR9	Anderson et al. (2013)
0.6	r_d/D_V	0.0672	0.0031	2012	WiggleZ	Blake et al. (2012)
0.697	$D_A(r_{d,fidd}/r_d)$	1499	77	2020	DECals DR8	Sridhar et al. (2020)
0.72	$D_V(r_{d,fidd}/r_d)$	2353	63	2017	SDSS-IV DR14	Bautista et al. (2018)
0.73	r_d/D_V	0.0593	0.0020	2012	WiggleZ	Blake et al. (2012)
0.81	D_A/r_d	10.75	0.43	2017	DES Year1	Abbott et al. (2019)
0.874	$D_A(r_{d,fidd}/r_d)$	1680	109	2020	DECals DR8	Sridhar et al. (2020)
1.48	$D_H \cdot r_d$	13.23	0.47	2020	eBoss DR16 BAO+RSD	Hou et al. (2021)
1.52	$D_V(r_{d,fidd}/r_d)$	3843	147.0	2017	SDSS-IV/DR14	Ata et al. (2018)
2.3	$H \cdot r_d$	34188	1188	2012	Boss Lya quasars DR9	Busca et al. (2013)
2.34	$D_H \cdot r_d$	8.86	0.29	2019	BOSS DR14 Lya in LyBeta	de Sainte Agathe et al. (2019)

L. Aresté Saló, D. Benisty,
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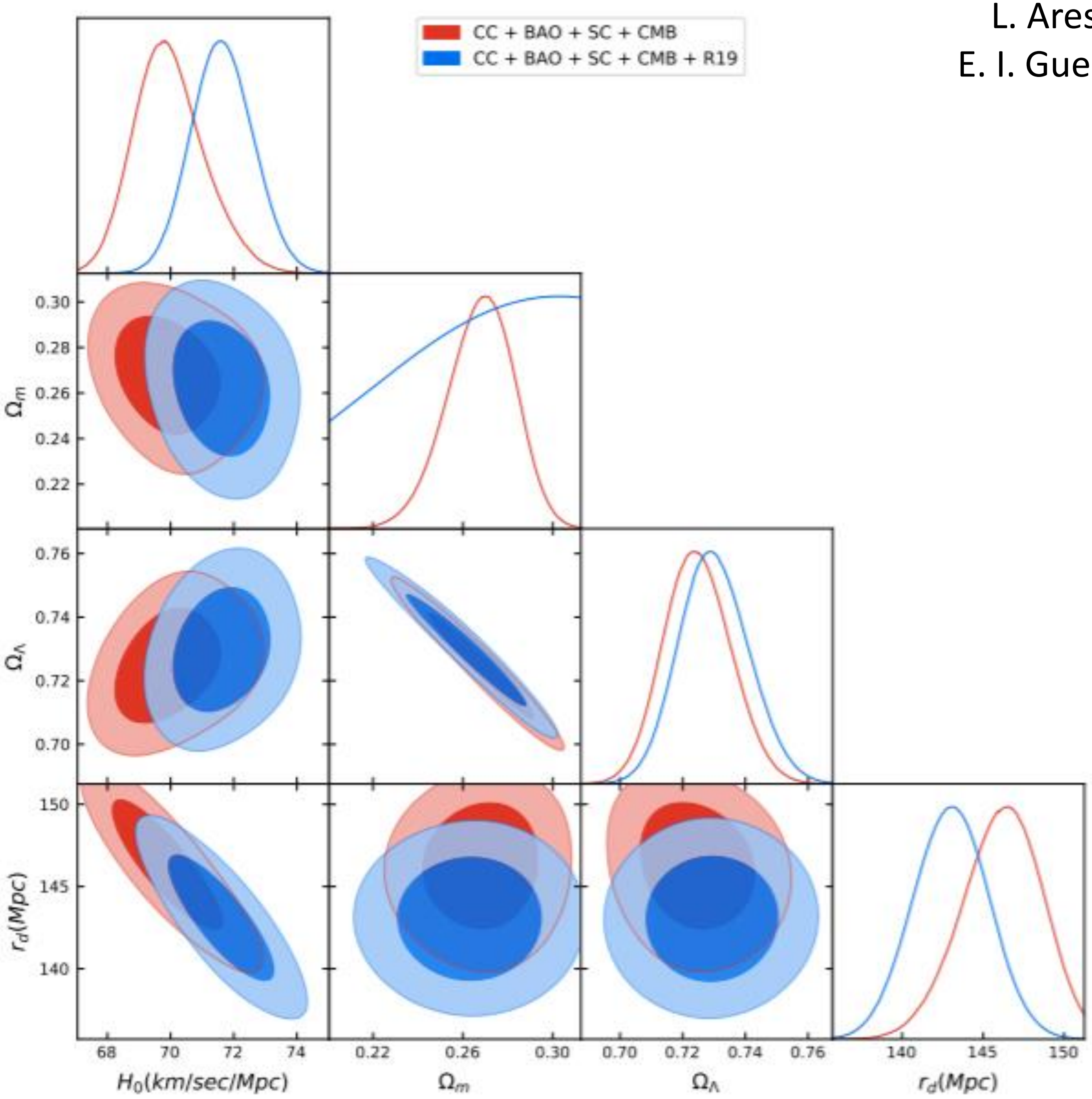
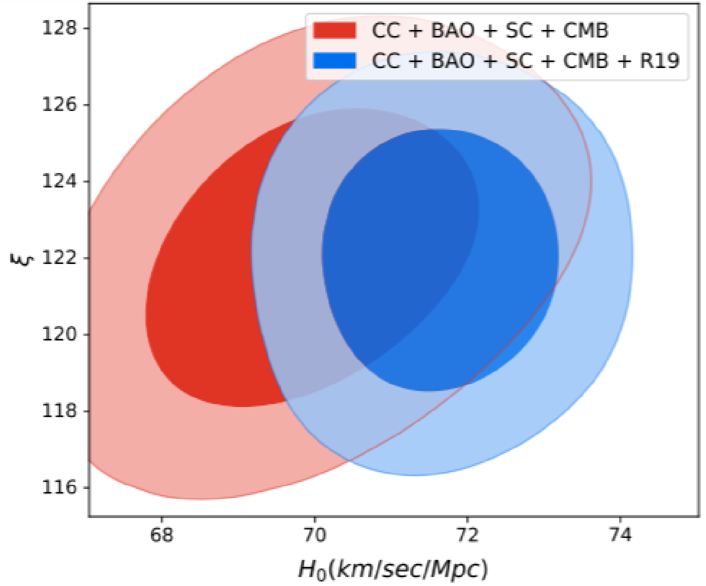
2102.09514

Data Fit

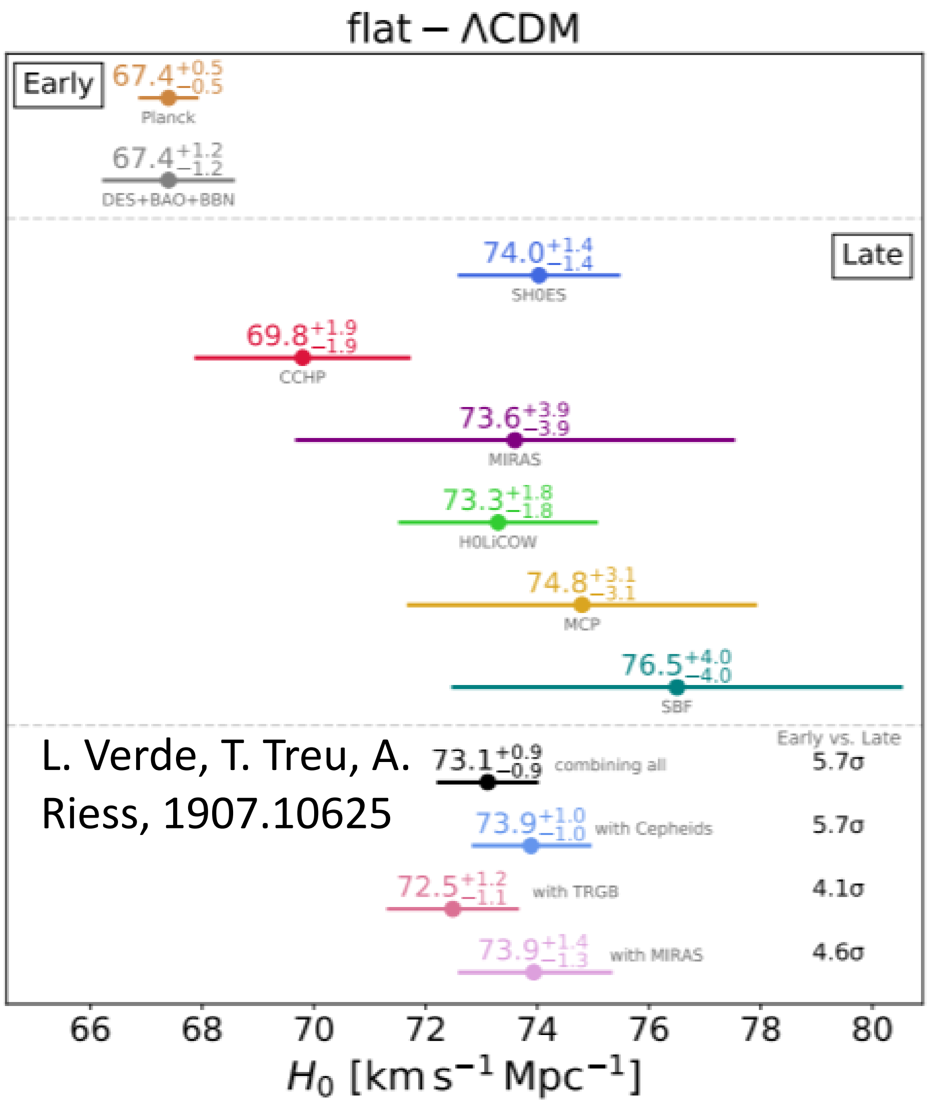




Parameter	LQI	LQI + SH0ES
$H_0(km/sec/Mpc)$	70.06 ± 1.123	71.75 ± 0.8885
φ_0/M_{pl}	22.72 ± 1.541	22.38 ± 1.395
$\dot{\varphi}_0/(H_0 M_{pl})\, 10^{-71}$	4.113 ± 2.635	5.279 ± 2.675
Ω_m	0.2679 ± 0.01286	0.2610 ± 0.01647
Ω_Λ	0.7250 ± 0.09131	0.7304 ± 0.01107
ξ	121.9 ± 1.865	122.0 ± 1.94
$r_d(Mpc)$	145.8 ± 2.363	143.0 ± 1.957

Standard problems in Cosmology

- The cosmological constant problem
- The origin of inflation
- Hubble tension (?)
- Do Inflation and Dark Energy connect?



Summary

- **An approximate Resonance function** for the Slow roll parameter predicts a good quintessential inflationary potential: especially for a Lorentzian.

- The slow roll parameters small and fit with the Planck 2018 data.

- Cosmological See-Saw Mechanism.

- Reheating mechanism comes from additional scalar field.