Lorentzian Quintessential Inflation

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SIXTEENTH MARCEL GROSSMANN MEETING
ON RECENT DEVELOPMENTS IN THEORETICAL AND EXPERIMENTAL GENERAL RELATIVITY, ASTROPHYSICS AND RELATIVISTIC FIELD THEORIES

2004.00339 awarded 2nd prize in Gravity Research Foundations
International Journal of Modern Physics D 2042002
Problems with the early universe theory

• The Flatness Problem: $k = 0$.

• The Horizon Problem
Signs for Dark Energy

• “Pantheon” - Type Ia Super Nova
  Riess 2018,2019

• Quasars

• Cosmic Chronometers

• Baryon Acoustic Oscillations

• Perturbations (CMB, DES, DESI, LSST)

G. Risaliti and E. Lusso Nature Ast. 2019
\[
\mathcal{L} = \frac{1}{2} R - \frac{1}{2} g^{\mu \nu} \phi_{,\mu} \phi_{,\nu} - V(\phi)
\]

\[
3H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi)
\]

\[
-3H^2 - 2\dot{H} = \frac{1}{2} \dot{\phi}^2 - V(\phi)
\]

- We can address the “dark energy” equation of state, by the slow roll approximation:
  \[
  \frac{1}{2} \dot{\phi}^2 \ll V(\phi)
  \]

- The Slow Roll parameters and the number of e-folds read:
  \[
  \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = \frac{V''}{V}, \quad N = \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi
  \]

The end of inflation \( \epsilon = 1, \ N = 60 \)
Quantum Fluctuations

- Perturbed inflaton field: $\phi(\eta, x) = \phi(\eta) + \frac{f(\eta, x)}{a(\eta)}$

- Mukhanov-Sasaki equation for the FT of the modes at early times:
  $$\ddot{f}_k + k^2 f_k = 0, \quad k \eta \gg 1$$

- The Power Spectrum: $\langle |f|^2 \rangle = \Delta_f^2(k, \eta) = \frac{k^3}{2\pi^2} |f_k|^2$

- The curvature power spectrum: $\Delta_R^2 = \frac{H^2}{8\pi\epsilon} \sim A_s k^{n_s-1}$, the spectral index reads: $n_s = 1 - 6\epsilon + 2\eta$

- Tensor perturbations read: $\Delta_h^2 = \frac{2H^2}{\pi^2} \sim A_t k^{n_t-1}$ with $n_t \approx -2\epsilon$. The scalar to tensor ratio: $r = \frac{A_t}{A_s} = 16\epsilon$
Starobinsky inflation

• Quantum corrections to GR are important for the early universe.

\[ \mathcal{L} = \frac{1}{2} R + \frac{R^2}{12M^2} \]

In the Einstein Frame:

\[ V(\phi) = \Lambda^4 \left(1 - e^{-\sqrt{2/3} \phi/M_p}\right)^2 \]
Predictions of Inflation

\begin{align*}
    n_s &= 1 - \frac{2}{N}, \\
    r &= \frac{12}{N^2}
\end{align*}

For Starobinsky Inflation

\[ 0.95 < n_s < 0.97, \quad r < 0.064 \]

Y. Akrami et al. [Planck Collaboration], arXiv:1807.06211
Quintessential Inflation


- Guendelman et al. 2015

\[ U_{\text{eff}}(\varphi) = \frac{(f_1 e^{-\alpha \varphi} - M_1)^2}{4\chi_2 \left[ f_2 e^{-2\alpha \varphi} + M_2 + \epsilon (f_1 e^{-\alpha \varphi} - M_1)^2 \right]} \]

\[ \varphi \rightarrow \varphi + \frac{1}{\alpha} \ln \lambda \]
Slow-roll condition

• For a general homogenous solution:

\[ \epsilon_{n+1} = \left| \frac{d\epsilon_n}{dN} \right| \ll 1, \quad \epsilon_0 = \frac{1}{H}, \quad N = \ln a \]

\[ \epsilon_1 \equiv -\frac{\ddot{H}}{H^2}, \quad \epsilon_2 \equiv \frac{\dddot{H}}{H\dot{H}} - \frac{2\dot{H}}{H^2}, \quad \epsilon_3 \equiv \left( \frac{\dddot{H} + \ddot{H}}{H\dot{H}} - \frac{2\dot{H}}{H^2} \right) \]
The Scale Factor Potential

• The potential reads:

\[ \dot{a}^2 + U(a) = 0 \]

• For \( \Lambda \)CDM:

\[
U(a) = -a^2 H_0^2 \left[ \Omega_{\Lambda}^{(0)} + \frac{\Omega_K^{(0)}}{a^2} + \frac{\Omega_m^{(0)}}{a^3} + \frac{\Omega_r^{(0)}}{a^4} \right]
\]

Dashed yellow - \( \Lambda \),
Dashed green – Cold DM, blue - \( \Lambda \)CDM
$V(\phi)$ is not $U(a)$

• The relations between those potentials read:

$$\phi(a) = - \int_{a_i}^{a_f} \frac{\sqrt{2U(a) - aU'(a)}}{a\sqrt{U(a)}} \, da,$$

$$V(\phi(a)) = - \frac{a U'(a) + 4U(a)}{2a^2}.$$

• The $U(a)$ continues to the right hand side, where the $V(\phi)$ can go forward and back.
Scale factor approach to inflation

The end of inflation corresponds to a minimal point. $U'(a_f) = 0$
Lorentzian Slow Roll

• We avoid the singularity with a small modification: \( \epsilon(N) = \frac{2\xi}{\pi} \frac{\Gamma}{4N^2 + \Gamma^2} \).

• \( 0 < \epsilon < 3 \) yields \( \Gamma < \frac{2\xi}{3\pi} \).

\[ \epsilon_2 = -\frac{8N}{\Gamma^2 + 4N^2}, \quad \text{Etc.} \]
The vacuum energy

• The vacuum energy term has an integration constant.

\[ H = \sqrt{\frac{\Lambda_0}{3}} \exp\left[-\frac{\xi}{\pi} \tan^{-1}\left(\frac{2N}{\Gamma}\right)\right]. \]

\[ \xi \approx 129, \quad \Lambda_0 = 1.7 \cdot 10^{-32} M_{pl}^4. \]

• With cosmological see-saw mechanism:

\[ H_\pm = \sqrt{\frac{\Lambda_0}{3}} \exp^{\mp \xi/2} \]
The observables

\[ N_f = \pm \sqrt{\frac{\Gamma}{4\pi}} (2\xi - \pi \Gamma), \]
\[ r = \frac{32\Gamma \xi}{\pi \Gamma^2 + 4\pi N_i^2}, \]
\[ n_s = \frac{\pi (\Gamma^2 + 4N_i(N_i + 2)) - 4\Gamma \xi}{\pi (\Gamma^2 + 4N_i^2)}. \]

- A uniform prior
  
  \( N \in [50; 70], \xi \in [100; 200], \Gamma \in [0; 1] \)

  yields:

  \[ r = 0.045^{+0.065}_{-0.053}, \quad n_s = 0.9624^{+0.0087}_{-0.011}, \]
The potential

\[ \phi = \int_0^N \sqrt{-2 \frac{H'}{H}} dN, \quad V(\phi) = HH' + 3H^2. \]

\[ N = \frac{\Gamma}{2} \sinh \left( \sqrt{\frac{\pi}{\xi \Gamma}} \phi \right), \]

\[ V(N) = \Lambda_0 e^{-\frac{2\xi}{\pi} \tan^{-1} \left( \frac{2N}{\Gamma} \right)} \left( 1 - \frac{2\Gamma \xi}{3\pi \Gamma^2 + 12\pi N^2} \right). \]

\[ V(\phi) = \Lambda_0 e^{-\frac{2\xi}{\pi} \tan^{-1} (\sinh x)} \left( 1 - \frac{2\xi}{3\pi \Gamma \sech^2 x} \right), \quad x \equiv \sqrt{\pi / \Gamma \xi \phi}. \]
CSSM: Simpler potentials

- \( V(\varphi) = \lambda M_{pl}^4 \exp \left[ -\frac{2\zeta}{\pi} \arctan \left( \sinh \left( \gamma \varphi / M_{pl} \right) \right) \right] \),

- The See Saw Mechanism is applied.
Reheating

• No oscillations in the potential.
• We introduce a new field with $\phi$ dependent mass:
  $$m_\sigma^2 = \mu^2 \exp[-2\alpha \phi]$$

$\phi_0$ is the velocity of the field
Where $m_\sigma \sim m_\sigma$

• After the analysis, the total density reads:
  $$\rho_\sigma = \left( \frac{\alpha \dot{\phi}_0}{\sqrt{2\pi \log[\alpha \phi_0/\mu]}} \right)^4 \frac{1}{a^4}$$

• The particle creation does not produce exactly a thermal spectrum yet.
Data Fit

• CMB distant priors (chen 2019)

• Hubble Diagram of: Type Ia supernova, Quasars, Gamma ray Bursts

• Cosmic Chronometers

• Baryon Acoustic Oscillations collation (Benisty & Staicova A&A 647, A38 (2021))
Standard problems in Cosmology

• The cosmological constant problem

• The origin of inflation

• Hubble tension (?)

• Do Inflation and Dark Energy connect?

L. Verde, T. Treu, A. Riess, 1907.10625
Summary

- An approximate Resonance function for the Slow roll parameter predicts a good quintessential inflationary potential: especially for a Lorentzian.

- The slow roll parameters small and fit with the Planck 2018 data.

- Cosmological See-Saw Mechanism.

- Reheating mechanism comes from additional scalar field.