GRAVITATIONAL LENSING BY WORMHOLES IN BINARY SYSTEMS

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Lensing by Wormholes

- The Ellis wormhole has zero mass at the spatial infinity, but it causes light deflection;
- Lensing by wormholes has been explored by several authors during the years;
- It was recently investigated as an observational probe of an exotic spacetime;
- The demagnification effect has been found as a distinctive signature which is not present in lensing from ordinary matter.

Demagnification Effect

Metrics falling as $1/r^n$ were investigated by Kitamura et al. (2013):

- the deflection angle falls down with the same exponent as the metric: $\alpha \sim 1/u^n$
  ($n > 1$ and u impact parameter);
- demagnification of the total lensed images could appear: $\beta > \frac{2}{n+1}$ (in units of θ_E and under a large-n approximation, β is the source position).

This demagnification effect may be evidence of an Ellis wormhole and it might be used for hunting the search for exotic matter for $n > 1$.

Violation of the weak energy condition

The implications for the energy-momentum tensor supporting this kind of metrics falling as $1/r^n$ were investigated by Bozza and Postiglione (2015).

They prove that:

- $0 < n < 1$ may describe galactic halos;
- $n > 1$ would be the signature of a violation of the weak energy condition $\rightarrow$existence of exotic matter;
- $n = 2$ correspond to the Ellis wormhole.

n is the ratio between tangential and radial pressure, $n = -2p_t/p_r$.

The Lens Equation

In a binary system composed by two lenses (binary lenses, A and B), the lens equation can be written as follows

$$\vec{\beta} = \vec{\theta} - \frac{\vec{\theta} - \vec{\theta_A}}{|\vec{\theta} - \vec{\theta_A}|^{n+1}} - \gamma^{m+1} \frac{\vec{\theta} - \vec{\theta_B}}{|\vec{\theta} - \vec{\theta_B}|^{m+1}}. \tag{1}$$

- θ is the angular position at which the image is observed;
- θ_E is the Einstein radius of the lens;
- $\gamma = \theta_{E,B}/\theta_{E,A}$ is the "strength ratio".

The lens equation allows us to find images, given the source position and a lens configuration.

Critical Curves and Caustics

- The number of images formed at a given source β depends on the source position;
- These regions with a different number of images are delimited by caustics;
- The condition $J(z) = 0$ (the Jacobian determinant of the lens map) defines the critical curves on the lens plane;
- By applying the lens map on critical points we find the corresponding points on the source plane, which form the caustics.

When a source crosses a caustic, a new pair of images is created on the corresponding point in the critical curve.

Cases and Topology Regimes I

We investigated 3 cases:

- equal-strength binary with $\gamma = 1$;
- unequal-strength binary with $\gamma = \sqrt{0.1}$
  (the bigger lens is the standard one with fixed $n = 1$);
- reversed unequal-strength binary with $\gamma = \sqrt{0.1}$
  (the standard lens is the smaller one).

In this presentation we show the equal-strength case with $\gamma = 1$, the full research can be found in:
Bozza V., Pietroni S., Melchiorre C., Universe 2020, 6(8), 106.

Cases and Topology Regimes II

For the standard binary Schwarzschild lens in the equal-strength case, we know that three topologies exist:

- close separation, for $s < s_{CI}$;
- intermediate separation, for $s_{CI} < s < s_{IW}$;
- wide separation, for $s > s_{IW}$;

and the two transitions are $s_{CI} = 1$ and $s_{IW} = 2\sqrt{2}$ in our units.

The three topologies exist for any value of m and n.

Our model contains 4 parameters: the indexes of the two potentials n, m, the separation between the two lenses s, and the ratio of the two Einstein radii γ.

Standard Binary Lenses

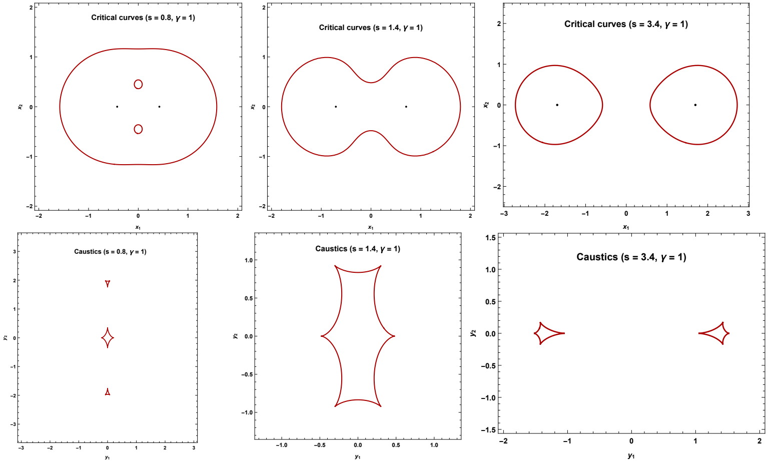

Figure: Critical curves and caustics in the equal-strength binary, in the standard binary lenses $m = n = 1$ for the three topologies.

Symmetric Binary Lenses With $1/r^n$ Potentials

Bozza & Melchiorre (2016) investigated the caustic topologies for binary lenses in the symmetric case with equal n for $0 \leq n \leq 3$, they find out:

- the existence of giant caustics for exotic matter ($n > 2$).

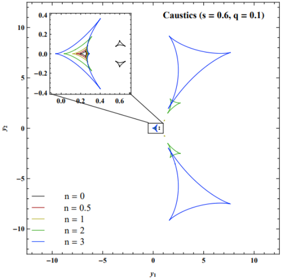

Figure: Giant caustics in the unequal-strength binary. Close separation.

Critical Curves and Caustics: Wide Separation

The red curve is for the standard Schwarzschild case $n = m = 1$, we keep $n = 1$ for the first lens and we see what happens when m varies in the second lens

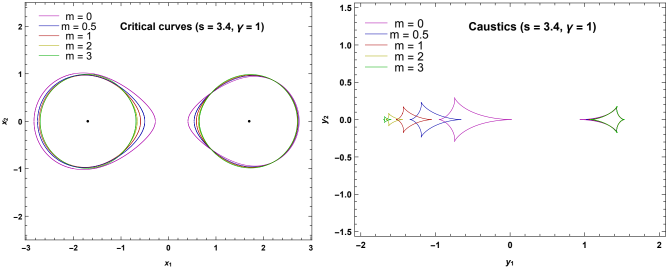

Figure: Critical curves and caustics in the equal-strength binary, wide separation with fixed $n = 1$ and variable m: $m = 0$ is the singular isothermal sphere already investigated by Shin and Evans, $m = 0.5$ is the galactic halo, $m = 2$ is the Ellis wormhole and $m = 3$ is for exotic matter.

Critical Curves and Caustics: Intermediate Separation

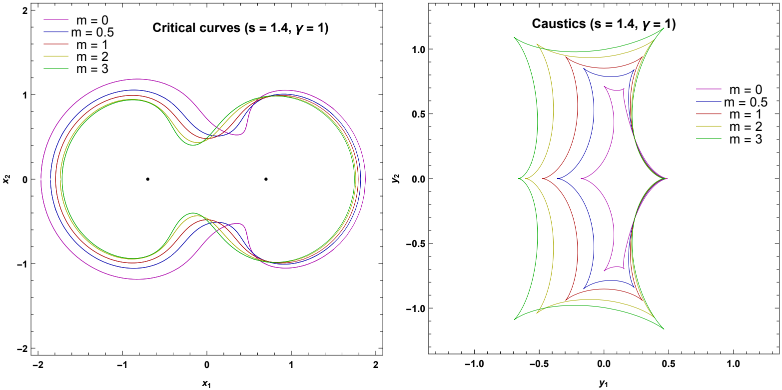

Figure: Critical curves and caustics in the equal-strength binary, intermediate separation with fixed $n = 1$ and variable m. The red curve is for the standard Schwarzschild case $n = m = 1$.

Critical Curves and Caustics: Close Separation

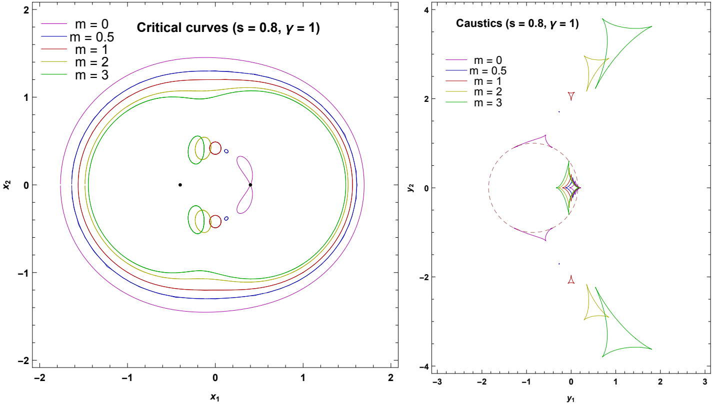

Figure: Critical curves and caustics in the equal-strength binary, close separation with fixed $n = 1$ and variable m. The red curve is for the standard Schwarzschild case $n = m = 1$. Dashed magenta circle indicates the pseudocaustic for $m = 0$.

The Elliptic Umbilic

In the range $0 \leq m < 1$ an *elliptic umbilic catastrophe* exists in the close separation. The value of s at which the catastrophe happens is

$$s_{euc} = \left(\frac{1-mn}{m+1}\right)^{\frac{1}{n+1}} \sqrt{1 + \frac{\gamma^2 (m+1)^{\frac{2}{n+1}}}{(n+1)^{\frac{2}{m+1}} (1-mn)^{\frac{2(m-n)}{(m+1)(n+1)}}}} \qquad (2)$$

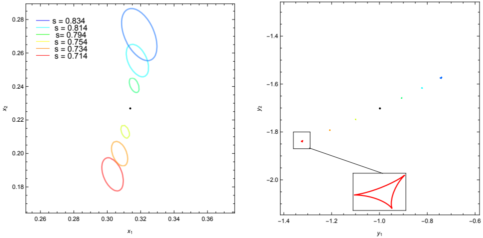

Figure: The elliptic umbilic catastrophe for $n = 2$, $m = 0.25$ The separation at which the catastrophe occurs is $s_{euc} = 0.774$.

Transitions Between Topologies

For any n, m, γ we found the boundaries for s_{CI}, given only numerically, and the analytical expression for s_{IW}.

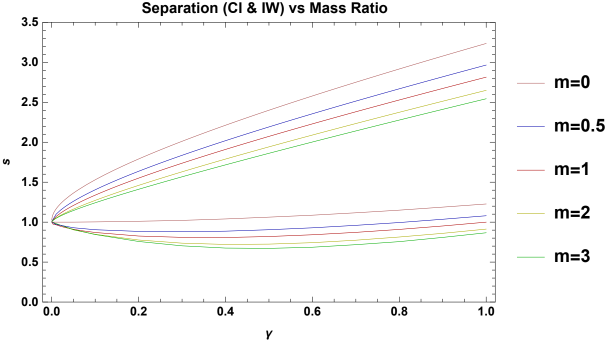

Figure: Critical values of the separation for s_{IW} transition (upper curves); numerical critical values of the separation for s_{CI} transition as a function of γ (lower curves) for $n = 1$ and variable m.

Extremely unequal-strength ratio limit I I

The caustic evolution in the extreme limit $\theta_{E,B} << \theta_{E,A}$, in the case of two Schwarzschild objects ($n = m = 1$), is the so-called "planetary" limit.

For the caustics of the perturbing object in the wide case we have an extension of the caustics in the parallel and in the vertical direction:

$$\Delta\zeta_{||,wide} = 2(n+1)\frac{\gamma}{s^{\frac{m(n+1)}{m+1}}\left(s^{n+1}-1\right)^{\frac{1}{m+1}}} \quad (3)$$

$$\Delta\zeta_{\perp,wide} = 2(n+1)\frac{\gamma}{s^{\frac{m(n+1)}{m+1}}\left(s^{n+1}+n\right)^{\frac{1}{m+1}}} \quad (4)$$

Extremely unequal-strength ratio limit II

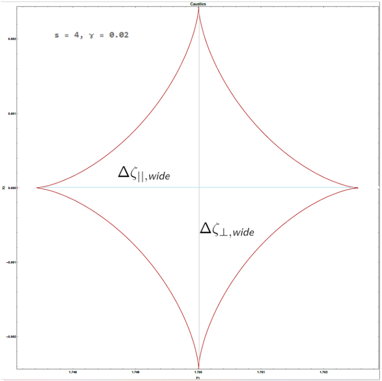

Figure: Caustic of the perturbing object in the wide separation for $s = 4$, $n = m = 1$, $\gamma = 0.02$: extension in the parallel and vertical direction.

Conclusions

These mixed binary lenses are important from the astrophysical point of view:

- in the investigation of pairs of galaxies with different halos $\mathbf{n,m<1}$;
- in the opening of a new channel in the search for wormholes when they appear in a non-isolated environment $\mathbf{n=1}$,$\mathbf{m=2}$;
- in the case in which one object is made up of exotic matter and the other one is a normal star $\mathbf{n>1}$,$\mathbf{m=1}$.