FIAS Frankfurt Institute for Advanced Studies

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Quantifying the S_8 tension: BA & ML

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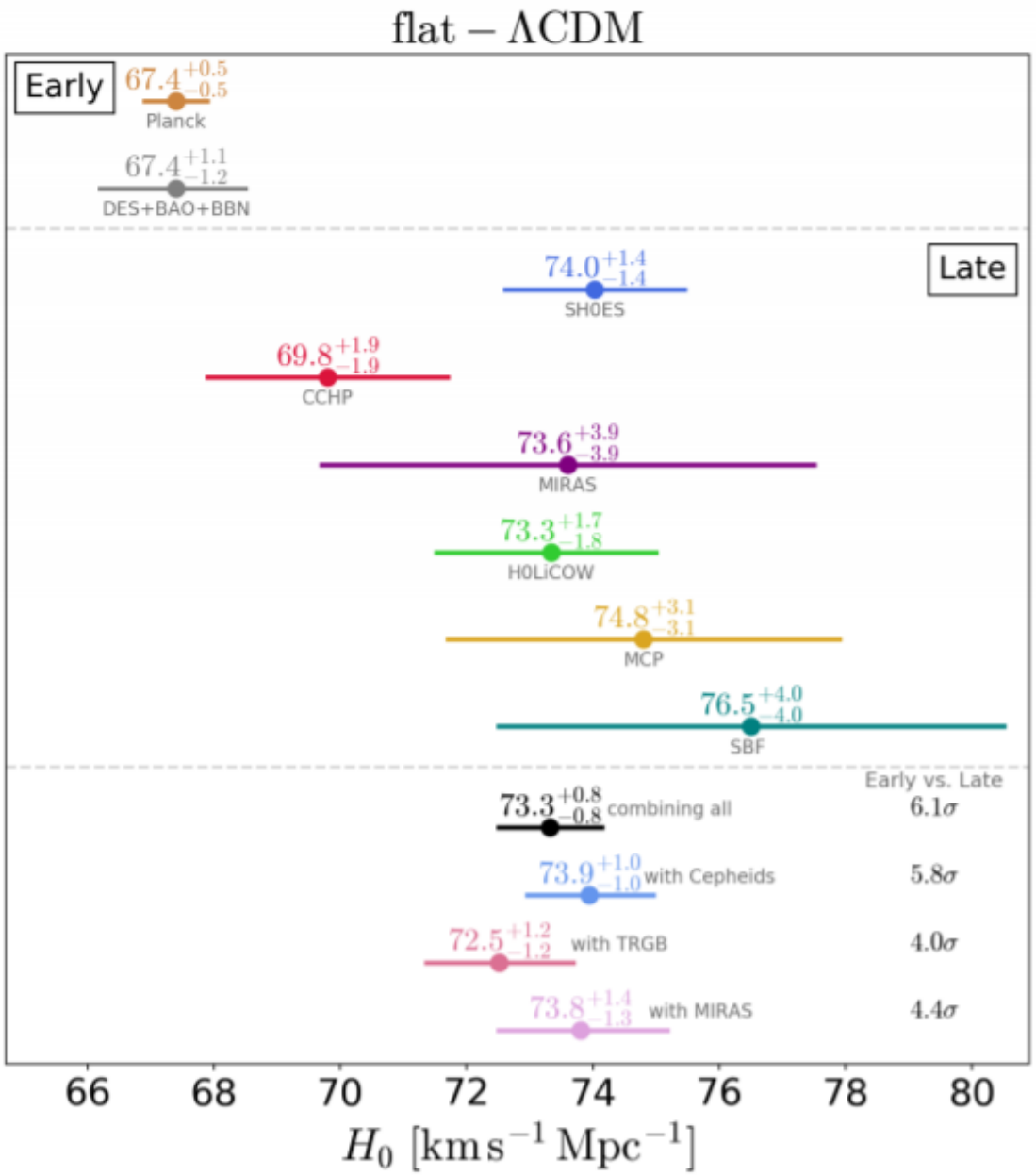
SIXTEENTH MARCEL GROSSMANN MEETING
ON RECENT DEVELOPMENTS IN THEORETICAL AND EXPERIMENTAL GENERAL RELATIVITY, ASTROPHYSICS AND RELATIVISTIC FIEL THEORIES

UNIVERSITY OF CAMBRIDGE

Tensions in Cosmology

- 4σ difference between the Hubble constant from the CMB vs. SH0ES.

- More then 3σ difference between early and late measurements (CMB & LSS)



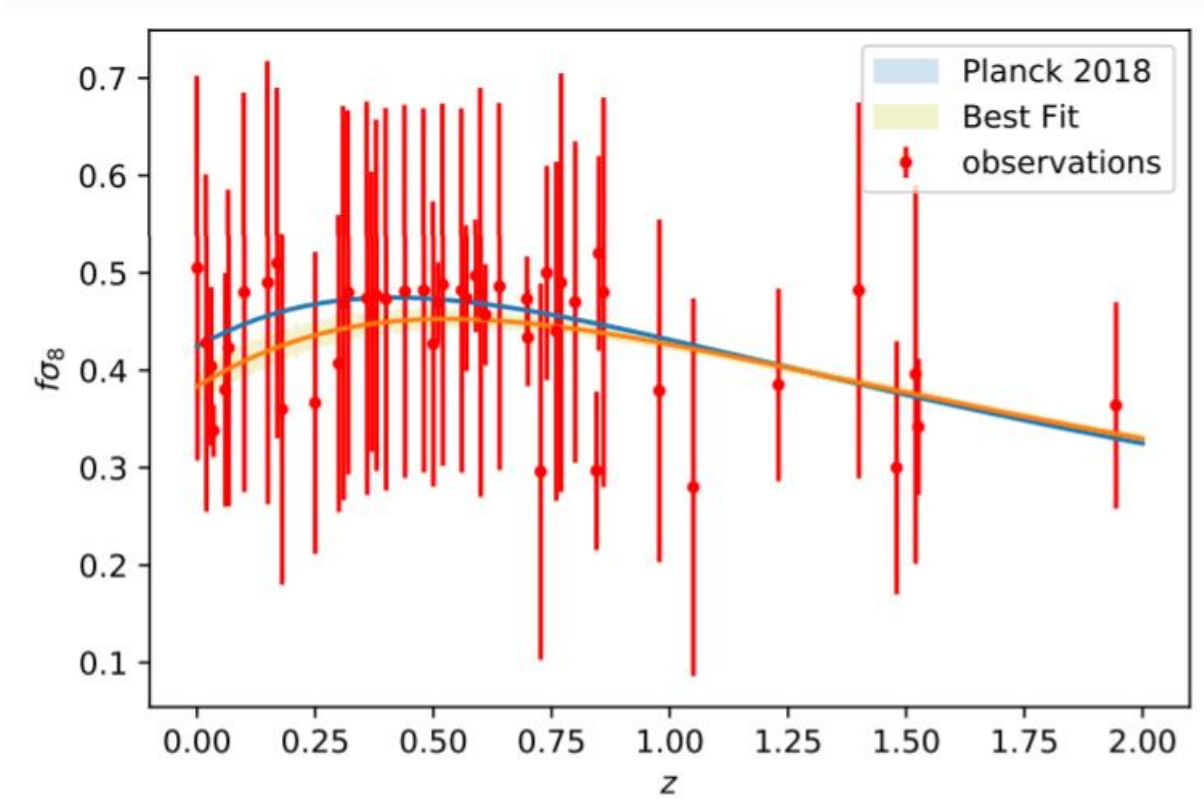
The S_8 tension

- σ_8 - matter fluctuations averaged in spheres of radius $8\ h^{-1}Mpc$

- The S_8 quantifying the amplitude of late-time matter fluctuations

$$S_8 = \sigma_8 \sqrt{\frac{\Omega_m}{0.3}}$$

- ~ 50 data points from the latest experiments: $0.01 \leq z \leq 1.994$



Homogenous Cosmology

- The Freidmann Eq. is normalized with:

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{-3(1+w)})$$

- The DE parametrization: $w = w_0 + (1-a)w_a$

- Radiation is ignored since the measurements are from the late universe.

Perturbative Cosmology

- Matter perturbations: $\delta_m = \delta\rho/\rho$
- The linear matter perturbations (in Fourier space):

$$\delta''_m(a) + \left(\frac{3}{a} + \frac{E'(a)}{E(a)}\right)\delta'_m(a) = \frac{3}{2}\frac{\Omega_m(a)}{a^2}\delta_m(a)$$

- assumption of small scales approximation (no k).

$$f(a) = \frac{d\log\delta_m}{d\log a}, \sigma_8(a) = \sigma_8\frac{\delta_m(a)}{\delta_m(1)}$$

$$f\sigma_8(a) = a\frac{\delta'_m(a)}{\delta_m(1)}\sigma_8$$

Uncorrelated and Correlated data

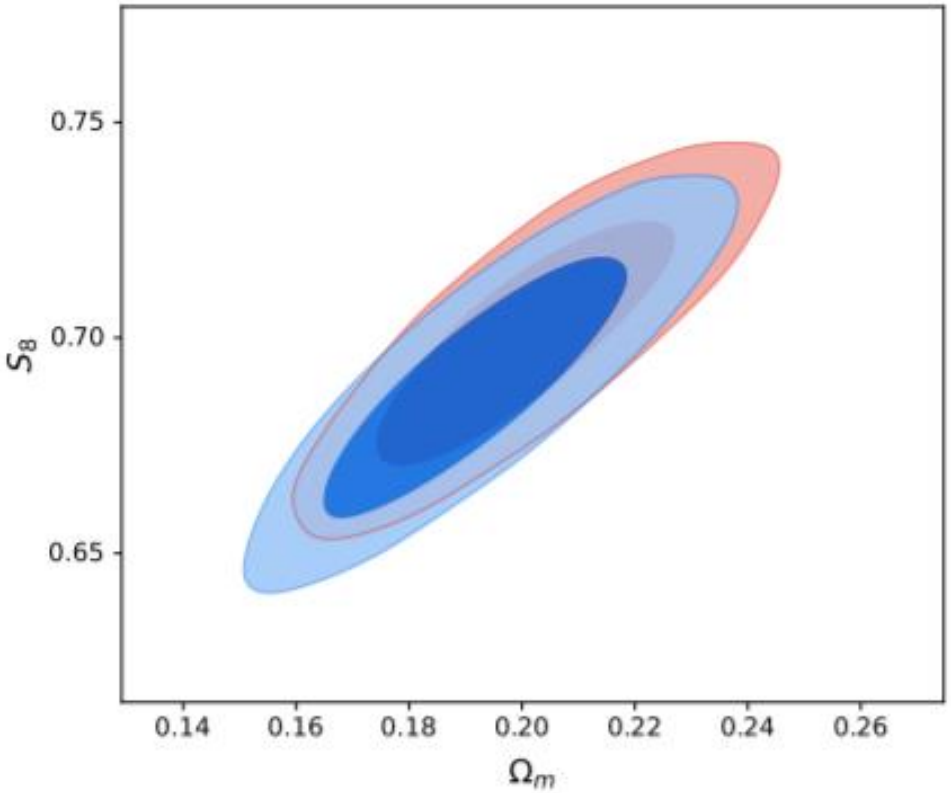
- Likelihood: $\mathcal{L} = \exp\left(-\frac{\chi^2}{2}\right)$
- $\chi^2 = \sum_{i=1}^{N} \left(\frac{f\sigma_8(d_i) - f\sigma_8(\theta_i)}{\sigma_y^{(i)}} \right)^2$ for uncorrelated points.
- Can be written as: $\chi^2 = V_i^T (d - \theta) M_{ij} V_i \ \ (d - \theta)$

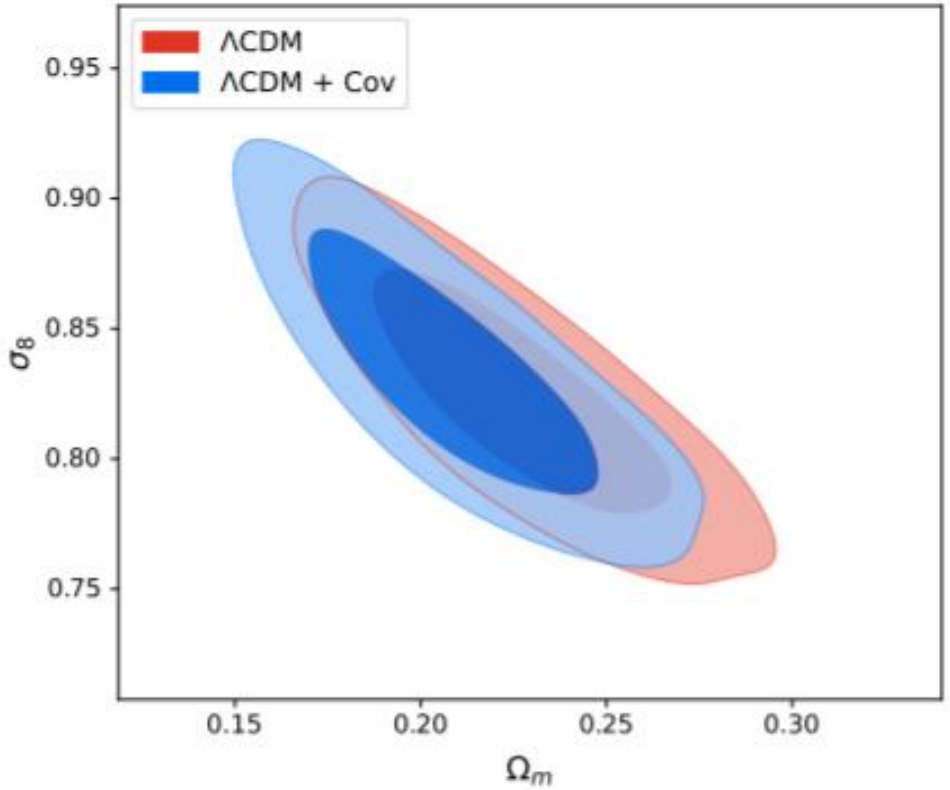
$M_{ij} = (\ldots \sigma_{ii} \ldots)$ for uncorrelated measurements.

- We test correlations, using $M_{ij} = 0.5\sigma_i\sigma_j$ with 25% random points.

Bayesian Analysis - **ΛCDM**

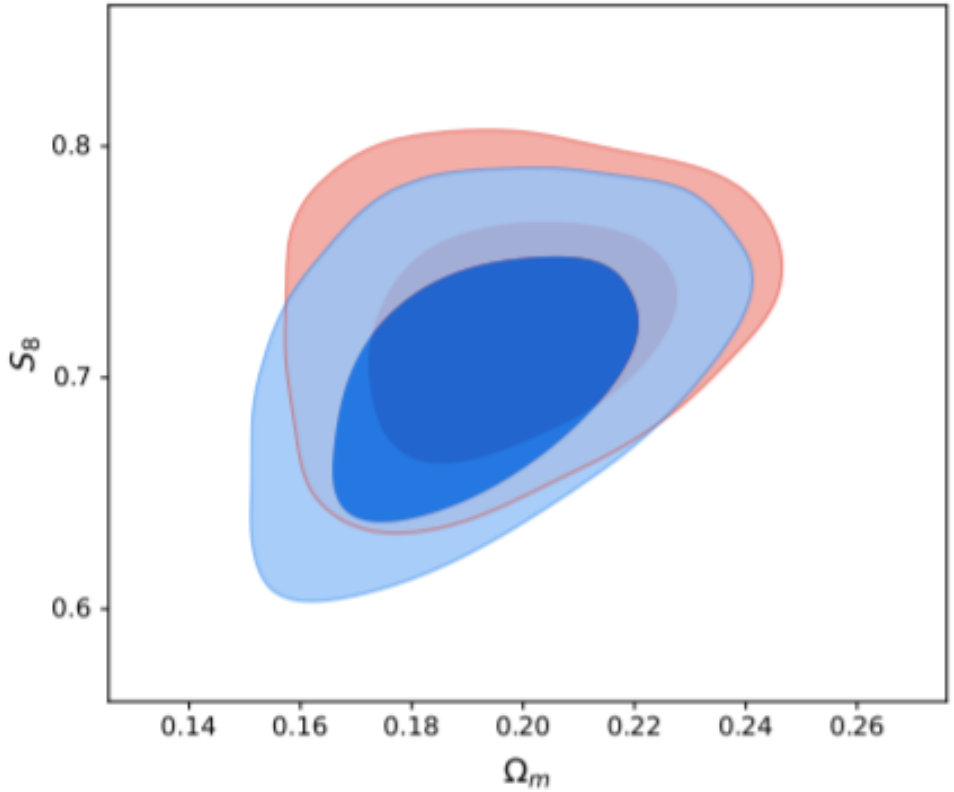
Model	Ω_m	σ_8	S_8	w
ΛCDM	$0.201^{+0.036}_{-0.033}$	$0.857^{+0.044}_{-0.042}$	$0.700^{+0.038}_{-0.037}$	-1
ΛCDM + Ran Cov	$0.192^{+0.037}_{-0.033}$	$0.863^{+0.046}_{-0.043}$	$0.690^{+0.040}_{-0.038}$	-1
Planck 2018	0.311 ± 0.019	0.811 ± 0.006	0.834 ± 0.016	-1

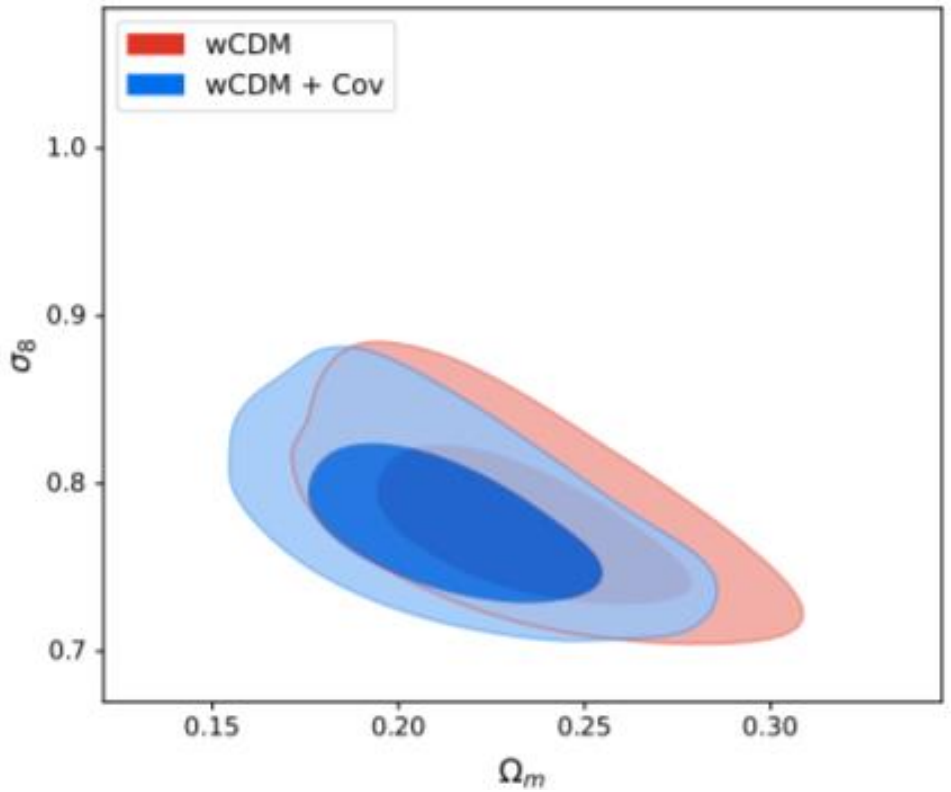




WCDM

Model	Ω_m	σ_8	S_8	w
ΛCDM	$0.201^{+0.036}_{-0.033}$	$0.857^{+0.044}_{-0.042}$	$0.700^{+0.038}_{-0.037}$	-1
ΛCDM + Ran Cov	$0.192^{+0.037}_{-0.033}$	$0.863^{+0.046}_{-0.043}$	$0.690^{+0.040}_{-0.038}$	-1
wCDM	$0.198^{+0.038}_{-0.035}$	$0.891^{+0.11}_{-0.095}$	$0.722^{+0.072}_{-0.067}$	$-0.90^{+0.29}_{-0.30}$
wCDM + Ran Cov	$0.192^{+0.038}_{-0.034}$	$0.879^{+0.10}_{-0.088}$	$0.701^{+0.077}_{-0.074}$	$-0.96^{+0.30}_{-0.31}$
Planck 2018	0.311 ± 0.019	0.811 ± 0.006	0.834 ± 0.016	-1





Gaussian Process Regression

- Unsupervised learning. Assumes Kernel distribution between two points.

- A Kernel dependent $cov\big(f(x), f(x')\big) = K(x, x')$

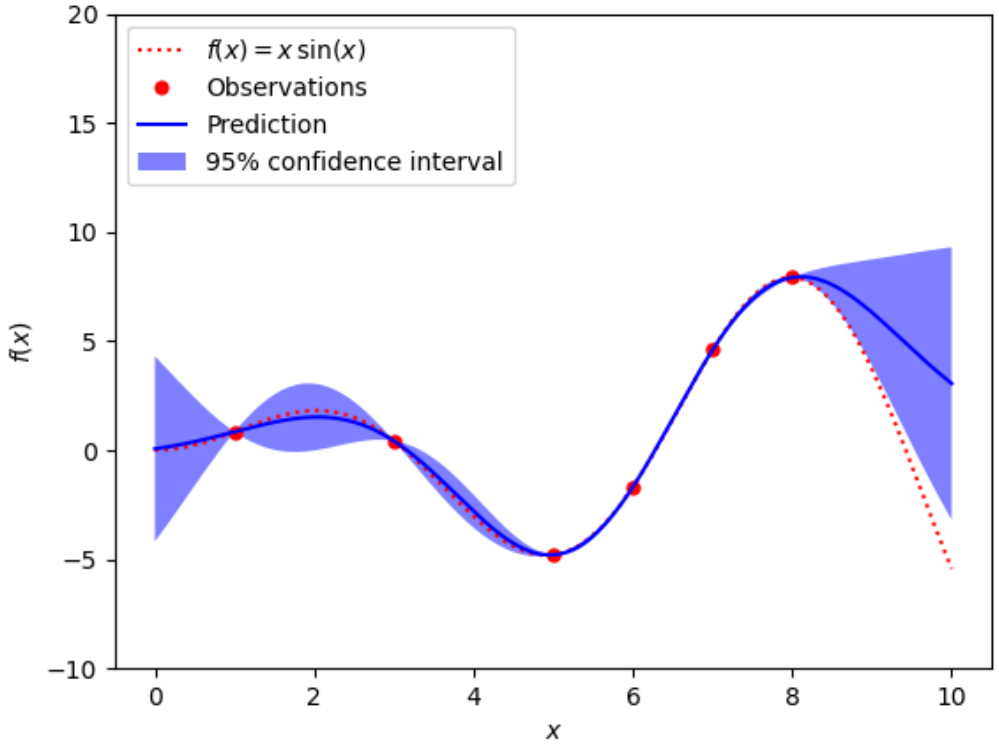
$$k(x, \tilde{x}) = \sigma_f^2 \exp(-\frac{(x-\tilde{x})^2}{2l^2}), \tag{13}$$

The Matern kernel with $\nu = 7/2$:

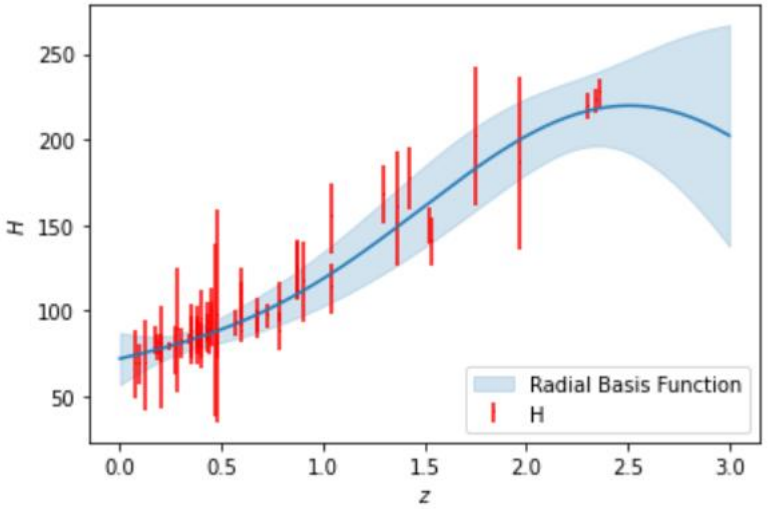
$$k(x, \tilde{x}) = \sigma_f^2 \exp(-\sqrt{7}\frac{|x-\tilde{x}|}{l}) \\ (1 + \sqrt{7}\frac{|x-\tilde{x}|}{l} + 14\frac{(x-\tilde{x})^2}{5l^2} + 7\sqrt{7}\frac{|x-\tilde{x}|^3}{15l^3}), \tag{14}$$

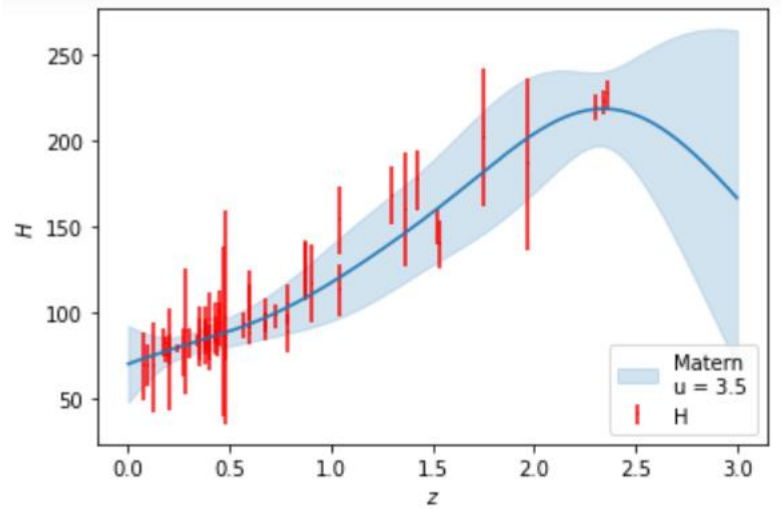
and Matern kernel with $\nu = 9/2$:

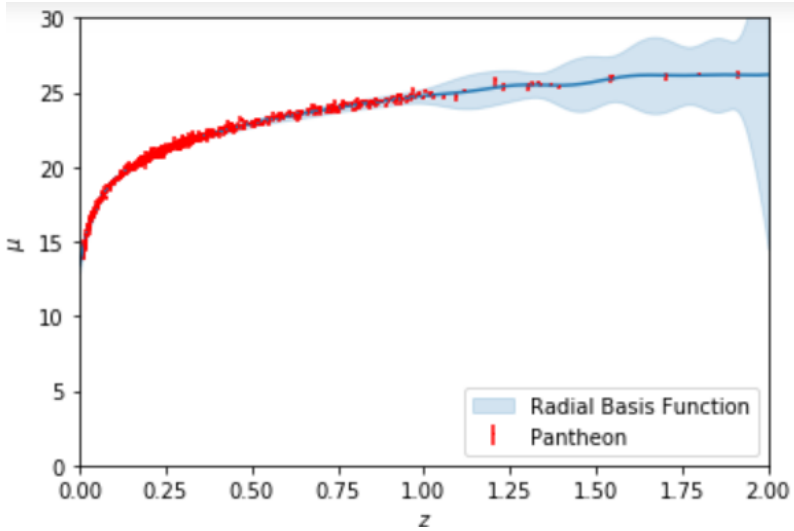
$$k(x, \tilde{x}) = \sigma_f^2 \exp(-3\frac{|x-\tilde{x}|}{l}) \\ (1 + 3\frac{|x-\tilde{x}|}{l} + 27\frac{(x-\tilde{x})^2}{7l^2} + 18\frac{|x-\tilde{x}|^3}{7l^3} + 27\frac{(x-\tilde{x})^4}{35l^4}). \tag{15}$$

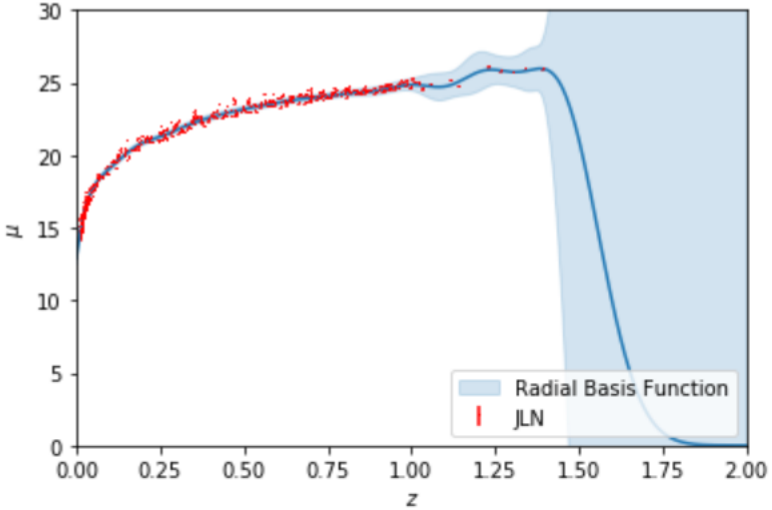


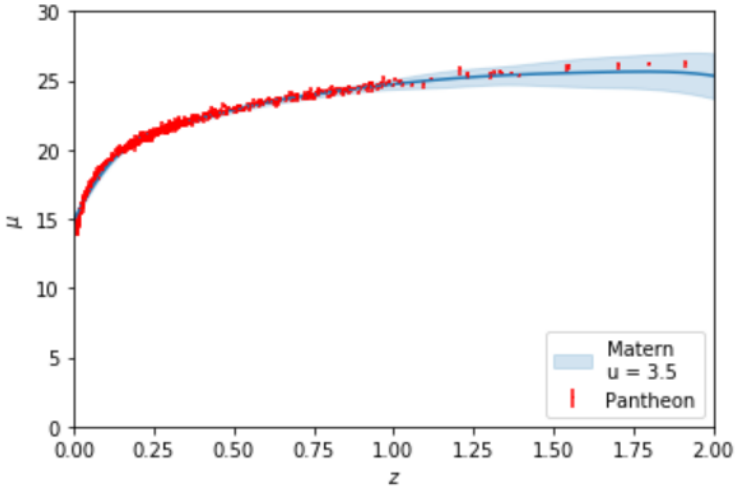
Kernel Dependent

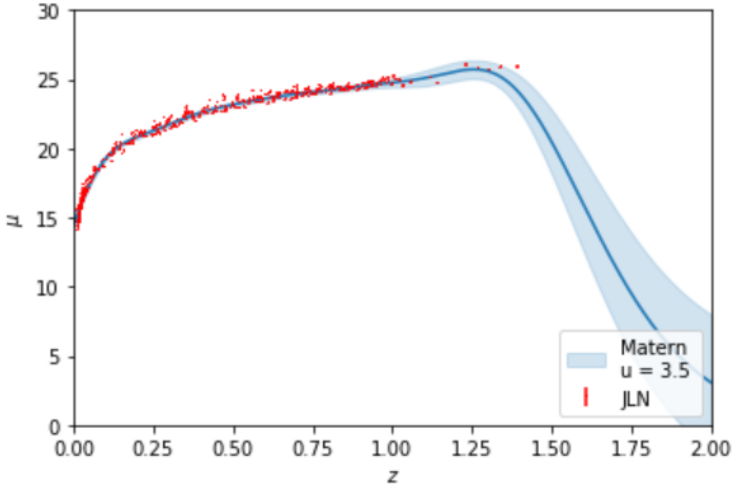




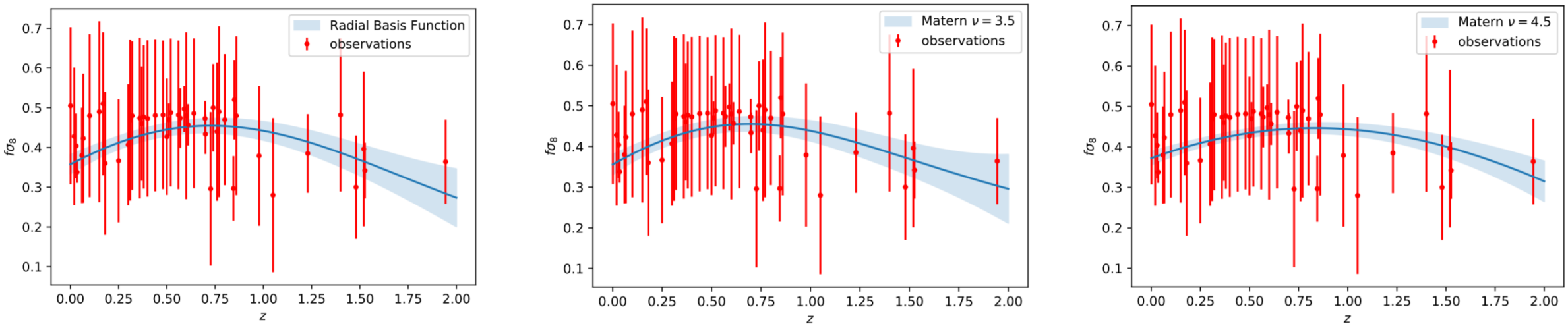








GP for the $f\sigma_8$ data

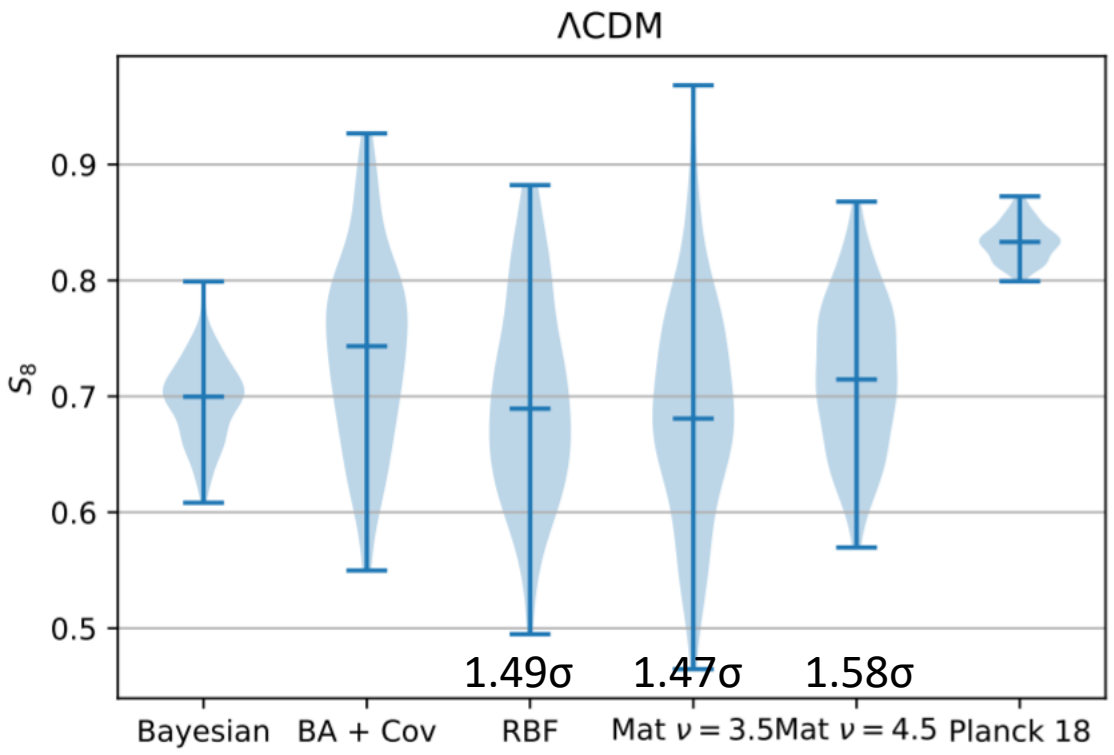


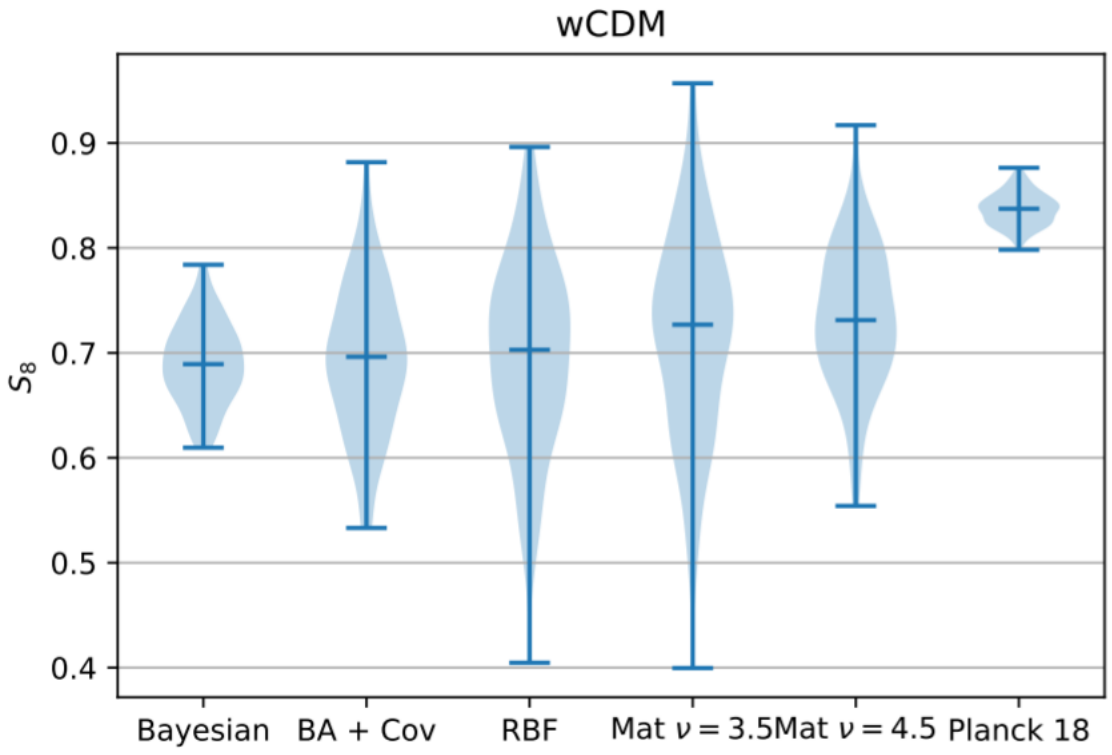
Kernel	RBF	Matern $\nu = 3.5$	Matern $\nu = 4.5$
$f\sigma_8(0)$	0.358 ± 0.0241	0.356 ± 0.025	0.372 ± 0.017

$f\sigma_8(0.001) = 0.505 \pm 0.0852$ MTF Mon. Not. Roy. Astron. Soc. 471, 3135 (2017)

$f\sigma_8(0.02) = 0.428 \pm 0.04652$ 6dFGS JCAP 1705, 015 (2017)

Results for the S_8 values





ΛCDM	RBF	Matern $\nu = 3.5$	Matern $\nu = 4.5$
σ_8	$0.77^{+0.21}_{-0.16}$	$0.77^{+0.21}_{-0.16}$	$0.79^{+0.19}_{-0.17}$
Ω_m	$0.27^{+0.12}_{-0.11}$	$0.26^{+0.13}_{-0.11}$	$0.27^{+0.12}_{-0.10}$
S_8	$0.707^{+0.085}_{-0.085}$	$0.701^{+0.089}_{-0.089}$	$0.731^{+0.063}_{-0.062}$

ΛCDM	RBF	Matern $\nu = 3.5$	Matern $\nu = 4.5$
σ_8	$0.77^{+0.21}_{-0.16}$	$0.77^{+0.21}_{-0.16}$	$0.79^{+0.19}_{-0.17}$
Ω_m	$0.27^{+0.12}_{-0.11}$	$0.26^{+0.13}_{-0.11}$	$0.27^{+0.12}_{-0.10}$
S_8	$0.707^{+0.085}_{-0.085}$	$0.701^{+0.089}_{-0.089}$	$0.731^{+0.063}_{-0.062}$

Final Results

- With B.A there is a clear tension with Planck measurement.
- With ML the tension is reduces to $\sim 1\ \sigma$ for some kernels.
- Still an open window to new physics…