Hubble Speed from First Principles

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Distances in Cosmology

Distances are set by the metric that describes our space-time; in a cosmological context, this implies that their scaling with time is related to the expansion rate of the Universe.

Comoving distance:

\[ \chi(z) = \frac{1}{H_0} \int_0^z \frac{d\tilde{z}}{E(\tilde{z})} \]

where I will call the cosmological model:

\[ E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_m (1 + z)^3 + \Omega_r (1 + z)^4 + \Omega_{DE} X(z) \]

\( X \) represents all contributions to the expansion that are not matter or radiation, e.g. cosmological constant \((X=1)\), DE, modified gravity, spatial curvature, etc..
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Comoving distance is not a direct observable. There are two types of distances measured in an expanding Universe:

**Luminosity distance**
- Standardized candles/sirens
  - SN Ie (GWs)

\[
m_B = M_B + 5 \log \left( \frac{d_L}{10\text{pc}} \right) + 25
\]
\[
M_B = 5 \log H_0 - 5a_B - 25
\]
\[
m_B = 5 \log \left( \frac{H_0 d_L}{10\text{pc}} \right) - 5a_B
\]

**Angular diameter distance**
- BAO (CMB peaks)
  - Standard rulers

\[
\theta_s(z) = \frac{r_s}{d_A(z)}
\]
\[
\Delta z = H(z) r_s
\]
In any metric theory of gravity, if photons propagate along null geodesics and obey number conservation:

\[ \eta(z) \equiv \frac{d_L(z)}{(1 + z)^2 d_A(z)} = 1 \]

Etherington Philos. Mag. 15, 761 (1933).
Distance Duality Relation

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Let us set \( \eta = 1 \) and rearrange the DDR as follows:

\[ H_0 = \frac{1}{(1 + z)^2} \frac{H_0 d_L(z)}{H(z) d_A(z)} H(z) \]

- \( H(z) d_A(z) \): from a combination of line-of-sight and transverse BAO measurements, without the need of an external anchor for the sound horizon at drag epoch [7 data points from BOSS]
- Cosmic chronometers (CC) measurements provide \( H(z) \) data that is free from calibration and does not depend on the underlying cosmological model [a compilation of 30 measurements]
- \( H_0 d_L \) can be obtained from measurements of SNIa without the need of calibration [Pantheon]
**Distance Duality Relation**

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$$
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$$

$$
H(z) = -\frac{1}{1 + z} \frac{\Delta z}{\Delta t}
$$

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$\Delta z$: measured spectroscopically

$\Delta t$: std clocks: passively evolving stellar population
Combining BAO and CC first, and then folding in SNIa, we can get a *calibration- and model-independent* estimate of the Hubble parameter:

\[
H_0 = \frac{1}{(1 + z)^2} \frac{[H_0 d_L(z)]^\text{SNIa}}{[d_A(z)]^\text{BAO+CC}}
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\]

Some technicalities:

- the three data set cover roughly same range, \(0 \leq z \leq 2.5\), but data points are not exactly at same redshifts
- we choose the redshifts of BAO data points as reference ones, and perform GP fit to SNIa and CC data to get corresponding data points (including error bars), i.e. PDFs
H_0 from Distance Duality Relation

- at each of the 7 redshifts, we draw 10k realizations of \([H_0d_L]^{\text{SNIe}}\) and 10k realizations of \([d_A]^{\text{BAO+CC}}\). We combine them together following the DDR equation to get realizations (PDFs) of H_0 at each redshift.

- resulting mean values are consistent within error bars, but they are different at each redshift, with two being notably lower…(next slide!)

- we finally multiply the 7 PDFs to get one PDF for H_0 from which we can extract mean and variance using the inverse transform sampling.

- Overall the method is a combination of GP and MCMC, we refer to it as GPMC.

- It is important to highlight that we work strictly within the redshift range of the data, without performing any extrapolation.
$H_0$ from Distance Duality Relation

$H_0 = 69.5 \pm 1.7 \text{km/s/Mpc}$
\(H_0\) from Distance Duality Relation
correspondingly we infer the absolute magnitude to be $M_B = -19.355 \pm 0.054$

i.e a shift of $|\delta M_B| = 0.138 \pm 0.067$ wrt SH0ES value.

in agreement with $\mu_{\text{Cepheid}} - \mu_{\text{TRGB}} = -0.139 \pm 0.024$ mag  [Efstathiou, arXiv:2007.10716]
A redshift trend in $H_0$?

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A redshift trend in $H_0$?

Assuming the validity of the DDR, is equivalent to assuming that the two probes are consistent tracers of cosmic expansion.

If this was not the case, the $H_0$ inferred from the previous procedure would not be a constant, but it would rather show some, unphysical trend in redshift.
perhaps a mild dynamical feature in $H(z)$