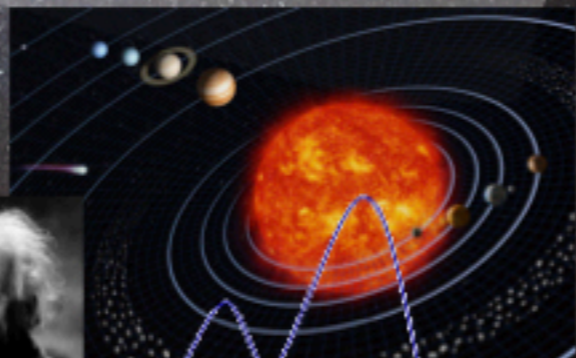


**Testing the general relativistic nature of the Milky Way Rotation Curve with data from the ESA satellite Gaia**

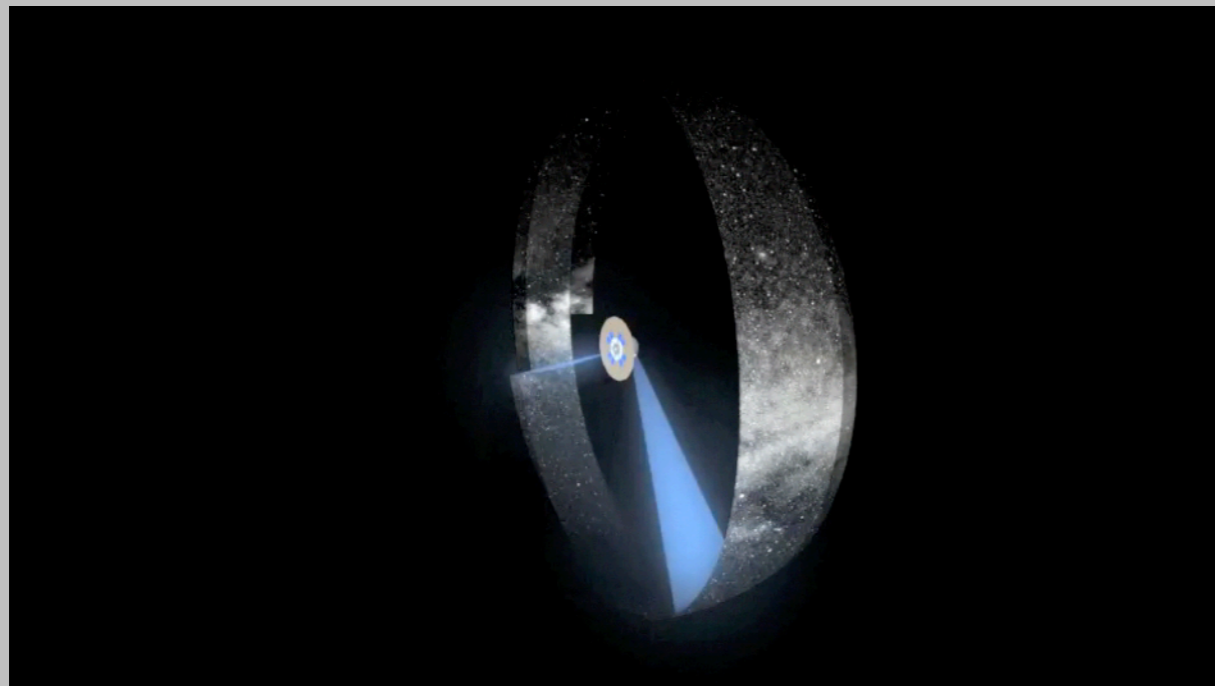
Mariateresa Crosta  
INAF  
Astrophysical Observatory of Torino

16th Marcel Grossmann Meeting, PT5  
*Dragging is never draggy: MAss and CHarge flows in GR*  
July 5 - 10 2021



$$g = \eta + h$$

Gaia is the ESA cornerstone mission, a wide European effort involving almost 450 scientists, launched in 2013.



The DPCT hosts the systems of the Astrometric Verification Unit (AVU), run by ALTEC (To) under the scientific supervision of the astrometric group INAF-OATo for ASI



Size at completion ~ 2 PB

AVU is in charge, for DPAC, of the verification, through the Global Sphere Reconstruction (GSR), of the absolute astrometry achieved through the baseline astrometric model



## Gaia Data Processing and Analysis Consortium (DPAC)



Cambridge, UK

Geneva, Switzerland

Toulouse, France

ESAC, Spain

Barcelona, Spain

Turin, Italy

Small external contributions from:  
Algeria, Brazil, Chile, Israel, United States, European Southern Observatory





**Gaia measures  
position (direction and distance) & velocity  
of over 1 billion stars in our Galaxy  
with an accuracy of up to 10 millionths-of-  
arcsecond**

**Science with one/two billion objects in 3 dimension,  
from structure and evolution of the MW to GR tests**

## Astrometry

positions  
proper motions  
parallaxes

end-of-mission astrometric  
accuracies better than  $5-10\mu\text{s}$   
(brighter stars)  
 $130-600\mu\text{s}$  (faint targets)

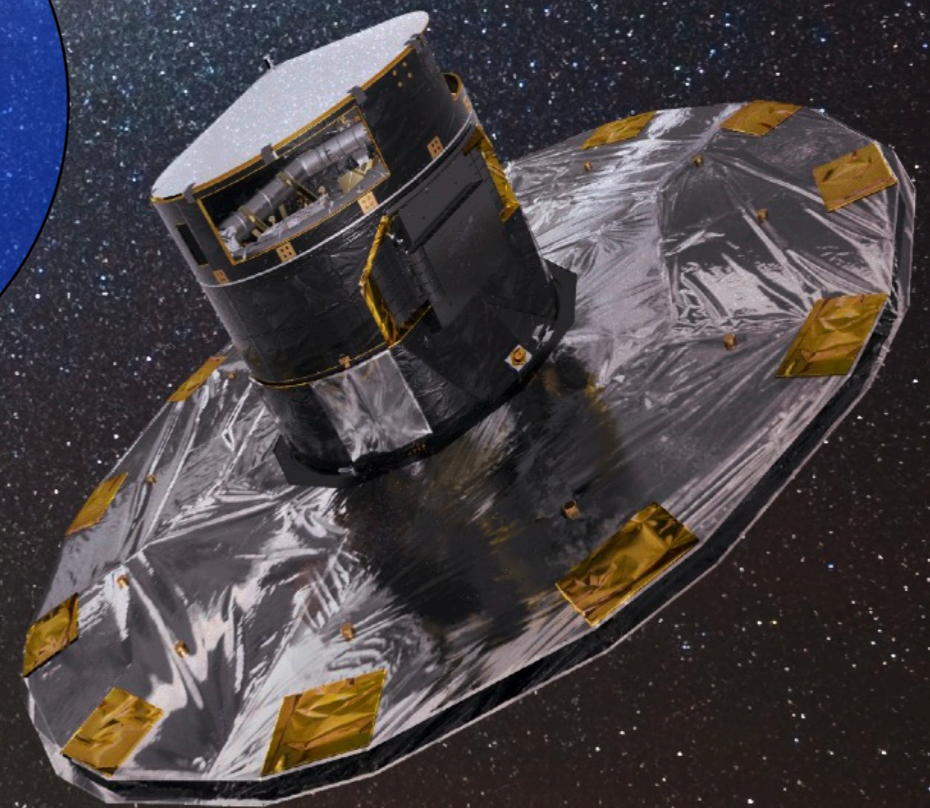
## Photometry

spectral classification  
photometric distances  
brightness  
temperature  
mass  
age  
chemical composition  
 $G < 20.7 \text{ mag}$

## Spectrometry

radial velocity  
chemical abundances

$G\_RVS = 16.2$



## Early Data Release 3 in numbers

<https://www.cosmos.esa.int/web/gaia/early-data-release-3>



### GAIA EARLY DATA RELEASE 3

**1 811 709 771**  
stellar positions

**1 806 254 432**  
brightness  
in white light

**1 542 033 472**  
brightness  
in blue light

**1 540 770 489**  
colour

**1 467 744 818**  
parallax and  
proper motions

**1 614 173**  
extragalactic  
sources

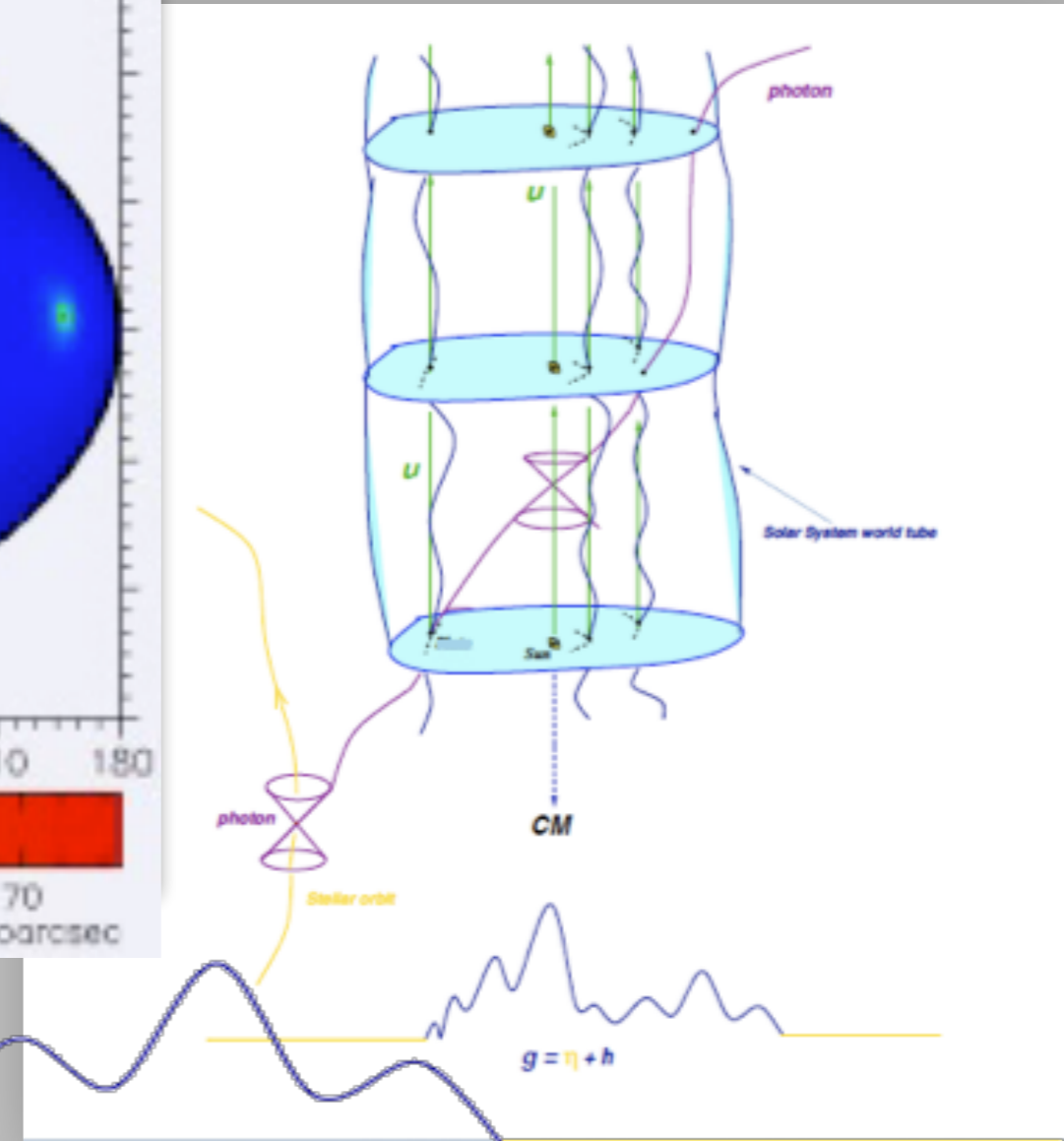
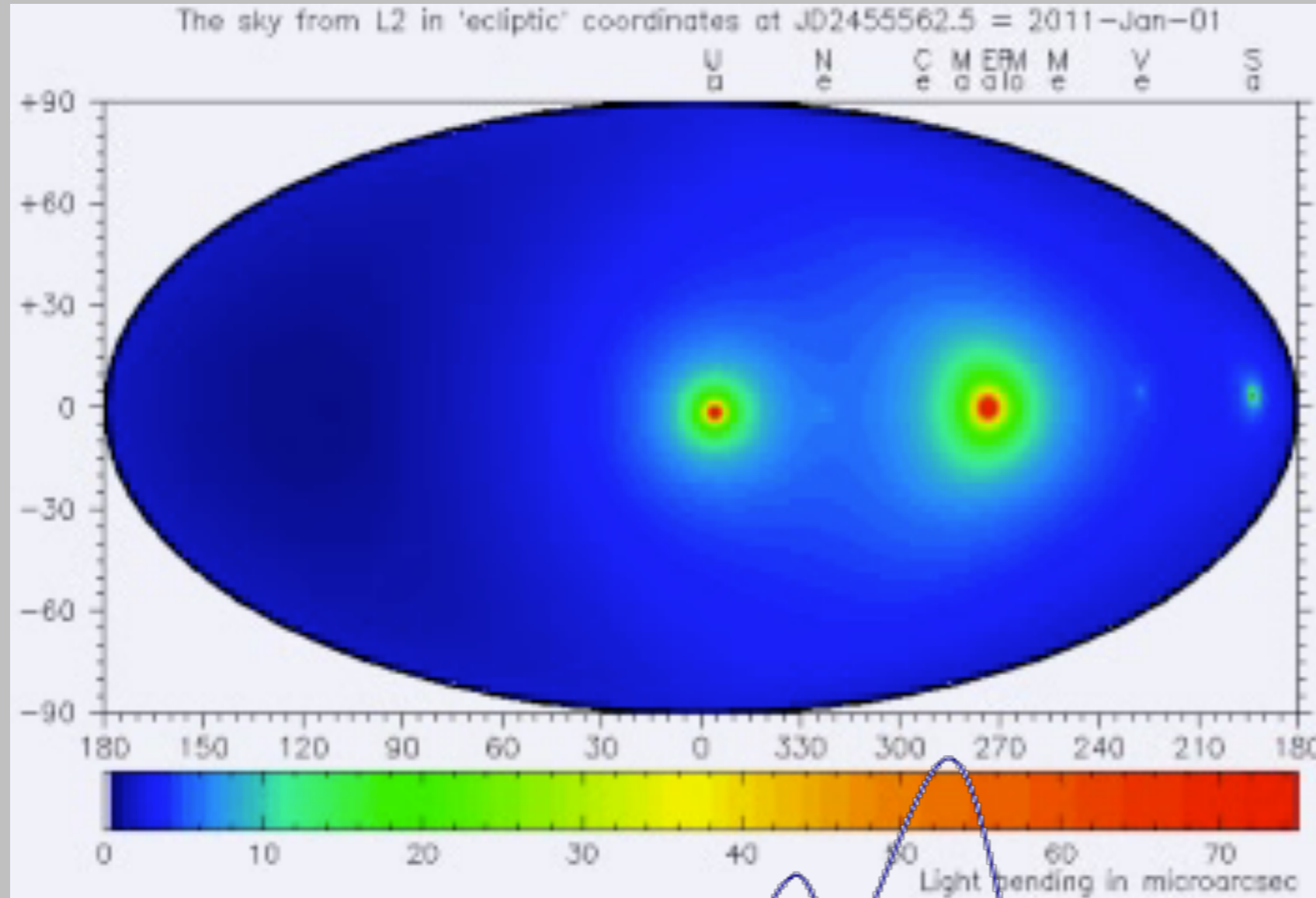
**1 554 997 939**  
brightness  
in red light

#SpaceCare #ExploreFarther



# Gaia-observer laboratory: the Solar System

micro-arcsecond accuracy+ dynamical gravitational fields  
relativistic models of light propagation:  
**RELATIVISTIC ASTROMETRY**



2 independent GR models (GREM and RAMOD)-> the Consortium constituted for the Gaia data reduction (DPAC) agreed to set up, respectively, two independent global sphere solutions: AGIS and GSR.

$+h$  perturbations at  $\mu$ -arcsec due to the solar system bodies. Off-diagonal terms are included (IAU metric)



# Gaia: the onset of gravitational astrometry era

Gaia is delivering a **relativistic kinematic**

For the Gaia-like observer the weak gravitational regime turns out to be "strong" when one has to perform high accurate measurements

- the position and velocity data, comprising the outputs of the Gaia mission, are fully GR compliant —>> Given a relativistic approach for the data analysis and processing, any subsequent exploitations should be consistent with the precepts of the theory underlying the astrometric model.

**A fully relativistic model for the Milky Way (MW) structure should be pursued!**

The **GR picture of the MW** can ensure a strong and coherent **Local Cosmology laboratory** against which any model of the Galaxy can be fully tested

- **Local Cosmology:** how well distances and kinematics at the scale of the Milky Way disk compare with the Lambda-CDM model predictions



In the most advanced simulations  $\Lambda$ -CDM cosmology assumes an average FLRW evolution while growth in structure is treated by Newtonian N-body simulations:

*“Friedman tells space how to curve and Newton tells mass how to move”*

[Alan A. Coley, David L. Wiltshire](#)

General Relativity (GR) is only partially considered

**Missing: ray-tracing to obtain true observables!**

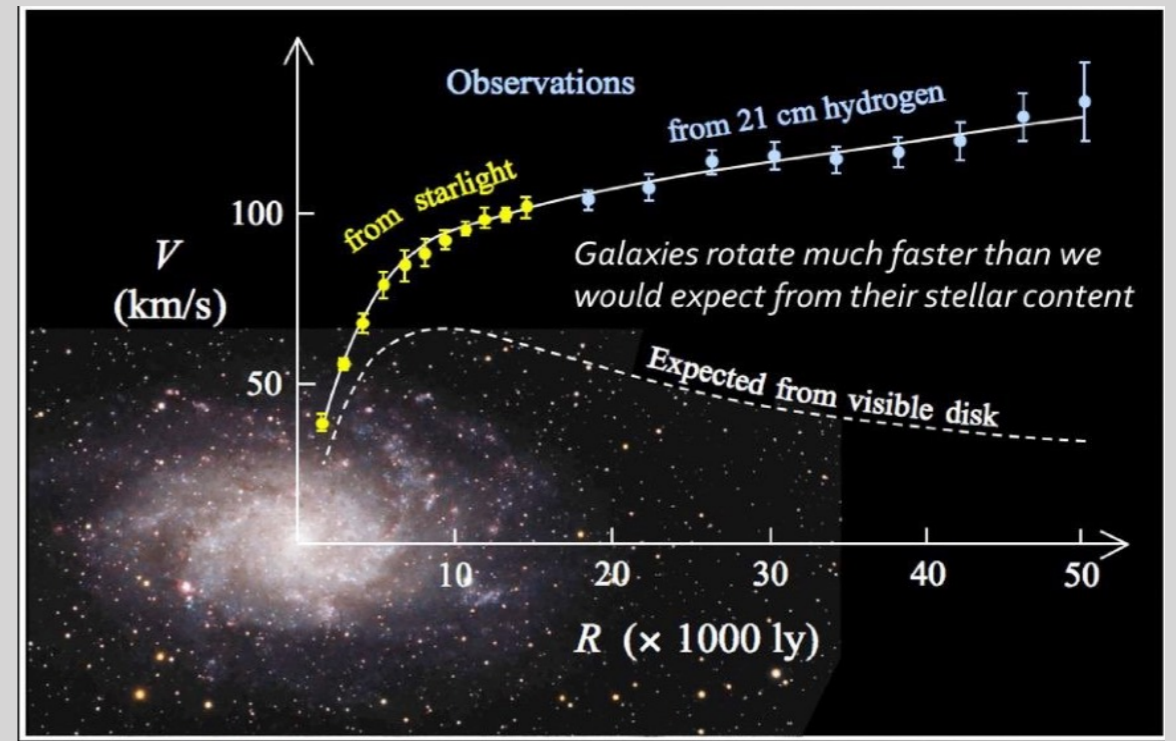
*M. Crosta, MGM16-PT5, 5 July 2021*

# Flat Galactic rotation curves at kpc scale as first GR test for the MW

Flat rotation curves in disk galaxies - a longest outstanding problem in astronomy - provide the main observational support to the hypothesis of surrounding dark matter. Adding a “dark matter” halo allows a good fit to data

Stellar kinematics, as tracer of gravitational potential, is the most reliable observable for gauging different matter components

By routinely scanning individual sources throughout the whole sky, Gaia directly measures the (relativistic) kinematics of the stellar component



Rotation curves are distinctive features of spiral galaxies like our Milky Way, a sort of a kinematical/dynamical signature, like the HR diagram for the astrophysical content.

-> the rotation curve of the MW used as a first test for a GR Galaxy with the Gaia DR2 data

# weak field regime @Milky Way scale

---

In general one assumes that:

**gravitational potential or “relativistic effects” at the MW scale are usually “small”, then**

✓negligible..

✓locally Newton approximation is retained valid at each point..

but  $(v_{\text{Gal}}/c)^2 \sim 0,69 \times 10^{-6} \text{ (rad)} \sim 100 \text{ mas}$

$(v_{\text{Gal}}/c)^3 \sim 0,57 \times 10^{-9} \text{ (rad)} \sim 120 \mu\text{as}$

the individual DR2 astrometric error is  $\leq 100 \mu\text{as}$   
throughout most of its magnitude range



**“weakly” relativistic effect could be relevant**

**need to compare the GR model and the classical one**

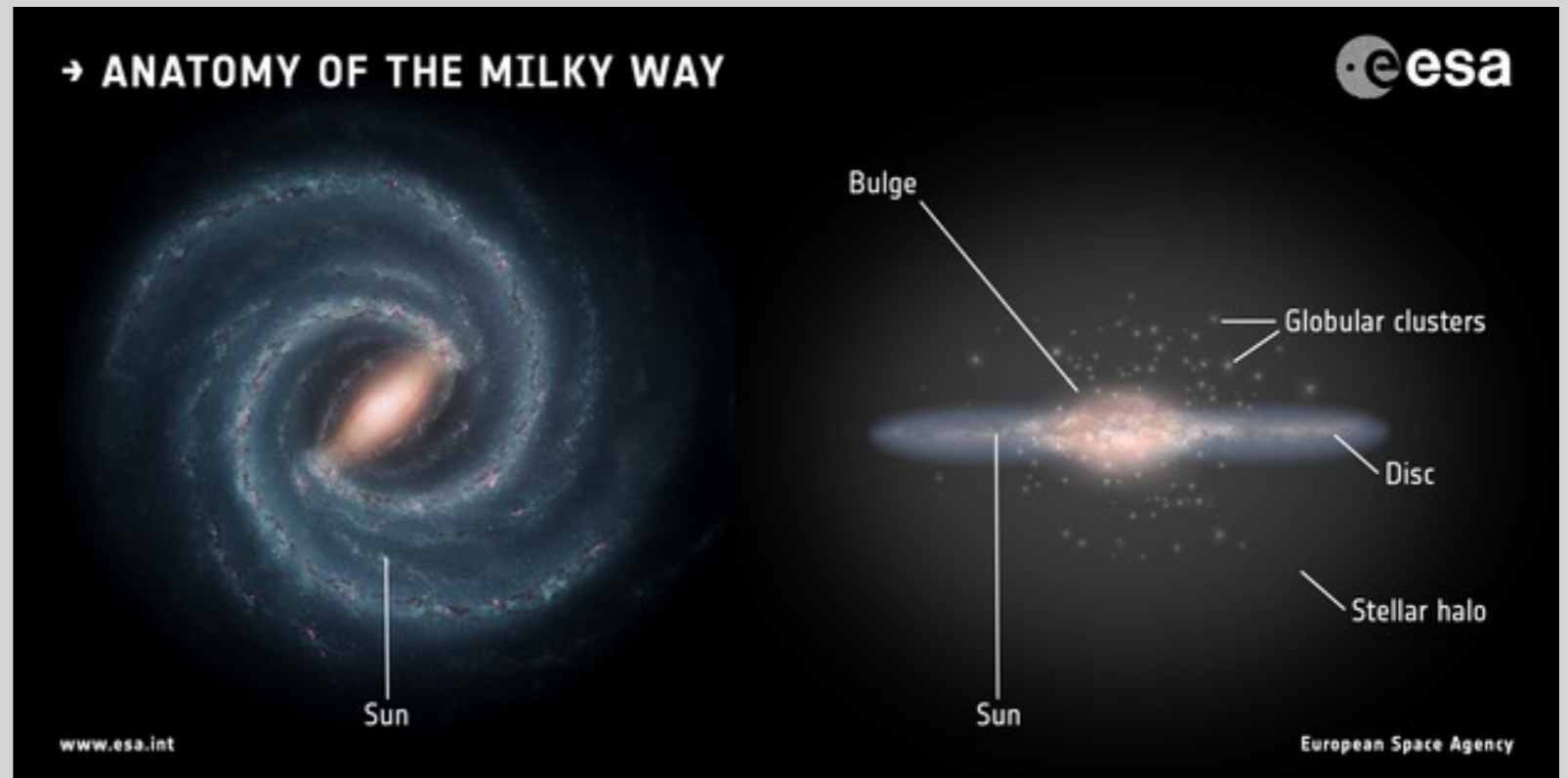
**The small curvature limit in General Relativity may not coincide with the Newtonian regime**



# “Classic” Milky Way (MWC) model with Dark matter halo

- Newtonian limit applied for Galactic dynamics  
-> Poisson's equation

$$\nabla^2 \Phi = 4\pi G \rho$$



## 1. Plummer **bulge**

$$\rho_b = \frac{3b_b^2 M_b}{4\pi(r^2 + b_b^2)^{5/2}}$$

bulge spherical radius

**$b_b = 0.3$  kpc**

Pouliasis, E., Di Matteo, P., & Haywood, M. 2017, A&A, 598, A66

## 2. Miyamoto-Nagai **thin and thick disks**

$$\rho_d(R, z) = \frac{M_d b_d^2}{4\pi} \frac{\left[ a_d R^2 + (a_d + 3\sqrt{z^2 + b_d^2})(a_d + \sqrt{z^2 + b_d^2})^2 \right]}{\left[ R^2 + (a_d + \sqrt{z^2 + b_d^2})^2 \right]^{5/2} (z^2 + b_d^2)^{3/2}}$$

**$b_{td} = 0.25$  kpc and  $b_{Td} = 0.8$  kpc**

Bovy, J. 2015, ApJs, 216, 29 Korol, Rossi & Barausse (2019) McMillan, P. J. 2017, MNRAS, 465, 76-94

## 3. Navarro-Frank-White DM **halo**

$$\rho_h(r) = \rho_0^{halo} \frac{1}{(r/A_h)(1 + r/A_h)^2}$$

Navarro, J. F., Frenk, C. S. and White, S. D. M. 1996, ApJ, 462, 563

$M_b$ ,  $M_{td}$ ,  $M_{Td}$ ,  $a_{td}$ ,  $a_{Td}$ ,  $b_d$ ,  $\rho_0^{halo}$  and  $A_h$  correspond to the bulge mass, the masses and the scale lengths/heights of the thin and thick disks, the halo scale density, and the halo radial scale

$$\nabla^2 \Phi_{tot} = 4\pi G(\rho_b + \rho_{td} + \rho_{Td} + \rho_h) \quad \rightarrow \quad V_c^2 = R \left( d\Phi_{tot} / dR \right)$$

**MWC velocity profile**

# A GR model for the Milky Way

Einstein equation are very difficult to solve analytically and Galaxy is a multi-structured object making it even the more difficult to detail a metric for the whole Galaxy

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = - dt^2 + 2Nd\phi dt + (r^2 - N^2)d\phi^2 + e^\nu(dr^2 + dz^2)$$

**Galactic metric-disk**

1. Stationarity and axisymmetry spacetime
2. Reflection symmetry (around the galactic plane)
3. The disk is an equilibrium configuration of a pressure-less rotating perfect fluid (a GR dust)
4. The masses inside a large portion of the Galaxy interact only gravitationally and reside far from the central bulge region
5. The rotational curve is due to the angular-momentum sustained stellar population
6. Stars = dust grains, co-moving with the Gaia-observer

The function  $N(r,z)$  was constrained by Balasin & Grumiller (BG) to the separation ansatz  $N(r,z) = R(r)F(z)$  and the reflection symmetry assumption.

$$N(r, z) = V_0(R_{out} - r_{in}) + \frac{V_0}{2} \sum_{\pm} \left( \sqrt{(z \pm r_{in})^2 + r^2} - \sqrt{(z \pm R_{out})^2 + r^2} \right)$$

(Balasin and Grummiller, Int.J. Mod. Phys., 2008)

- \*  $r_{in}$  = bulge size
  - \*  $R_{out}$  = extension of the MW disk-> Galaxy size
  - \*  $V_0$  = velocity in the flat regime
- $|z| < r_{in}$

Einstein field Eq. from the metric disk

$$r\partial_z\nu + \partial_r N\partial_z N = 0$$

$$2r\partial_r\nu + (\partial_r N)^2 - (\partial_z N)^2 = 0$$

$$2r^2(\partial_r\partial_r\nu + \partial_z\partial_z\nu) + (\partial_r N)^2 + (\partial_z N)^2 = 0$$

$$r(\partial_r\partial_r N + \partial_z\partial_z N) - \partial_r N = 0$$

$$(\partial_r N)^2 + (\partial_z N)^2 = kr^2\rho e^\nu$$

$$\rho(R, z) = e^{-\nu(R,z)} \frac{1}{8\pi R^2} [(\partial_R N(R, z))^2 + (\partial_z N(R, z))^2]$$

# The Gaia observer linked to the gravitational dragging

## Observer in circular motion

$$u^\alpha = \Gamma (k^\alpha + \beta m^\alpha) \quad \beta \text{ constant angular velocity (with respect to infinity), } \Gamma \text{ normalization factor}$$

or

$$u^\alpha = \gamma \left( e_{\hat{0}}^\alpha + \zeta^{\hat{\phi}} e_{\hat{\phi}}^\alpha \right) \quad \text{ZAMO frames = locally non-rotating observers, zero angular momentum with respect to flat infinity and move on worldlines orthogonal to the hypersurfaces } t=\text{constant}$$

$\gamma$  Lorentz factor

orthonormal frame adapted to the ZAMO  $Z^\alpha = (1/M)(\partial_t - M^\phi \partial_\phi) \quad M = r/\sqrt{(r^2 - N^2)}, \quad M^\phi = N/(r^2 - N^2)$

(de Felice and Bini, "Classical measurements in curved space-time")

$$\zeta^{\hat{\phi}} = \frac{\sqrt{g_{\phi\phi}}}{M} (\beta + M^\phi)$$

$$ds^2 = -M^2 dt^2 + (r^2 - N^2)(d\phi + M^\phi dt)^2 + e^\nu(dr^2 + dz^2)$$

$$\zeta^{\hat{\phi}} = \frac{N(r, z)}{r}$$

if static (as the observer in BCRS, Gaia catalogue)

$$|V(r, z)| = N(r, z)/r \propto g_{0\phi}$$

**V: spatial velocity of the co-rotating dust as seen by an asymptotic observer at rest wrt to the center of the Galaxy (or the rotation axis)**

**Gravitational dragging working at disk scale**

**The question before us: the MW rotation curve, dark matter or geometry driven?**

# Data sample: full reconstruction of disc kinematics based on DR2 data only

- i. **Complete Gaia DR2 astrometric dataset** ( $\alpha, \delta, \mu_\alpha, \mu_\delta, \text{parallax}$ )
- ii. **Parallaxes good to 20%** (i.e.  $\text{parallax\_over\_error} \geq 5$ )  
—> parallaxes to better than 20% allow to deal with similar (quasi-gaussian) statistics when transforming to distances
- iii. **Gaia-measured velocity along the line of sight, i.e. radial velocity, with better than 20% uncertainties** from Gaia DR2

**i.+ii.+iii. —> proper 6D reconstruction of the phase-space location occupied by each individual star as derived by the same observer**

- iv. Only for Early Type stars, **cross-matched entry in the 2MASS catalog** following Poggio et al. (2018)  
—> for the actual materialization of the sample

1. Full transformation (including complete error propagation) from the ICRS equatorial to heliocentric galactic coordinates
2. then translation to the galactic center

**very homogenous sample of 5277 early type stars and 325 classical type I Cepheids.**

99.4 % of the sample in  $4,9 \leq r \leq 15,8$  kpc (a range of 11 kpc) and below 1 kpc from the galactic plane (characteristic scale height for the validity of the BG model)

*to date the best angular-momentum sustained stellar population of the Milky Way that better traces its observed RC!*

# MCMC fit to the Gaia DR2 data - Classical (MWC) and GR (BG) RC

Best fit estimates as the median of the posteriors and their 1 $\sigma$  level credible interval

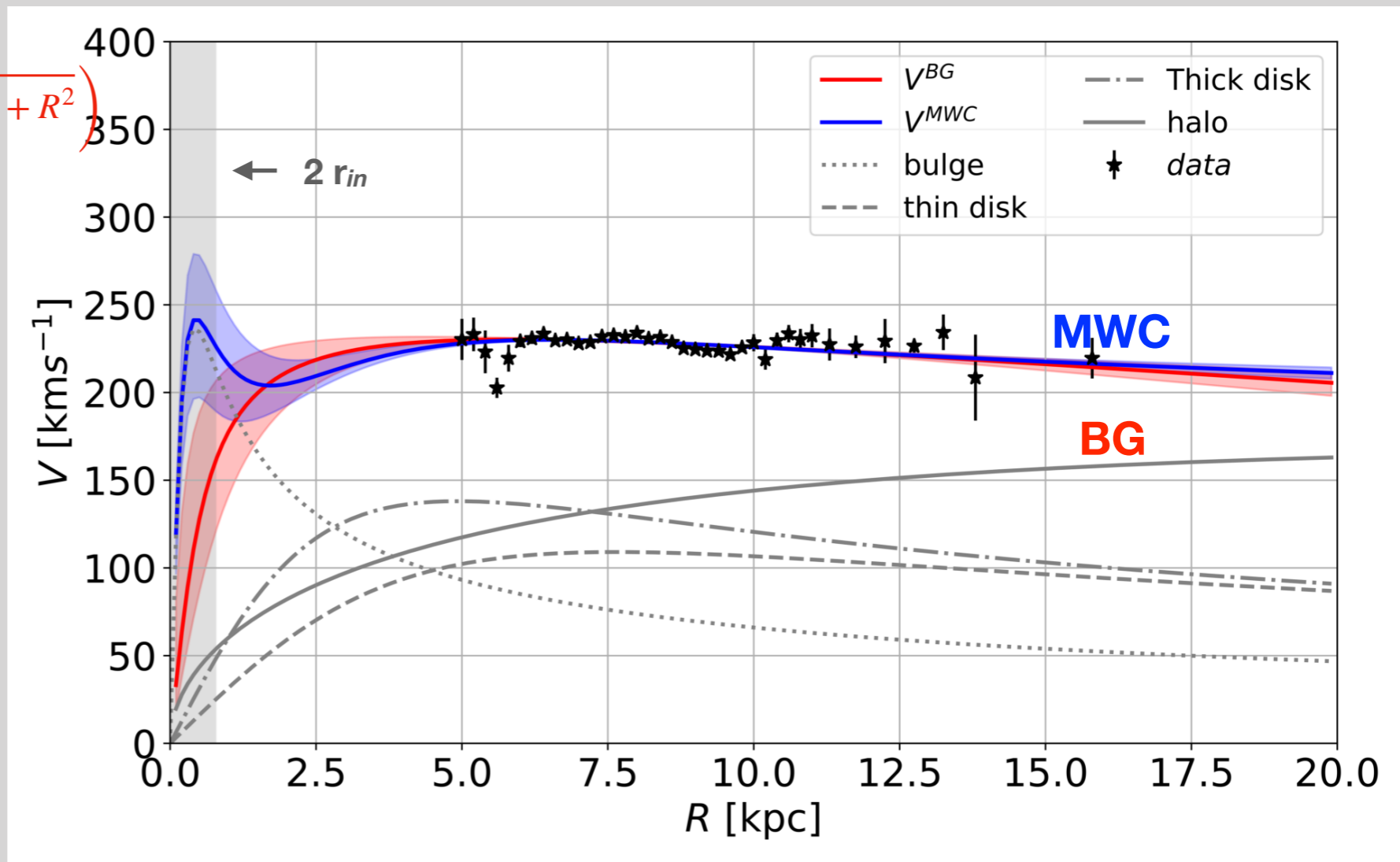
$$V_{\phi}^{BG}(R) = \frac{V_0}{R} \left( R_{out} - r_{in} + \sqrt{r_{in}^2 + R^2} - \sqrt{R_{out}^2 + R^2} \right)$$

BG model	$\theta$	$\sigma_{\theta}^{-}$	$\sigma_{\theta}^{+}$
$r_{in}$ [kpc]	0.39	<b>b<sub>b</sub>=0.3 kpc!</b>	+0.36
$R_{out}$ [kpc]	47.87	-14.80	+23.96
$V_0$ [km/s]	263.10	-16.44	+25.93
$e^{v_0}$	0.083	-0.014	+0.014

MWC model	$\theta$	$\sigma_{\theta}^{-}$	$\sigma_{\theta}^{+}$
$M_b$ [ $10^{10} M_{\odot}$ ]	1.0	-0.4	+0.4
$M_{td}$ [ $10^{10} M_{\odot}$ ]	3.9	-0.4	+0.4
$M_{Td}$ [ $10^{10} M_{\odot}$ ]	4.0	-0.5	+0.5
$a_{td}$ [kpc]	5.2	-0.5	+0.5
$a_{Td}$ [kpc]	2.7	-0.4	+0.4
$\rho_0^{halo}$ [ $M_{\odot} pc^{-3}$ ]	0.009	-0.003	+0.004
$A_h$ [kpc]	17	-3	+4

**Both models fit the data!**

Colored area= reliability intervals of the fitted curves



For our likelihood analysis the two models appear almost identically consistent with the data.

**Weak field GR off-diagonal term mimic DM in MW!**

Ref: On testing CDM and geometry-driven Milky Way rotation curve models with *Gaia* DR2- Crosta M., Giammaria M., Lattanzi M. G., Poggio E., MNRAS, Volume 496, Issue 2, August 2020, Pages 2107–2122

For both models, the errors due to the Bayesian analyses are at least one order of magnitude lower than the resulting uncertainties of the parameters.

For the BG free parameters uniform prior distributions (first general relativistic model fitted to data)

For MWC normal prior distributions (comparison of our bayesian analysis with the most recent observational estimates)

# The baryonic density profile via Einstein field eq.

According to the relativistic model  
 $0.083 \pm 0.006$   
**solar masses/cubic parsec**

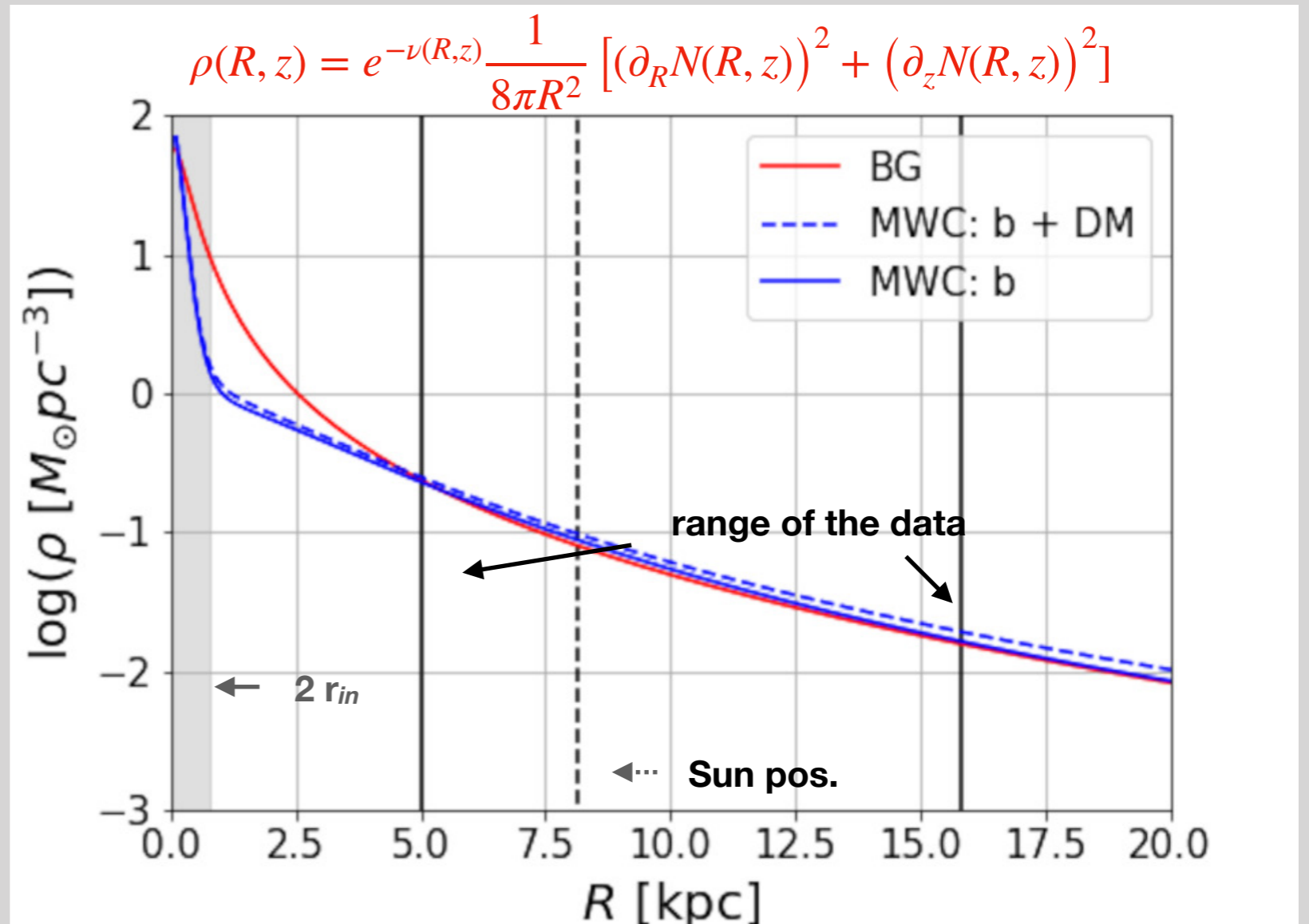
**In agreement**, with current independent estimates

$0.077 \pm 0.007 M_{\text{sun}} \text{pc}^{-3}$   
 (Bienayme et al. 2014, A&A, 571)

$0.084 \pm 0.012 M_{\text{sun}} \text{pc}^{-3}$   
 (McKee et al. 2015, ApJ, 814, 13)

$0.098 \pm 0.006 M_{\text{sun}} \text{pc}^{-3}$   
 (Garbari et al. 2012MNRAS, 425, 1445)

As expected in the disk region ( $z \sim 0$ ), for MWC the dominant matter is baryonic, while DM is a minor component there, i.e.  
 $\rho_{\text{DM}} \sim 0.01 M_{\odot} \text{pc}^{-3}$



Density profile of the MW at  $z=0$  derived from 100 random draws from the posterior distribution of the fit

$$\log \mathcal{L} = -\frac{1}{2} \sum_i \left( \frac{[V_\phi(R_i) - V_\phi^{exp}(R_i|\theta)]^2}{\sigma_{V_\phi}^2} + \log(\sigma_{V_\phi}^2) \right) - \frac{1}{2} \left( \frac{[\rho(R_\odot) - \rho^{exp}(R_\odot|\theta)]^2}{\sigma_{\rho_\odot}^2} + \log(\sigma_{\rho_\odot}^2) \right)$$

# Dragging effect vs. halo effect

The relativistic dragging effect has no newtonian counterpart, thus we compared:

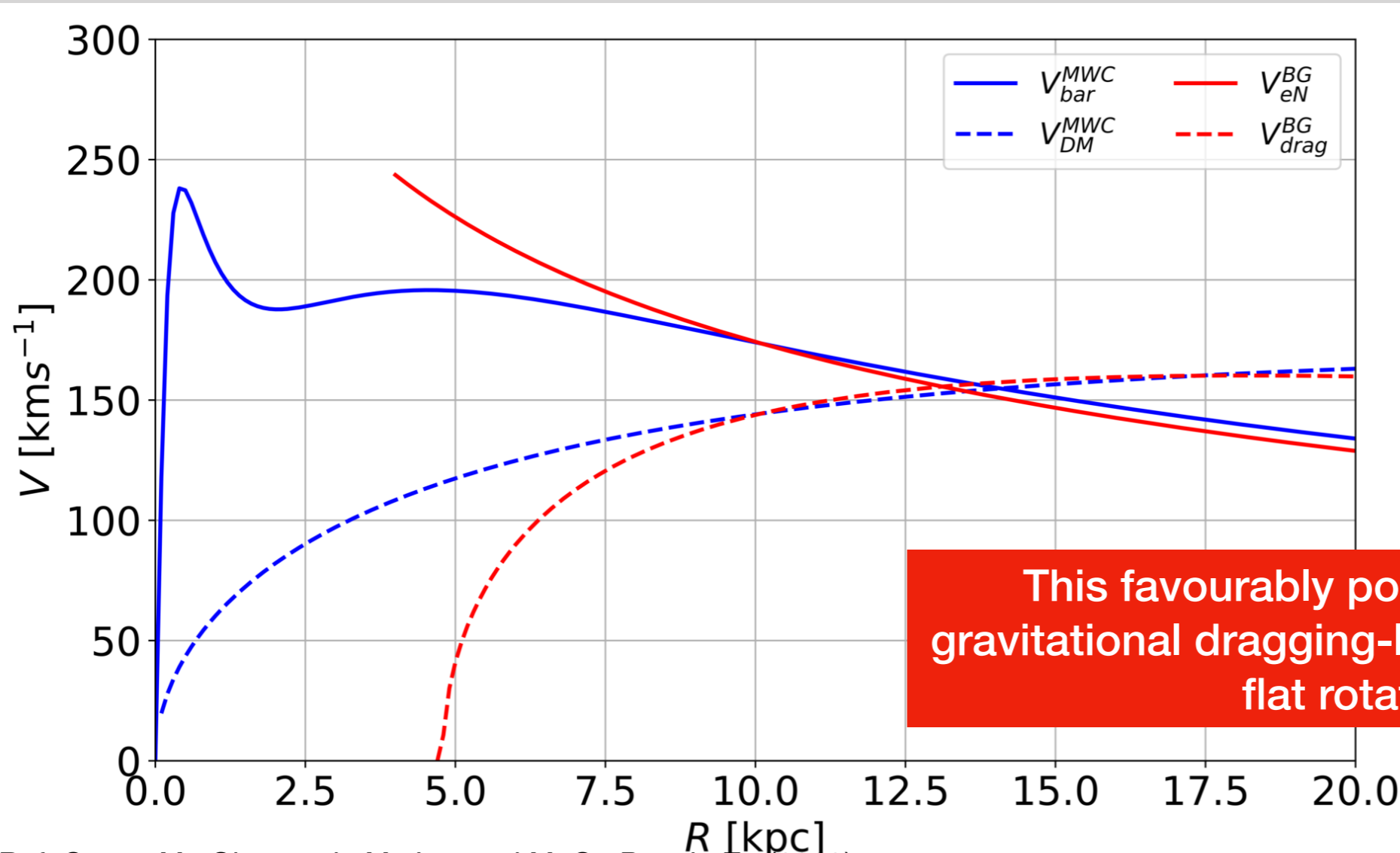
- (i) the MWC baryonic-only contribution with the effective Newtonian profile (Binney & Tremaine 1988) calculated by using the BG density:  $V_{eN}^{BG}$
- (ii) the MWC dark matter-only contribution (halo) with the "dragging curve" traced by subtracting  $V_{eN}^{BG}$  to  $V_{BG}$

$$\sum_i (V_{eN}^{BG}(R_i, k) - V_{eN}^{MWC}(R_i))^2 / N \quad |z_k| < r_{in}$$

For the effective BG disk half-thickness  $|z|_{eff}$ , the minimization process yields  **$|z|_{eff}=0.215\text{kpc}$**   $b_{td} = 0.25 \text{ kpc!}$

$$V_{drag}^{BG}(R_i; |z|_{eff}) = \sqrt{(V^{BG}(R))^2 - (V_{eN}^{BG}(R; |z|_{eff}))^2}$$

amount of rotational velocity across the MW plane due to gravitational dragging



$R < 5 \text{ kpc}$  could be the breaking point for the direct applicability of the BG model to the Milky Way, as it calls for a more suitable relativistic description of its central regions

This favourably points to the fact that a gravitational dragging-like effect could sustain a flat rotation curve

# ***Hypotheses non fingo & Occam's razor?***

Our interpretation of the fitted relativistic velocity profile with Gaia DR2  
**depends only on the background geometry**

**DM:** does not absorb or emit light but it exerts and responds only to the gravity force; it enters the calculation as extra mass (halo) required to justify the flat galactic rotational curves.

**GR:** a gravitational dragging "DM-like" effect driving the Galaxy velocity rotation curve could imply that geometry - unseen but perceived as manifestation of gravity according to Einstein's equation - is responsible of the flatness at large Galactic radii.

By setting a coherent GR framework, one can effectively establish

*“ Mass tells space how to curve and  
space tells mass how to move ”*

i.e. to what extent the MW structure is dictated by the standard theory of gravity

the “ether” was cured by a new kinematics (i.e. special relativity) instead of “new” dynamic as inspired by the FitzGerald-Lorentz contraction phenomena (“extra molecular force”)

*“We know that electric forces are affected by the motion of the electrified bodies relative to the ether and it seems a not improbable supposition that the molecular forces are affected by the motion and that the size of the body alters consequently.”* FitzGerald, Science, 1889



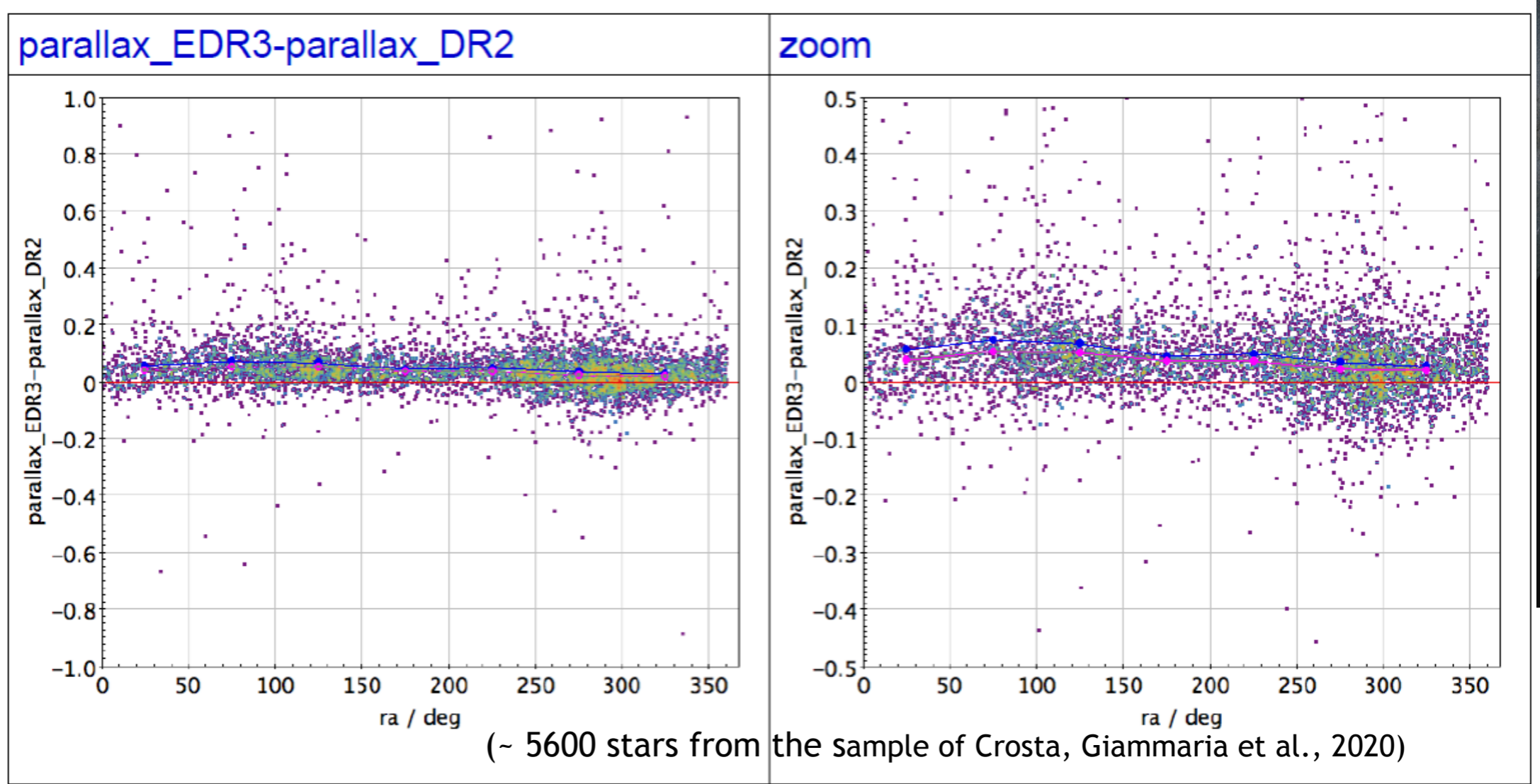
# Gaia EDR3 - Milky Way

Milky Way (MW) as a product of a cosmological evolution at  $z=0$  ?

Early Data Release 3  
34  
1.8 billion  
b

It also includes  
although for "only  
tempera

Image credit: ESA/Gaia  
Image license: CC BY-SA



# Next improvements

In 2022, at the time of the Gaia 3rd release, DR3, extension of test with the rotation curve by another 2-4 kpc (including both sides, inner and outer, of the MW disk). The Local Cosmology group in INAF-OATo (Lattanzi, Re Fiorentin, Bucciarelli, Poggio, Spagna, Drimmel, Vecchiato) is focusing on:

## For the observational side

- Increase the sample: Gaia eDR3/DR3 (2022) + spectroscopic surveys (e.g. SDSS, APOGEE, LAMOST, RAVE, GES - Gaia ESO Survey, GALAH)
- Match with observations toward the Galactic center
- Expected sample size to increase from current 6000 to more than 100 thousands upper main sequence disc stars, with the addition of early-type B stars.

## For the theoretical side

- Improve the model: new solutions & new observables of the Einstein Field Equation (i.e. metric solutions to describe the Galaxy); a more consistent mathematical solution of a relativistic velocity profile; a study, e.g., of the class of Lewis and Papapertou metrics in attempt to encompass all the different MW structures and to fit different conformal factors with the Gaia data (as we did for the density in BG case)
- Extend the MW “geometry” to other galaxies, including also relativistic kinematic (e.g. acceleration versus MOND)
- Comparison with N-body (cosmological) simulations also with numerical relativity (e.g. Einstein-Vlasov system solvers). The use of Gaia data must be parallel with the utilisation of the most advanced cosmological simulations with baryonic matter (gas and stars)

**With more physically appropriate metrics, along with adequate solution, the Galaxy can play a reference role for other galaxies, much like the Sun for stellar models**

**Stay tuned!**

**Thank you for your attention**