FIAS Frankfurt Institute for Advanced Studies

Ben-Gurion University of the Negev

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SIXTEENTH MARCEL GROSSMANN MEETING

ON RECENT DEVELOPMENTS IN THEORETICAL AND EXPERIMENTAL GENERAL RELATIVITY, ASTROPHYSICS AND RELATIVISTIC FIEL THEORIES

Covariant Canonical Gauge Theory of Gravity

David Benisty

In a collaboration with D.Vasak and J.Struckmeier

UNIVERSITY OF CAMBRIDGE

Fundamental features of geometry

- The metric $g^{\alpha\beta}$ - the infinitesimal distance between two points:

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

- The connection $\Gamma^\lambda_{\alpha\beta}$ - a curve with covariantly constant tangent vector:

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0$$

- No Torsion :

$$\Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{\beta\alpha}$$

$$\Gamma^\alpha{}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + K^\alpha{}_{\mu\nu} + L^\alpha{}_{\mu\nu}$$

Metric Formalism vs. Palatini Formalism

Metric formalism:

- Dependence only on the metric and its derivatives.
- Christoffel Symbol $\{^{\rho}_{\mu\nu}\} = \frac{1}{2} g^{\rho\lambda}(g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda})$

Palatini Formalism:

- The connection is being an independent degree of freedom in the action.

Is there any correspondence between those formulations?

Metric Formalism

- Field equation

$\frac{1}{2}\boldsymbol{R^2}$:

$$R\left(R^{\mu\nu}-\frac{1}{4}g^{\mu\nu}R\right)-\nabla^{\mu}\nabla^{\nu}R+g^{\mu\nu}\ \nabla^{2}R$$

$\frac{1}{2}\boldsymbol{R_{\mu\nu}R^{\mu\nu}}$:

$$R^{\mu\alpha}R_{\alpha}^{\nu}-\frac{1}{4}g^{\mu\nu}\left(R_{\alpha\beta}R^{\alpha\beta}\right)-\frac{1}{2}\nabla_{\gamma}(\nabla^{\mu}R^{\nu\gamma}+\nabla^{\nu}R^{\mu\gamma})+\frac{1}{2}\nabla^{2}R^{\mu\nu}+\frac{1}{4}\nabla^{2}Rg^{\mu\nu}$$

$\frac{1}{2}\boldsymbol{R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}}$:

$$R^{\mu\alpha\beta\gamma}R_{\alpha\beta\gamma}^{\nu}-\frac{1}{4}g^{\mu\nu}(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})+\left(\nabla_{\alpha}\nabla_{\beta}+\nabla_{\beta}\nabla_{\alpha}\right)R^{\mu\alpha\nu\beta}$$

Palatini formalism

- Field equation:

$\frac{1}{2}\boldsymbol{R^2}$:

$$R\left(R^{\mu\nu}-\frac{1}{4}g^{\mu\nu}R\right)$$

$\frac{1}{2}\boldsymbol{R_{\mu\nu}R^{\mu\nu}}$:

$$R^{\mu\alpha}R_{\alpha}^{\nu}-\frac{1}{4}g^{\mu\nu}(R_{\alpha\beta}R^{\alpha\beta})$$

$\frac{1}{2}\boldsymbol{R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}}$:

$$R^{\mu\alpha\beta\gamma}R_{\alpha\beta\gamma}^{\nu}-\frac{1}{4}g^{\mu\nu}(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})$$

- Γ variation:

$$K^{\mu\nu}_{\lambda(\alpha)}=(g^{\mu\nu}\nabla_{\lambda}-\frac{1}{2}\delta^{\nu}_{\lambda}\nabla^{\mu}-\frac{1}{2}\delta^{\mu}_{\lambda}\nabla^{\nu})R$$

$$K^{\mu\nu}_{\lambda(\beta)}=\nabla_{\lambda}R^{\mu\nu}-\frac{1}{4}\delta^{\mu}_{\lambda}\nabla^{\nu}R-\frac{1}{4}\delta^{\nu}_{\lambda}\nabla^{\mu}R$$

$$K^{\mu\nu}_{\lambda(\beta)}=\nabla_{\sigma}R_{\lambda}^{\mu\sigma\nu}+\nabla_{\sigma}R_{\lambda}^{\nu\sigma\mu}$$

A difference in squared gravity terms

- Palatini Formalism:

$\frac{1}{2}\boldsymbol{R^2}$:

$$R\left(R^{\mu\nu} - \frac{1}{4}g^{\mu\nu}R\right)$$

$\frac{1}{2}\boldsymbol{R_{\mu\nu}R^{\mu\nu}}$:

$$R^{\mu\alpha}R_{\alpha}^{\nu} - \frac{1}{4}g^{\mu\nu}(R_{\alpha\beta}R^{\alpha\beta})$$

$\frac{1}{2}\boldsymbol{R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}}$:

$$R^{\mu\alpha\beta\gamma}R_{\alpha\beta\gamma}^{\nu} - \frac{1}{4}g^{\mu\nu}(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})$$

- Contribution for the metric formalism:

$$-\nabla^{\mu}\nabla^{\nu}R + g^{\mu\nu}\,\nabla^2 R$$

$$-\frac{1}{2}\nabla_{\gamma}(\nabla^{\mu}R^{\nu\gamma} + \nabla^{\nu}R^{\mu\gamma}) + \frac{1}{2}\nabla^2 R^{\mu\nu} + \frac{1}{4}\nabla^2 R$$

$$+(\nabla_{\alpha}\nabla_{\beta} + \nabla_{\beta}\nabla_{\alpha})R^{\mu\alpha\nu\beta}$$

The metricity constraint

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David Benisty, Eduardo Guendelman

The theorem:

$$\mathcal{L}(g)_{\,2^{\text{order}}} \Leftrightarrow \mathcal{L}(g,\Gamma) + k^{\alpha\beta\gamma} g_{\alpha\beta;\gamma\ 1^{\text{order}}}$$

- $\delta k^{\alpha\beta\gamma}$: $\quad g_{\alpha\beta;\gamma} = 0 \quad \Rightarrow \quad \Gamma^{\rho}_{\mu\nu} = \{^{\rho}_{\mu\nu}\}$
- $\delta \Gamma^{\alpha}_{\beta\gamma}$: $\quad \frac{\delta \mathcal{L}(\kappa)}{\delta \Gamma^{\rho}_{\mu\nu}} = -k^{\alpha\mu\nu} g_{\rho\alpha} - k^{\alpha\nu\mu} g_{\rho\alpha}$
- $\delta g^{\mu\nu}$: $\quad G^{\mu\nu}_{(\kappa)} = \frac{\delta \mathcal{L}(\kappa)}{\delta g_{\mu\nu}} = -k^{\mu\nu\lambda}_{;\lambda}$

The contribution for the metric E.o.M

$$g^{\rho\sigma}\frac{\partial\mathcal{L}(\kappa)}{\partial\Gamma^{\rho}_{\mu\nu}} = -k^{\sigma\mu\nu} - k^{\sigma\nu\mu}$$

$$g^{\rho\nu}\frac{\partial\mathcal{L}(\kappa)}{\partial\Gamma^{\rho}_{\mu\sigma}} = -k^{\nu\mu\sigma} - k^{\nu\sigma\mu}$$

By changing the indices:

$$g^{\rho\mu}\frac{\partial\mathcal{L}(\kappa)}{\partial\Gamma^{\rho}_{\nu\sigma}} = -k^{\mu\nu\sigma} - k^{\mu\sigma\nu}$$

- The new contribution:

$$\frac{\delta\mathcal{L}(\kappa)}{\delta g_{\sigma\nu}} = \frac{1}{2}\nabla_{\mu}\left(g^{\rho\sigma}\frac{\partial\mathcal{L}(\kappa)}{\partial\Gamma^{\rho}_{\mu\nu}} + g^{\rho\nu}\frac{\partial\mathcal{L}(\kappa)}{\partial\Gamma^{\rho}_{\mu\sigma}} - g^{\rho\mu}\frac{\partial\mathcal{L}(\kappa)}{\partial\Gamma^{\rho}_{\nu\sigma}}\right)$$

A correspondence for squared gravity terms

$$G^{\mu\nu}_{(\gamma)\ 2\mathbf{order}} = G^{\mu\nu}_{(\gamma)\ 1\mathbf{order}} + G^{\mu\nu}_{(\kappa)}$$

$$\frac{\delta \mathcal{L}(\kappa)}{\delta g_{\sigma\nu}} = \frac{1}{2}\nabla_\mu\left(g^{\rho\sigma}\frac{\partial \mathcal{L}(\kappa)}{\partial \Gamma^\rho_{\mu\nu}} + g^{\rho\nu}\frac{\partial \mathcal{L}(\kappa)}{\partial \Gamma^\rho_{\mu\sigma}} - g^{\rho\mu}\frac{\partial \mathcal{L}(\kappa)}{\partial \Gamma^\rho_{\nu\sigma}}\right)$$

$$K^{\mu\nu}_{\lambda(\alpha)} = \left(g^{\mu\nu}\nabla_\lambda - \frac{1}{2}\delta^\nu_\lambda\nabla^\mu - \frac{1}{2}\delta^\mu_\lambda\nabla^\nu\right)R$$

$$G^{\mu\nu}_{(\alpha)\ 2\mathbf{order}} = G^{\mu\nu}_{(\alpha)\ 1\mathbf{order}} + \alpha\left[-\nabla^\mu\nabla^\nu R + g^{\mu\nu}\Box R\right]$$

$$K^{\mu\nu}_{\lambda(\beta)} = \nabla_\lambda R^{\mu\nu} - \frac{1}{4}\delta^\mu_\lambda\nabla^\nu R - \frac{1}{4}\delta^\nu_\lambda\nabla^\mu R$$

$$G^{\mu\nu}_{(\beta)\ 2\mathbf{order}} = G^{\mu\nu}_{(\beta)\ 1\mathbf{order}} + \beta\left[-\frac{1}{2}\nabla_\gamma(\nabla^\mu R^{\nu\gamma} + \nabla^\nu R^{\mu\gamma}) + \frac{1}{2}\Box R^{\mu\nu} + \frac{1}{4}\Box R\right]$$

$$K^{\mu\nu}_{\lambda(\beta)} = \nabla_\sigma R_\lambda^{\ \mu\sigma\nu} + \nabla_\sigma R_\lambda^{\ \nu\sigma\mu}$$

$$G^{\mu\nu}_{(\gamma)\ 2\mathbf{order}} = G^{\mu\nu}_{(\gamma)\ 1\mathbf{order}} + \gamma\left[(\nabla_\alpha\nabla_\beta + \nabla_\beta\nabla_\alpha)R^{\mu\alpha\nu\beta}\right]$$

A path integral approach

First order Formalism

$$\mathcal{Z} = \int \mathcal{D}k\mathcal{D}g\mathcal{D}\Gamma e^{i\int d^4x\sqrt{-g}(\mathcal{L}(g,\Gamma)+k^{\alpha\beta\gamma}g_{\alpha\beta;\gamma})}$$

$$= \int \delta(g_{\alpha\beta;\gamma})\mathcal{D}g\mathcal{D}\Gamma e^{i\int d^4x\sqrt{-g}\mathcal{L}(g,\Gamma)}$$

$$\sim \int \delta(\Gamma^{\rho}_{\mu\nu} - \{^{\rho}_{\mu\nu}\})\mathcal{D}g\mathcal{D}\Gamma e^{i\int d^4x\sqrt{-g}\mathcal{L}(g,\Gamma)}$$

$$= \int \mathcal{D}g\, e^{i\int d^4x\sqrt{-g}\mathcal{L}(g)}$$

Second order Formalism

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J. Struckmeier, J. Muench, D. Vasak, J. Kirsch, M. Hanauske, and H. Stoecker

Hamiltonian for dynamic spacetime

$k^{\alpha\beta\gamma}$ as a metric conjugate momenta:

$$\tilde{L} = \tilde{k}^{\alpha\beta\gamma} g_{\alpha\beta,\gamma} - \widetilde{H}$$

For a gauge theory that includes general mapping of space time $x \to X$, we identify the connection $\gamma^{\eta}_{\alpha\beta}$ as the gauge field of these C.T.

Condition for canonical transformation under a dynamical spacetime.

$$S = \int (\tilde{k}^{\alpha\beta\gamma} g_{\alpha\beta,\gamma} - \tilde{q}_{\alpha}^{\beta\gamma\delta} \gamma^{\alpha}_{\beta\gamma,\delta} - \widetilde{H} - \tilde{F}^{\beta}_{1,\beta}) \; d^4x$$
$$= \int (\widetilde{K}^{\alpha\beta\gamma} G_{\alpha\beta,\gamma} - \frac{1}{2} \tilde{Q}_{\alpha}^{\beta\gamma\delta} \Gamma^{\alpha}_{\beta\gamma,\delta} - \widetilde{H}') d^4X$$

$\tilde{F}_1^{\beta}$ is the generating function.

Gauge Hamiltonian

$$F_3^{\mu} = -k^{\alpha\beta\mu} G_{\xi\lambda} \frac{\partial X^{\xi}}{\partial x^{\alpha}} \frac{\partial X^{\lambda}}{\partial x^{\beta}} - q_{\eta}^{\alpha\beta\mu} \left(\Gamma_{\xi\lambda}^{\tau} \frac{\partial x^{\eta}}{\partial X^{\tau}} \frac{\partial X^{\xi}}{\partial x^{\alpha}} \frac{\partial X^{\lambda}}{\partial x^{\beta}} + \frac{\partial x^{\eta}}{\partial X^{\tau}} \frac{\partial X^{\tau}}{\partial x^{\alpha} \partial x^{\beta}} \right)$$

Yields the transformation rules for the metric:

$$G_{\mu\nu} = g_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial X^{\nu}} \frac{\partial x^{\beta}}{\partial X^{\mu}},$$

and the connection:

$$\Gamma_{\alpha\beta}^{\kappa} = \gamma_{\eta\tau}^{\xi} \frac{\partial x^{\eta}}{\partial X^{\tau}} \frac{\partial X^{\xi}}{\partial x^{\alpha}} \frac{\partial X^{\lambda}}{\partial x^{\beta}} + \frac{\partial x^{\xi}}{\partial X^{\alpha} \partial X^{\beta}} \frac{\partial X^{\kappa}}{\partial x^{\xi}}$$

rotation

In-homogeneous part

$$L = k^{\alpha\beta\gamma} g_{\alpha\beta;\gamma} - \frac{1}{2} q_{\alpha}^{\beta\gamma\delta} R_{\beta\gamma\delta}^{\alpha} - H_{Dyn}(g, \mathrm{q}, \mathrm{k}) + L_m$$

The equations of motion

$$L = k^{\alpha\beta\gamma} g_{\alpha\beta;\gamma} - \frac{1}{2} q_{\alpha}^{\beta\gamma\delta} R^{\alpha}_{\beta\gamma\delta} - H_{Dyn}(g, \mathrm{q}, \mathrm{k}) + L_m$$

$$k^{\alpha\beta\gamma} : \; g_{\alpha\beta;\gamma} = \frac{\partial H_{Dyn}}{\partial k^{\alpha\beta\gamma}}$$

$$\gamma^{\mu}_{\nu\rho}: \; (k^{\alpha\mu\nu} + k^{\alpha\nu\,\mu}) g_{\alpha\rho} = -\frac{1}{2} \nabla_{\beta} \left(q_{\rho}^{\mu\beta\nu} + q_{\rho}^{\nu\beta\mu} \right)$$

$$q^{\sigma}_{\mu\nu\rho}: \; -\frac{1}{2} R^{\sigma}_{\mu\nu\rho} = \frac{\partial H_{Dyn}}{\partial q^{\sigma}_{\mu\nu\rho}}$$

$$g^{\mu\nu}: \; T^{\mu\nu} = g^{\mu\nu} \left(-k^{\alpha\beta\gamma} g_{\alpha\beta;\gamma} + \frac{1}{2} q_{\alpha}^{\beta\gamma\delta} R^{\alpha}_{\beta\gamma\delta} \right) + 2k^{\mu\nu\lambda}_{;\lambda} - \frac{2}{\sqrt{-g}} \frac{\partial \widetilde{H}_{Dyn}}{\partial g^{\mu\nu}}$$

$$2k^{\mu\nu\lambda}_{;\lambda} = -\nabla_{\gamma} \nabla_{\alpha} (q^{\mu\gamma\nu\alpha} + q^{\nu\gamma\mu\alpha})$$

Sample H_{Dyn}

$$L = k^{\alpha\beta\gamma} g_{\alpha\beta;\gamma} - \frac{1}{2} q_{\alpha}^{\beta\gamma\delta} R^{\alpha}_{\beta\gamma\delta} - H_{Dyn}(g, \mathrm{q}, \mathrm{k})$$

For a Hamiltonian with a quadratic momentum q:

$$\tilde{\mathcal{H}}_{\mathrm{Dyn}} = \frac{1}{4g_1} \tilde{q}_{\eta}{}^{\alpha\xi\beta} \tilde{q}_{\alpha}{}^{\eta\tau\lambda} g_{\xi\tau} g_{\beta\lambda} \frac{1}{\sqrt{-g}} - g_2 \tilde{q}_{\eta}{}^{\alpha\eta\beta} g_{\alpha\beta} + g_3 \sqrt{-g}.$$

The equation of motions: (k,Γ,q,g)

k:

$$g_{\alpha\beta;\gamma} = -\frac{\partial \tilde{H}_{\mathbf{Dyn}}}{\partial \tilde{k}^{\alpha\beta\gamma}} = 0$$

q:

$$q_{\eta\alpha\xi\beta} = g_1 (R_{\eta\alpha\xi\beta} - R_{\eta\alpha\xi\beta}|_{\mathrm{max}}),$$

$$R_{\eta\alpha\xi\beta}|_{\mathrm{max}} = g_2 (g_{\eta\xi} g_{\alpha\beta} - g_{\eta\beta} g_{\alpha\xi})$$

D. Benisty E. I. Guendelman
D.Vasak J.Struckmeier

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Γ: $$\frac{1}{2}\nabla_\mu(\tilde{q}_\beta^{\alpha\eta\mu} + \tilde{q}_\beta^{\eta\alpha\mu}) = -(\tilde{k}^{\lambda\alpha\eta} + \tilde{k}^{\lambda\eta\alpha})g_{\lambda\beta}$$

The energy momentum tensor:

$$T^{\mu\nu} = \frac{1}{\kappa^2}G^{\mu\nu} + g_1 Q^{\mu\nu} + g_1 S^{\mu\nu} + g^{\mu\nu}\Lambda$$

where:

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$$

$$\boldsymbol{L = g_1 R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} + \frac{1}{2}R + \Lambda}$$

$$Q^{\mu\nu} = R^{\mu\alpha\beta\gamma}R^{\nu}_{\alpha\beta\gamma} - \frac{1}{4}g^{\mu\nu}R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$$

$$g_1 = \frac{3}{16\pi G\Lambda}, \qquad g_2 = \frac{1}{3}\Lambda.$$

$$S^{\mu\nu} = (\nabla_\alpha\nabla_\beta + \nabla_\beta\nabla_\alpha)R^{\mu\alpha\nu\beta}$$

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David Benisty, Eduardo I. Guendelman, David Vasak, Jurgen Struckmeier, Horst Stoecker
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$H_{Dyn}\,(\mathrm{g},\mathrm{q})$ without breaking metricity

Similar definitions for q tensors as Ricci tensor and Ricci scalar:

$$q^{\lambda}_{\alpha\lambda\beta} = q_{\alpha\beta}\,,\;\; q = q_{\alpha\beta}\,g^{\alpha\beta}$$

Lead to the general combination of the covariant Hamiltonian:

$$H_{Dyn}(g,\mathrm{q}) = g_0 - \frac{g_1}{2}q - \frac{g_{21}}{4}q^2 - \frac{g_{22}}{4}q^{\mu\nu}q_{\mu\nu} - \frac{g_{23}}{4}q^{\lambda}_{\alpha\beta\gamma}\,q^{\alpha\beta\gamma}_{\lambda}$$

$$L = k^{\alpha\beta\gamma}g_{\alpha\beta;\gamma} - \frac{1}{2}q^{\beta\gamma\delta}_{\alpha}R^{\alpha}_{\beta\gamma\delta} - H_{Dyn}(g,\mathrm{q}) + L_m$$

The E.o.M behave as in the second order formalism, and the energy momentum tensor is conserved.

The equation of motions

$$H_{Dyn}(g,\mathrm{q}) = g_0 - \frac{g_1}{2}q - \frac{g_{21}}{4}q^2 - \frac{g_{22}}{4}q^{\mu\nu}q_{\mu\nu} - \frac{g_{23}}{4}q^{\lambda}_{\alpha\beta\gamma}q^{\alpha\beta\gamma}_{\lambda}$$

The E.o.M:

$\mathbf{k}^{\alpha\beta\gamma}$: $g_{\alpha\beta;\gamma} = \frac{\partial H_{Dyn}}{\partial k^{\alpha\beta\gamma}} = 0 \qquad \Rightarrow \qquad \gamma^{\mu}_{\nu\rho} = \begin{Bmatrix} \mu \\ \nu\rho \end{Bmatrix}$

$\mathbf{q}^{\sigma}_{\mu\nu\rho}$: $R^{\sigma}_{\mu\nu\rho} = \frac{\partial H_{Dyn}}{\partial q^{\sigma}_{\mu\nu\rho}} = g_{23}q^{\sigma}_{\mu\nu\rho} + \delta^{\rho}_{\nu}\left(g_{22}q_{\mu\rho} + g_{\mu\rho}(g_{21}q + g_1)\right)$

Trace δ^{ν}_{ρ}:

$$R_{\mu\rho} = (g_{23}+4g_{22})\,q_{\mu\rho} + 4g_{\mu\rho}(g_{21}q + g_1)$$

Another trace $g^{\mu\rho}$:

$$R = (g_{23}+4g_{22} + 16g_{21})\,q + 16g_1$$

The momentum of the connection

$$q^{\sigma}_{\mu\nu\rho} = \alpha_1 R^{\sigma}_{\mu\nu\rho} + \delta^{\rho}_{\nu}\left(\alpha_2 R_{\mu\rho} + g_{\mu\rho}(\alpha_3 q + \alpha_4)\right)$$

$$\alpha_1 = \frac{1}{g_{23}}$$

$$\alpha_2 = -\frac{1}{g_{23}}\frac{g_{22}}{g_{23} + 4g_{22}}$$

$$\alpha_3 = -\frac{1}{16g_{21} + 4g_{22} + g_{23}}\frac{g_{21}}{g_{23} + 4g_{22}}$$

$$\alpha_4 = -\frac{g_1}{16g_{21} + 4g_{22} + g_{23}}$$

Legendre Transform

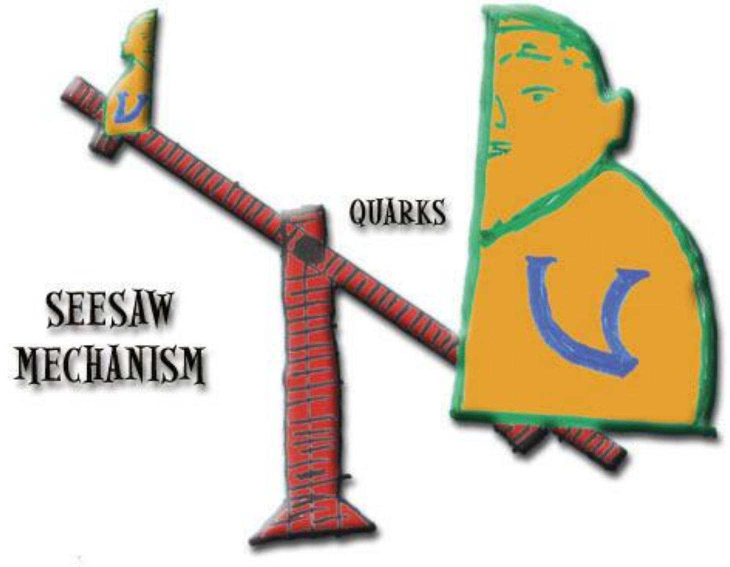
$$-\mathcal{L} = \frac{\alpha_1}{4} R^{\lambda}_{\alpha\beta\gamma} R_{\lambda}^{\alpha\beta\gamma} + \frac{\alpha_2}{4} R_{\mu\nu} R^{\mu\nu} + \frac{\alpha_3}{4} R^2 + \frac{\alpha_4}{4} R + \Lambda$$

The cosmological constant:

$$\Lambda = 4g_1\alpha_4 + g_0 = -\frac{g_1^2}{16g_{21} + 4g_{22} + g_{23}} + g_0$$

Cosmological seesaw mechanism

$$16g_{21} + 4g_{22} + g_{23} \gg 1$$



Gauss Bonnet

$$-\mathcal{L} = R^{\lambda}_{\alpha\beta\gamma} R_{\lambda}^{\alpha\beta\gamma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 - \frac{\alpha_4}{4} R - \Lambda$$

$$g_{21} = 1, \qquad g_{22} = -4, \qquad g_{23} = 15$$

Conformal gravity

$$\mathcal{L} = -\frac{1}{4} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}$$

$$g_{21} = 1, \qquad g_{22} = 10, \qquad g_{23} = -35$$

$$g_1 = g_0 = 0$$

Jose Beltr´an Jim´eneza, Lavinia Heisenberg, Tomi Koivisto
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Coincident General Relativity

$$\mathcal{Q} = g^{\mu\nu}\left(\left\{ {\alpha \atop \beta\mu} \right\} \left\{ {\beta \atop \nu\alpha} \right\} - \left\{ {\alpha \atop \beta\alpha} \right\} \left\{ {\beta \atop \mu\nu} \right\} \right).$$

$$\mathcal{S} = \int \mathrm{d}^n x \left[\frac{1}{2}\sqrt{-g}\mathbb{T} + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \lambda^\alpha{}_{\mu\nu} \nabla_\alpha g^{\mu\nu} \right]$$

$$\mathcal{S}_G = \int \mathrm{d}^n x \left[\frac{1}{2}\sqrt{-g} f(\mathcal{Q}) + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \lambda_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu} \right]$$

Summary

- Modified theories of gravity –

A connection between 2 formulations.

- Hamiltonian gauge theory of gravity –

Momenta for the metric and the connection.

- Quantum Gravity –

No higher derivatives in the action.