FIAS Frankfurt Institute for Advanced Studies

אוניברסיטת בן-גוריון בנגב
Ben-Gurion University of the Negev

MG16 5-10 JULY 2021
SIXTEENTH MARCEL GROSSMANN MEETING
ON RECENT DEVELOPMENTS IN THEORETICAL AND EXPERIMENTAL GENERAL RELATIVITY, ASTROPHYSICS AND RELATIVISTIC FIEL THEORIES

Unified DE-DM from Dynamical Space Time Cosmology

David Benisty

Under the Supervision of Prof. Eduardo Guendelman

Based on the papers:

- **Unified dark energy and dark matter from dynamical spacetime**

Phys.Rev. D98 (2018) no.2, 023506

- **Dark energy and dark matter unification from dynamical space time: observational constraints and cosmological implications**

JCAP 1906 (2019) 003

- **Inflation compactification from dynamical spacetime**

Phys.Rev. D98 (2018) no.4, 043522

- **Interacting Diffusive Unified Dark Energy and Dark Matter from Scalar Fields**

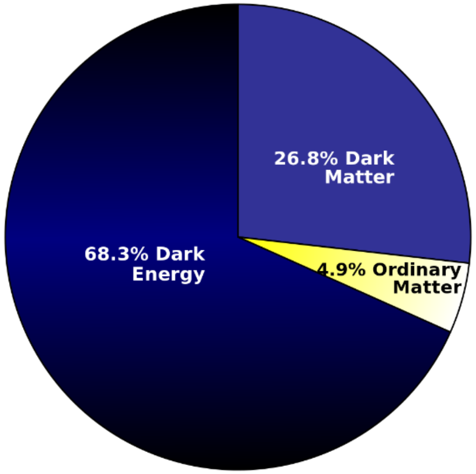
Eur.Phys.J. C77 (2017) no.6, 396

- **Unification of dark energy and dark matter from diffusive cosmology**

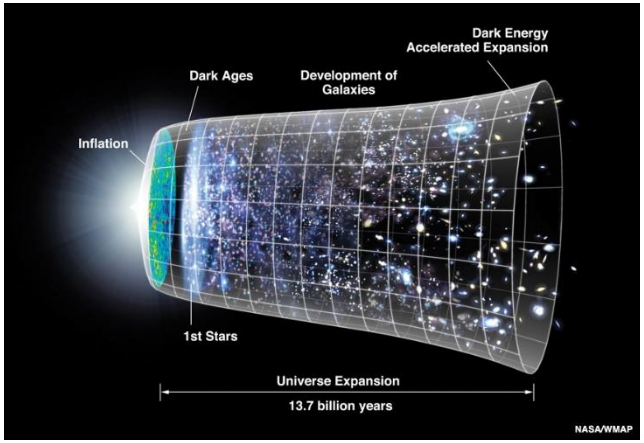
Phys.Rev. D99 (2019) no.12, 123521

Dark Energy & Dark Matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \Lambda g_{\mu\nu} + T_{\mu\nu}$$



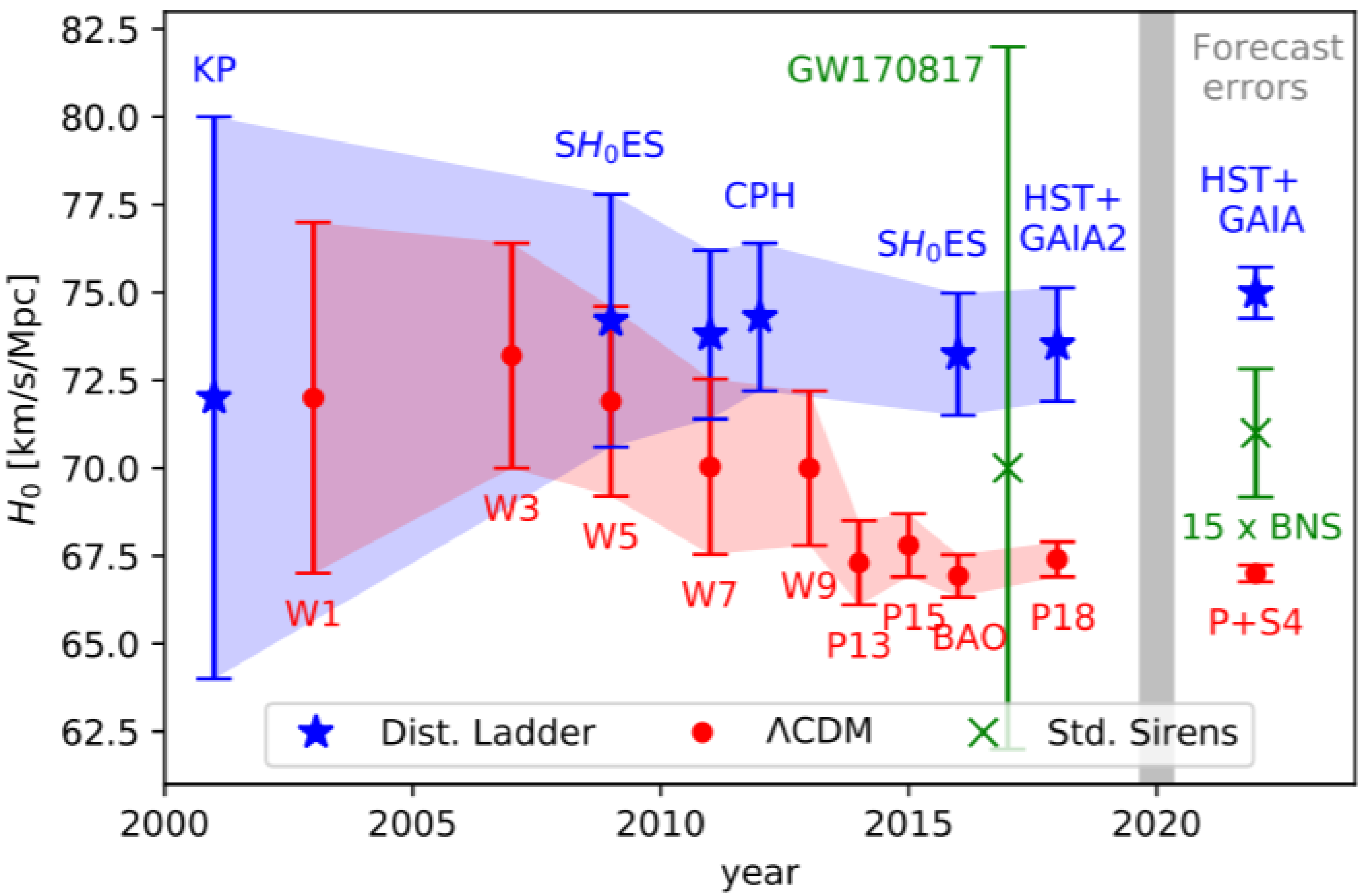
- Why is the observed value so many orders of magnitude smaller than that expected in QFT?



- Why is it of the same order of magnitude as the matter density of the universe at the present time?

Hubble Constant Tension

A sign for Dynamical DE?



Quintessence

PHYSICAL REVIEW D 97, 043503 (2018)
Jean-Baptiste Durrive, Junpei Ooba,
Kiyotomo Ichiki, Naoshi Sugiyama

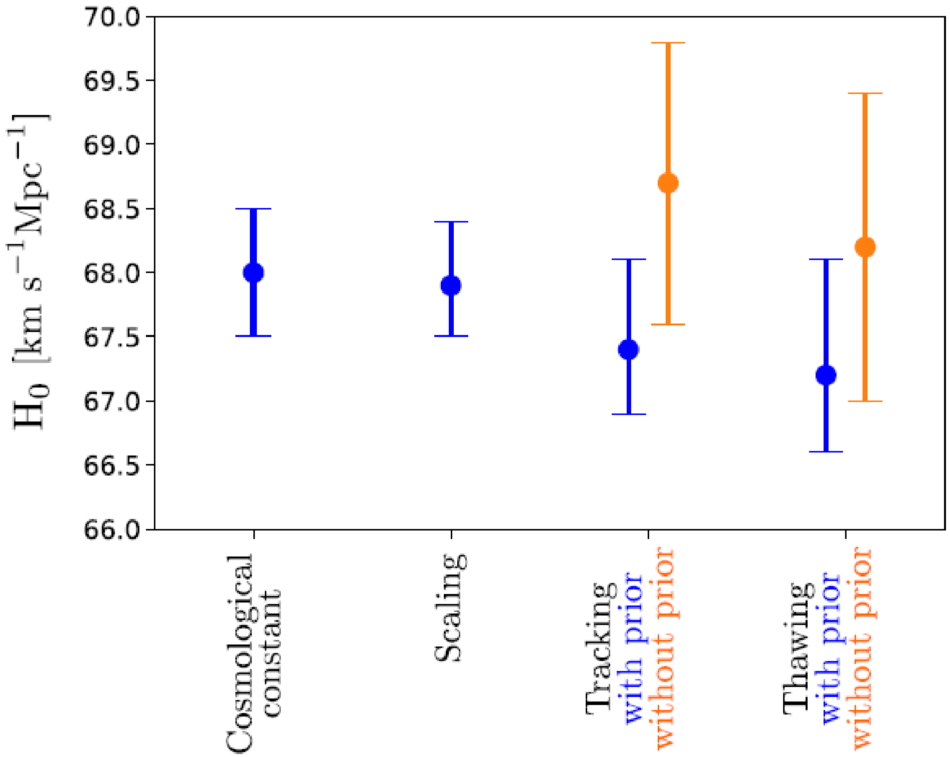
- May describe dynamical dark energy

Ratra, P.; Peebles, L. (1988)
Physical Review D. 37 (12) 3406

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_{\rm m},$$

$$\begin{cases} P_\phi = \frac{\dot\phi^2}{2} - V(\phi) \\ \rho_\phi = \frac{\dot\phi^2}{2} + V(\phi). \end{cases}$$

$$\begin{cases} 3H^2M_{\rm pl}^2 = \rho_\phi + \rho_{\rm m} \\ \ddot\phi + 3H\dot\phi + V_{,\phi} = 0 \\ \dot\rho_m + 3H\rho_{\rm m} = 0 \end{cases}$$



DOI: 10.1103/PhysRevD.81.043520
Gao, Martin Kunz,
Andrew R. Liddle, David Parkinson

Unified DE&DM different from quintessence

- A basic stress energy tensor unified DE – DM from a single scalar field.

$$T^{\mu\nu} = -\frac{1}{2}\phi^{,\mu}\phi^{,\nu} + U(\phi)g^{\mu\nu}$$

Demanding $\nabla_\mu T^{\mu\nu} = 0$ gives:

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + U'(\phi) = 0$$

- For $U(\phi) = Const$, and in FRW metric:

$$\dot{\phi} \sim \frac{1}{a^{3/2}}$$

DUST + DARK ENERGY

Exponential Potential

- The density equations:

$$3H^2 = \kappa^2[\tfrac{1}{2}\dot{\phi}^2 + \Lambda(\phi) + \rho_r + \rho_b],$$

$$U(\phi) = U_0 \exp -\lambda\phi$$

$$2\dot{H} + 3H^2 = -\kappa^2[-\Lambda(\phi) + \tfrac{1}{3}\rho_r],$$

- With the dimensionless quantities:

$$x \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \qquad y \equiv \frac{\kappa\sqrt{\Lambda}}{\sqrt{3}H},$$

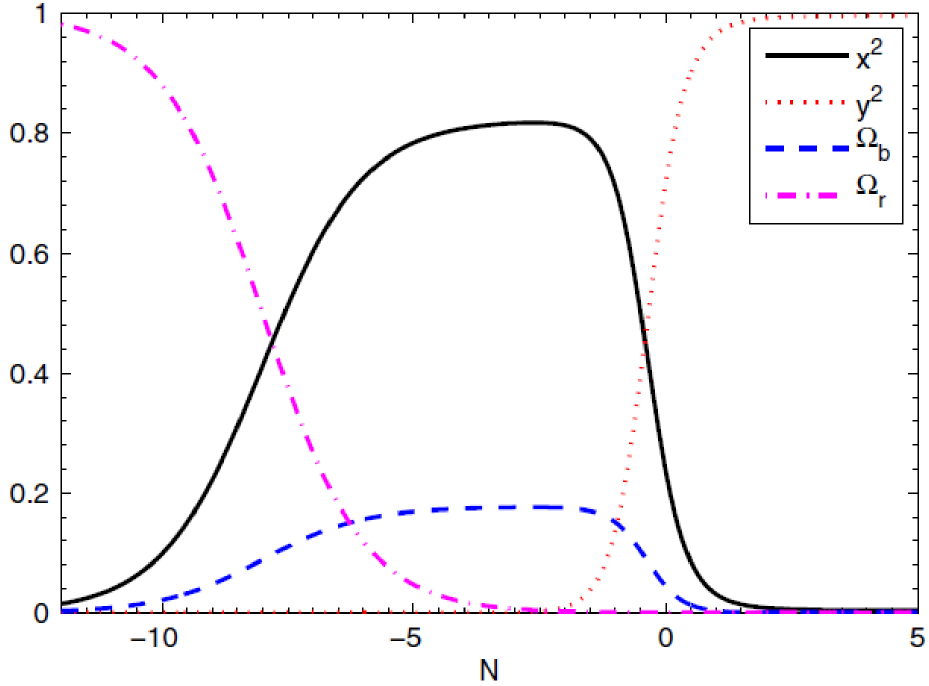
$$\sqrt{\Omega_b} \equiv \frac{\kappa\sqrt{\rho_b}}{\sqrt{3}H}, \qquad \sqrt{\Omega_r} \equiv \frac{\kappa\sqrt{\rho_r}}{\sqrt{3}H}.$$

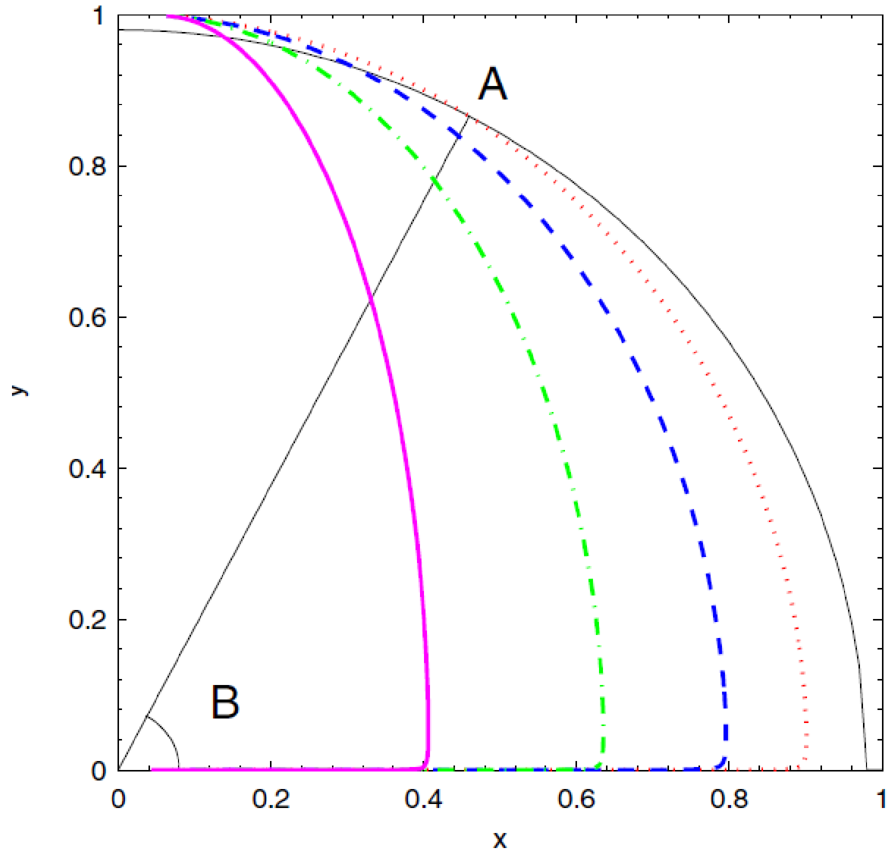
- Gives the Dynamical System Method:

$$\frac{dx}{dN} = -\frac{3}{2}x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x\left[1 - y^2 + \frac{1}{3}(1 - x^2 - y^2 - \Omega_b)\right],$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y\left[1 - y^2 + \frac{1}{3}(1 - x^2 - y^2 - \Omega_b)\right],$$

Dynamical System Method





Name	x	y	$\sqrt{\Omega_b}$	Existence	Stability	Ω_ϕ	w_ϕ
(a)	0	0	0	All λ	Unstable node	0	-
(b)	x	0	$\sqrt{1-x^2}$	All λ	Saddle line segment for $0 \le x \le 1$	x^2	0
(c)	$\frac{\sqrt{6}}{3}\lambda$	$\sqrt{1-\frac{2}{3}\lambda^2}$	0	$\lambda^2 \le \frac{3}{2}$	Stable node for $\lambda < \frac{\sqrt{6}}{2}$	1	$-1+\frac{2}{3}\lambda^2$

A toy model

- We start with a simple action of one dimensional particle in a potential $V(x)$.

$$S = \int \dot{B}\left[\frac{1}{2}m\dot{x}^2 + V(x)\right]dt$$

- δB gives the total energy of a particle:

$$\frac{1}{2}m\dot{x}^2 + V(x) = E$$

- δx gives the condition for B:

$$\frac{\mathrm{d}}{dt}\left(m\dot{x}\dot{B}\right) = \dot{B}V'(x)$$

GRAVITATIONAL THEORY WITH A DYNAMICAL SPACE–TIME
E. I. GUENDELMAN Int.J.Mod.Phys. A25 (2010) 4081-4099

Dynamical time Theories

- L. multiplier which force a covariant conservation of a stress energy tensor:

$$S(\chi) = \int \sqrt{-g}\, \chi_{\mu;\nu} T^{\mu\nu}_{(\phi)}\, d^4x$$

- χ^λ - dynamical space-time vector field .

$\chi_{\mu;\nu} = \partial_\nu \chi_\mu - \Gamma^\lambda_{\mu\nu}\chi_\lambda$ in the metric formalism ($g_{\alpha\beta;\gamma} = 0$).

$$\pi_0 = \frac{\partial \mathcal{L}}{\partial \chi_0} = T^0_{0\,(\phi)}$$

- $T^{\mu\nu}_{(\phi)}$ - stress energy tensor. The variation according to χ gives a **conserved** diffusive energy: $\nabla_\mu T^{\mu\nu}_{(\chi)} = 0$, in addition to $T^{\mu\nu}_{(G)} = \frac{\delta S(\chi)}{\delta g^{\mu\nu}}$.

- Dynamical time is as T.M.T for $T^{\mu\nu}_{(\chi)} = g^{\mu\nu}\Lambda$.

Symmetries

- If the matter is coupled through its energy momentum tensor as:

$$T^{\mu\nu}_{(\chi)} \rightarrow T^{\mu\nu}_{(\chi)} + \lambda g^{\mu\nu}$$

the process will not affect the equations of motion.

- Similar to "normal ordering" in QFT.

$$\chi_{\mu} \rightarrow \chi_{\mu} + k_{\mu}$$

- Where k_{μ} is a Killing vector.

Phys.Rev. D98 (2018) no.2, 023506

David Benisty, Eduardo Guendelman

The action

The complete action

$$\mathcal{L} = -\frac{1}{2}R + \chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} - \frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi)$$

With the coupled stress energy tensor:

$$T^{\mu\nu}_{(\chi)} = -\frac{1}{2}\phi^{,\mu}\phi^{,\nu} + U(\phi)g^{\mu\nu}$$

The action contains three variables: $g^{\mu\nu}, \phi, \chi^{\mu}$

EoM

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right)$$

- $\delta\chi^\mu$ - A constraint on scalar field:

$$\ddot{\phi} + \frac{3}{2} H\dot{\phi} + U'(\phi) = 0$$

- $\delta\varphi$ - A current:

$$\ddot{\phi}(\dot{\chi}_0 - 1) + \dot{\phi}[\ddot{\chi}_0 + 3\mathcal{H}(\dot{\chi}_0 - 1)] = U'(\phi)(\dot{\chi}_0 + 3\mathcal{H}\chi_0) - V'(\phi)$$

- $\delta g^{\mu\nu}$ - Dark energy and dark matter:

Dark Energy is hidden!

$$\rho = (\dot{\chi}_0 - \frac{1}{2})\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2(1 - \dot{\chi}_0 - 3\mathcal{H}\chi_0) - V(\phi) - \chi_0\dot{\phi}\ddot{\phi}$$

$U(\phi) = Const$

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + U'(\phi) = 0 \quad \Longleftrightarrow \quad \ddot{\phi} = -\frac{3}{2}H\dot{\phi} \quad \textbf{or} \quad \dot{\phi}^2 = \frac{2\Omega_m}{a^3}$$

$$\boxed{\delta = \dot{\chi}_0 - 1}$$

$$\ddot{\phi}(\dot{\chi}_0 - 1) + \dot{\phi}[\ddot{\chi}_0 + 3\mathcal{H}(\dot{\chi}_0 - 1)] = U'(\phi)(\dot{\chi}_0 + 3\mathcal{H}\chi_0) - V'(\phi) \quad \Longleftrightarrow \quad \dot{\phi}(\dot{\delta} + \frac{3}{2}H\delta) = -V'(\phi)$$

$$\rho = (\dot{\chi}_0 - \frac{1}{2})\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2(1 - \dot{\chi}_0 - 3\mathcal{H}\chi_0) - V(\phi) - \chi_0\dot{\phi}\ddot{\phi}$$

$$\Longleftrightarrow$$

$$\rho = (\delta + \frac{1}{2})\dot{\phi}^2 + V(\phi),$$

$$p = \frac{\delta}{2}\dot{\phi}^2 - V(\phi),$$

matter $w = 0$

D. Cosmological constant $w = -1$

Negative 2D radiation $w = 1/2$

Constant potentials

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + U'(\phi) = 0 \quad \Longleftrightarrow \quad \dot{\phi}^2 = \frac{2\Omega_m}{a^3}$$

$$\dot{\phi}(\dot{\delta} + \frac{3}{2}H\delta) = -V'(\phi) \quad \Longleftrightarrow \quad \delta = -\kappa\, a^{-1.5}$$

$$\rho = (\delta + \frac{1}{2})\dot{\phi}^2 + V(\phi),$$

$$p = \frac{\delta}{2}\dot{\phi}^2 - V(\phi),$$

$$\Longleftrightarrow$$

$$\rho = \Omega_\Lambda - \frac{\Omega_\kappa}{a^{4.5}} + \frac{\Omega_m}{a^3}$$

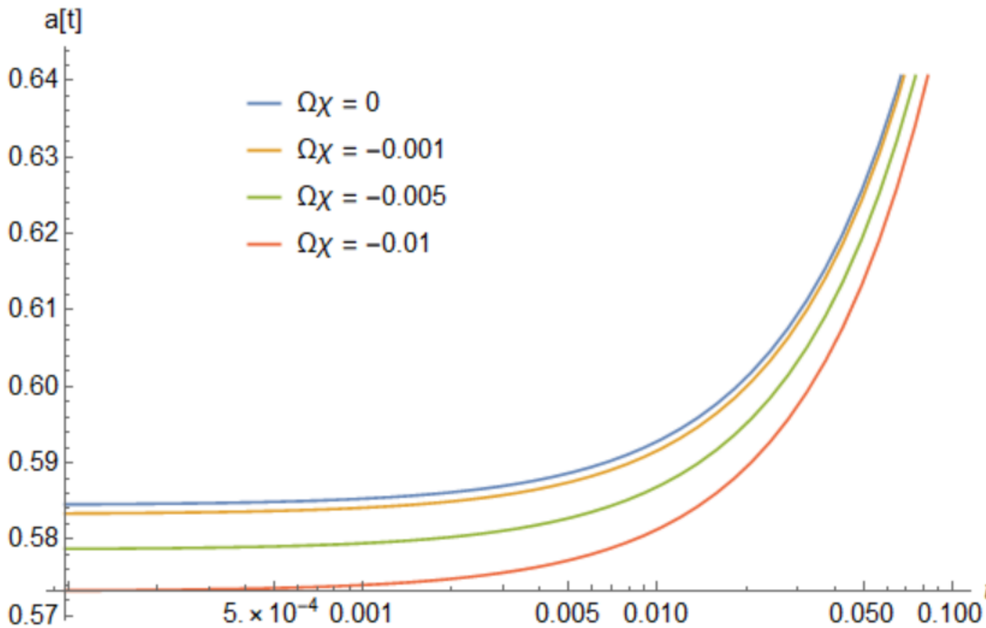
$$p = -\Omega_\Lambda - \frac{1}{2}\frac{\Omega_\kappa}{a^{4.5}}$$

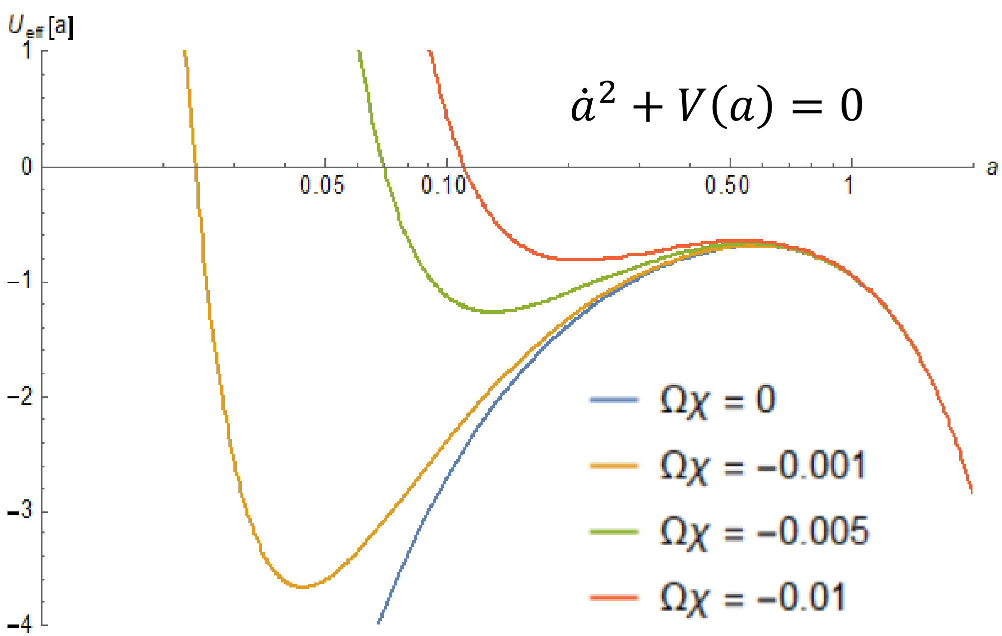
matter $w = 0$

D. Cosmological constant $w = -1$

Negative 2D radiation $w = 1/2$

A bouncing LCDM

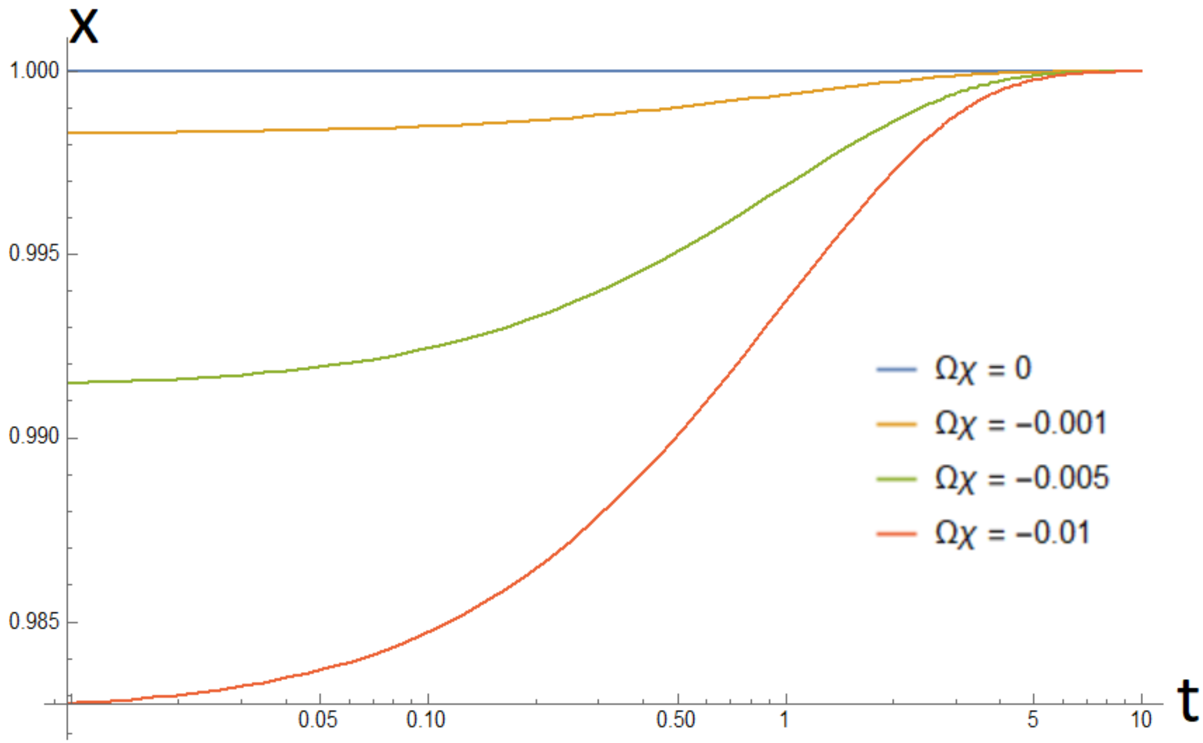




$$\dot{a}^2 + V(a) = 0$$

A dynamical time vs. cosmic time

- This correspondence between dynamical time and the cosmic time could be a solution of the time problem in quantum cosmology.



Exponential potential $U(\phi) = C, \quad V(\phi) = \Omega_\Lambda e^{-\beta\phi}$

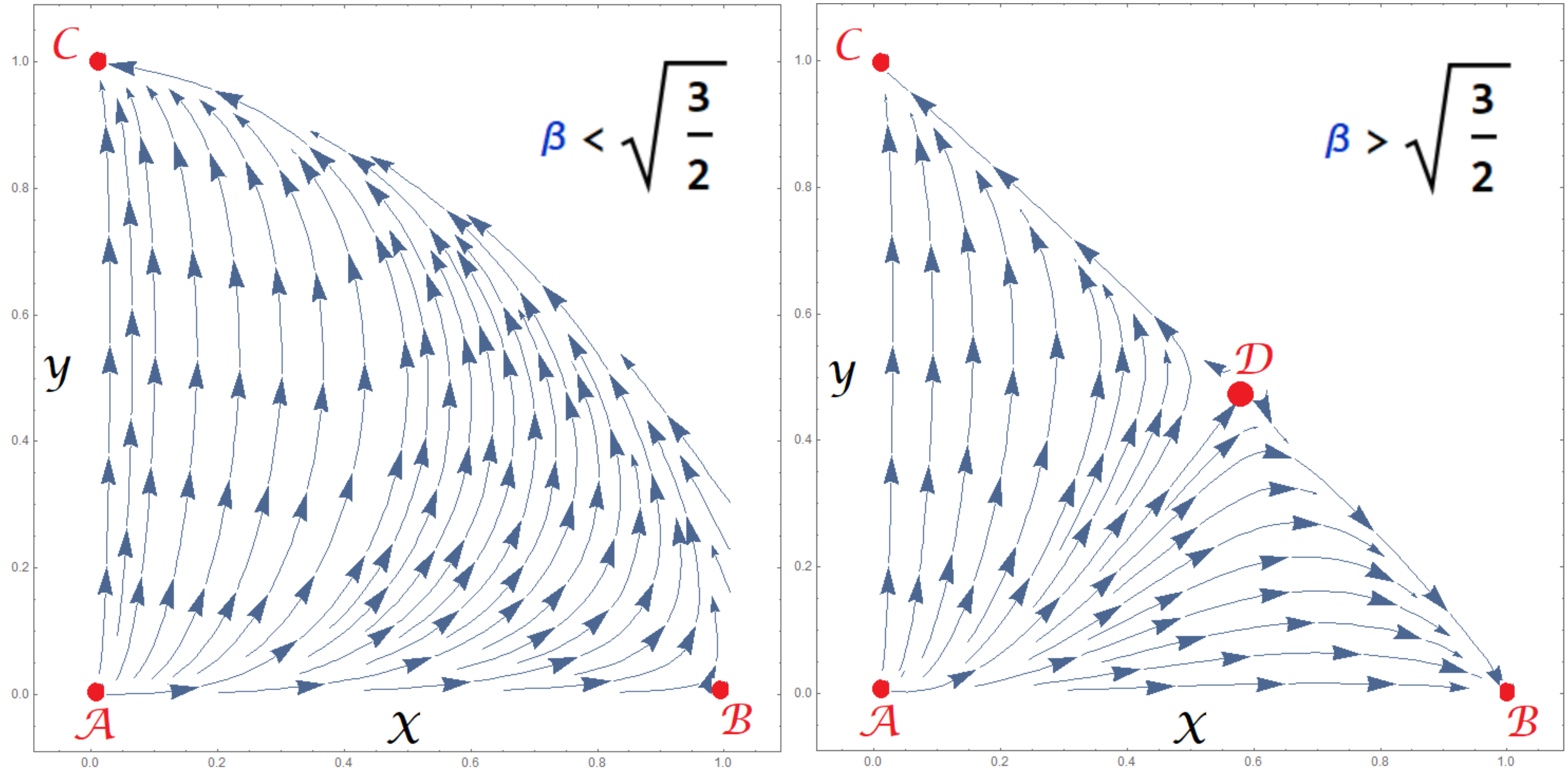
- The scalar field still behaves as dust: $\dot{\phi}^2 = \frac{2\Omega_m}{a^3}$

$$(1+2\delta)\frac{\dot{\phi}^2}{2} + V(\phi) = 3H^2$$

- In terms of dimensionless quantities:

$$(1+2\delta)x^2 + y^2 = 1$$

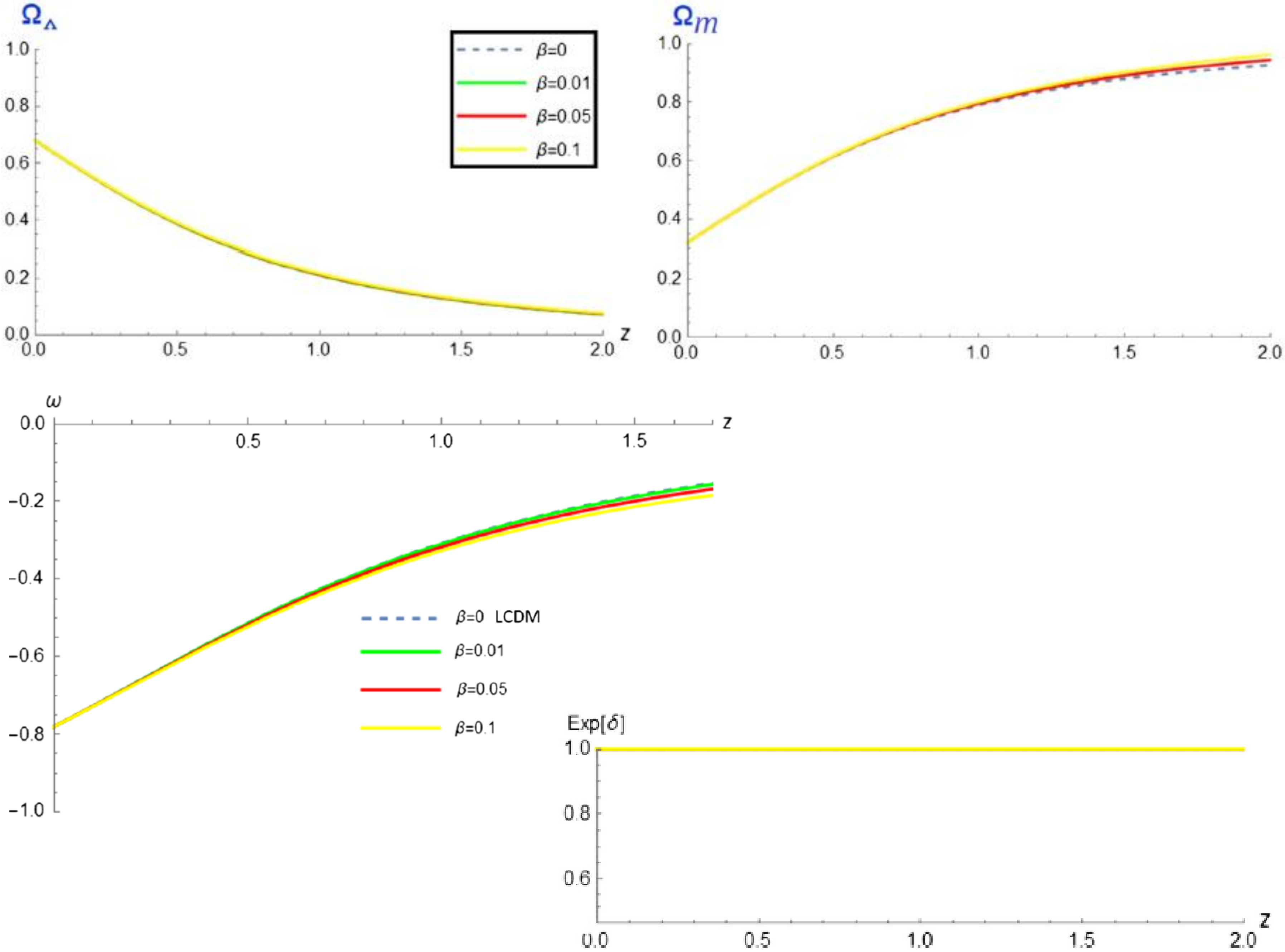
$$x = \frac{\dot{\phi}}{\sqrt{6}H}, \quad y = \frac{\sqrt{V(\phi)}}{\sqrt{3}H}$$



Name	existence	stability	universe
A	all β	unstable	-
B	all β	stable for $\beta > \sqrt{\frac{3}{2}}$	Dark Matter
C	all β	asymptotically stable	Dark Energy
D	$\beta > \sqrt{\frac{3}{2}}$	unstable saddle p.	unified DE-DM

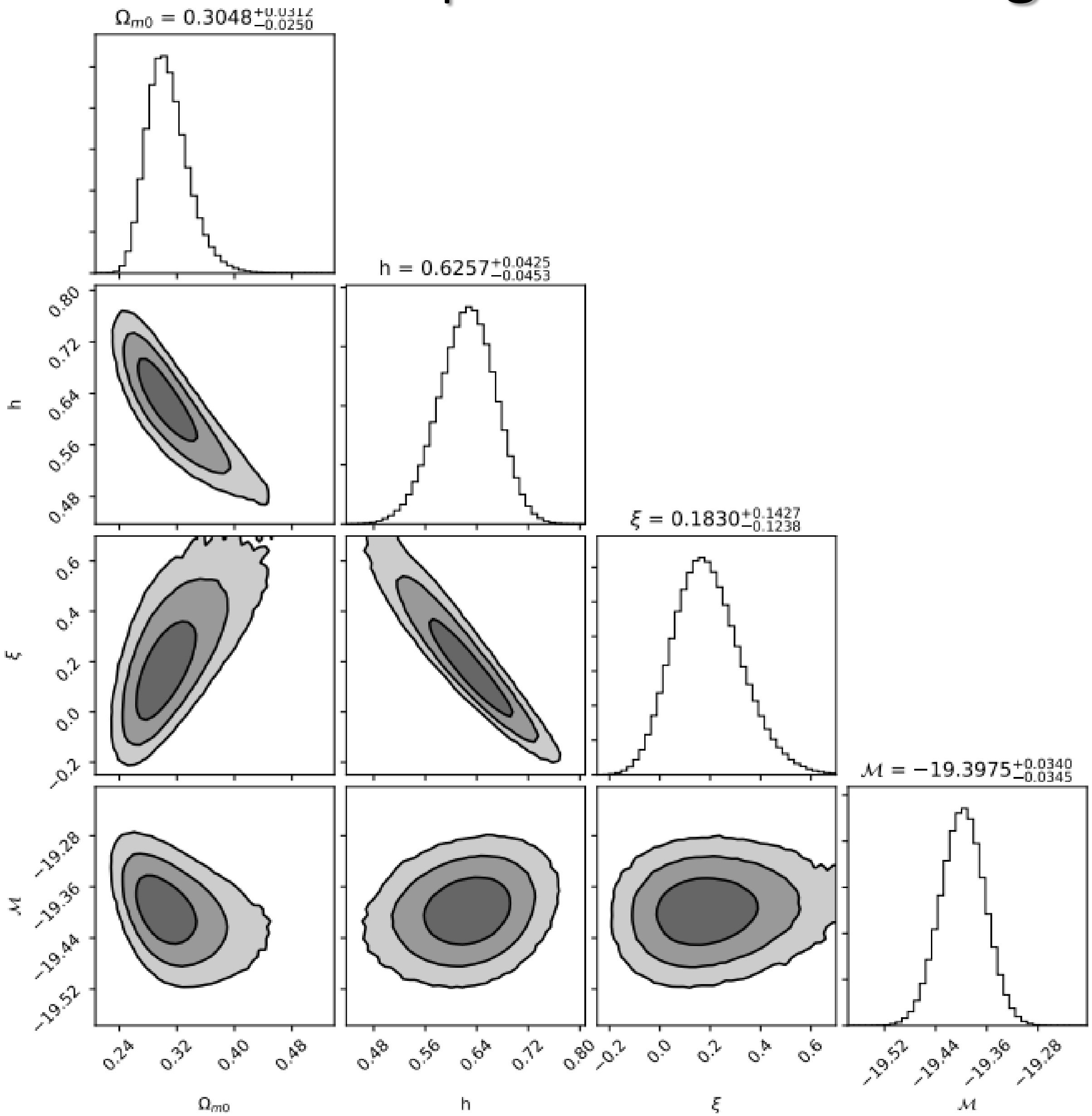
$$\frac{dx}{d\tau} = -\frac{3x}{4}(x^2 - 1 + 3y^2)$$

$$\frac{dy}{d\tau} = -\frac{y}{4}(-9 + 3x^2 + 9y^2 + 2\sqrt{6}x\beta)$$



Ω_Λ
β=0
β=0.01
β=0.05
β=0.1
z
Ω_m
z
ω
z
β=0 LCDM
β=0.01
β=0.05
β=0.1
Exp[δ]
z

Late time expansion data fitting



JCAP 1906 (2019) 003

F. K. Anagnostopoulos
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E.I. Guendelman

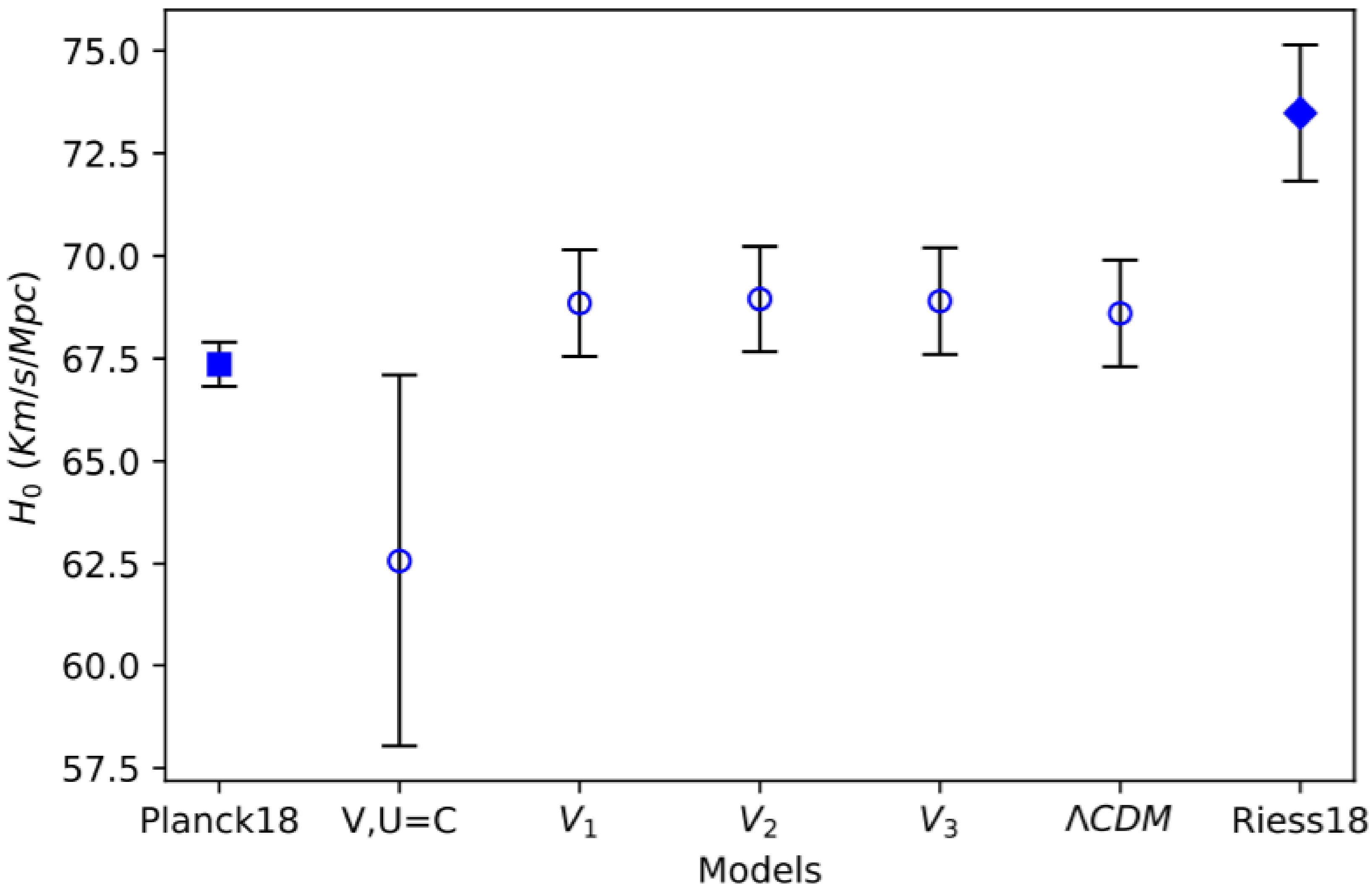
Better then LCDM for late time expansion data

Model	Ω_{m0}	h	α/κ	β	$\mathcal{M}$	$\chi^2_{\rm min}$
$V, Uconst.$	$0.305^{+0.032}_{-0.025}$	$0.6896^{+0.0126}_{-0.0124}$	$-0.183^{+0.125}_{-0.144}$	-	$-19.397^{+0.034}_{-0.035}$	84.049
V_1	$0.277^{+0.024}_{-0.023}$	$0.6885^{0.0130}_{0.0128}$	-	$-0.593^{+1.367}_{-1.355}$	$-19.390^{+0.034}_{-0.035}$	88.100
$V_2(cosine)$	0.270 ± 0.015	$0.6895^{+0.0128}_{-0.0127}$	–	1	-19.388 ± 0.035	87.954
V_3	$0.273^{+0.024}_{-0.023}$	$0.6890^{+0.0130}_{-0.0127}$	$1.152^{+1.370}_{-1.352}$	1	-19.389 ± 0.034	87.942
ΛCDM	$0.281^{+0.016}_{-0.015}$	0.686 ± 0.013	-	-	-19.403 ± 0.035	85.700

Model	AIC	ΔAIC	BIC	ΔBIC	DIC	ΔDIC
$V, Uconst$	92.470	0.518	102.470	2.955	88.473	0
V_1	96.521	4.569	106.5210	7.006	95.908	7.435
V_2	94.204	2.252	101.770	2.255	93.930	5.457
V_3	96.363	4.411	106.363	6.848	95.805	7.332
ΛCDM	91.952	0	99.515	0	91.671	3.198

TABLE V. The information criteria values AIC, BIC and DIC for the different potentials and the concordance model along with the corresponding differences $\Delta IC \equiv IC - IC_{\rm min}$.

H0 values



75.0
72.5
70.0
67.5
65.0
62.5
60.0
57.5
H_0 (Km/s/Mpc)
Planck18
V,U=C
V_1
V_2
V_3
ΛCDM
Riess18
Models

General Consideration of $T_{(\chi)}^{\mu\nu}$

- For a generalizes $T_{(\chi)}^{\mu\nu}$:

$$T_{(\chi)}^{\mu\nu} = -\frac{\lambda_1}{2}\phi^{,\mu}\phi^{,\nu} - \frac{\lambda_2}{2}g^{\mu\nu}\phi^{,\alpha}\phi_{,\alpha} + g^{\mu\nu}\,U(\phi)$$

With the "density and the pressure":

$$\rho_{(\chi)} = (\lambda_1 + \lambda_2)\frac{\dot{\phi}^2}{2} + U(\phi),$$

$$p_{(\chi)} = -\lambda_2\frac{\dot{\phi}^2}{2} - U(\phi).$$

The general action is again:

$$\mathcal{L} = -\frac{1}{2}R + \chi_{\mu;\nu}T_{(\chi)}^{\mu\nu} - \frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi)$$

S. Banerjee D. Benisty E.I. Guendelman
arXiv:1909.01982

The EoM

- The variation with respect to χ_μ: $\dot{\phi} = C_1 a^{-3\lambda_1/2\lambda},$
- Power Law $a \sim t^\alpha$

$$\rho = C_1^2 \frac{(\lambda + 3\alpha(\lambda_1 + 3\lambda))}{2(3\alpha(\lambda_1 - \lambda) + \lambda)} a^{-3\lambda_1/\lambda} + V.$$

- Exponential $a \sim e^{H_0 t}$

$$\rho = C_1^2 \frac{\lambda_1 + 3\lambda}{2(\lambda_1 - \lambda)} a^{-3\lambda_1/\lambda} + V$$

Mon.Not.Roy.Astron.Soc. 431 (2013) 923-929
J.A.S. Lima, S. Basilakos, Joan Sola

The Running Vacuum Model

- From QFT in curved spacetime:

$$\rho_\Lambda(H;\nu,\alpha) = \frac{3}{8\pi G}\left(c_0 + \nu H^2 + \frac{2}{3}\alpha H\right) + \mathcal{O}(H^4),$$

- For $G = Const$: $$\rho = \Omega_\Lambda^{(0)} + \frac{\Omega_m^{(0)}}{\xi} a^{-3\xi} + \frac{\Omega_r^{(0)}}{\xi'} a^{-4\xi'} \, .$$

$$\xi = \frac{1-\nu}{1-\alpha} \equiv 1 - \nu_{eff}\,, \quad \xi' = \frac{1-\nu}{1-\frac{4}{3}\alpha} \equiv 1 - \nu'_{eff}\,.$$

- Corresponds to DST cosmology with: $\xi = \lambda_1/\lambda, \quad \xi' = 1$

Inflation Compactification

Phys. Rev. D **31**, 2697 (1985)

Yasunari Tosa

- The metric: $ds^2 = -dt^2 + R(t)^2 dx^2 + r(t)^2 dy^2$
- The volume:

$$V = R^D r^d$$

- For separate equation of states, the compactification mechanism uses:

$$R(t) = V^{\frac{1}{D+d}} \exp[+\frac{1}{D+d}\sqrt{\frac{2Ed(D+d-1)}{D}}\int \frac{dt}{V}]$$

$$E = \frac{1}{2}\dot{V}^2 - \frac{D+d}{D+d-1}\Omega V^2$$

$$r(t) = V^{\frac{1}{D+d}} \exp[-\frac{1}{D+d}\sqrt{\frac{2ED(D+d-1)}{d}}\int \frac{dt}{V}]$$

Phys.Rev. D98 (2018) no.4, 043522
David Benisty, Eduardo Guendelman

Inflation Compactification With D.S.T

- For constant potentials, the density is: $$\Omega = \Omega_\Lambda + \frac{\Omega_m}{V} + \frac{\Omega_\kappa}{V^{3/2}}$$

$$E = \frac{1}{2}\dot{V}^2 + U_{eff}$$

with the appropriate effective potential:

$$U_{\text{eff}}(V) = -\frac{D+d}{D+d-1}(\Omega_\Lambda V^2 + \Omega_m V + \Omega_\kappa \sqrt{V})$$

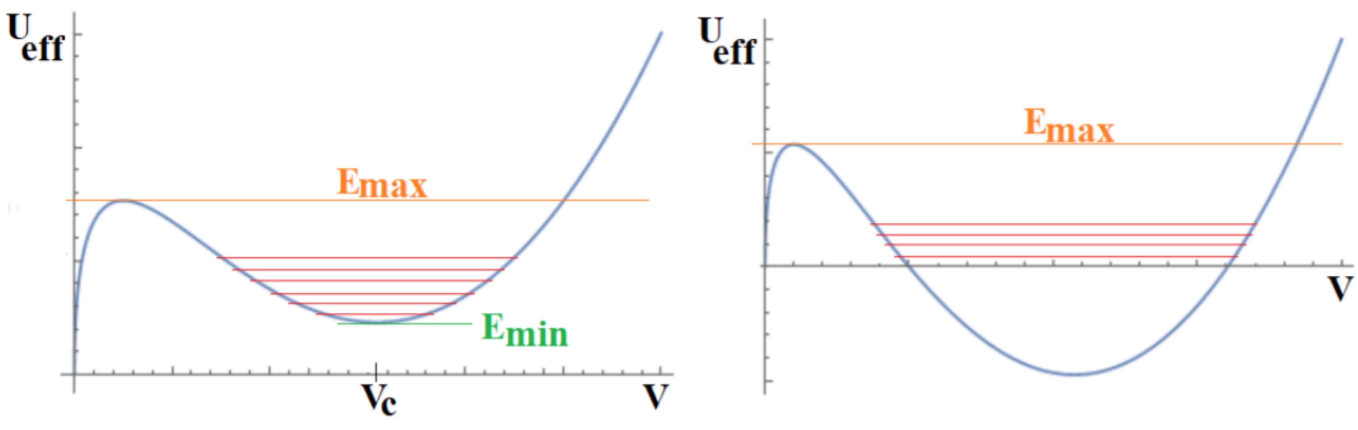
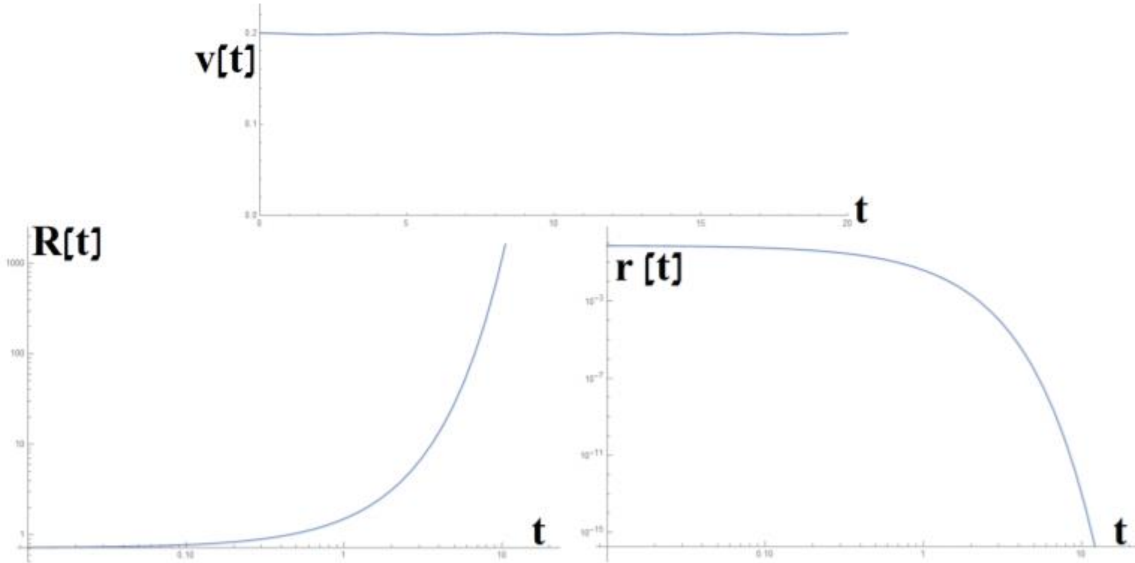
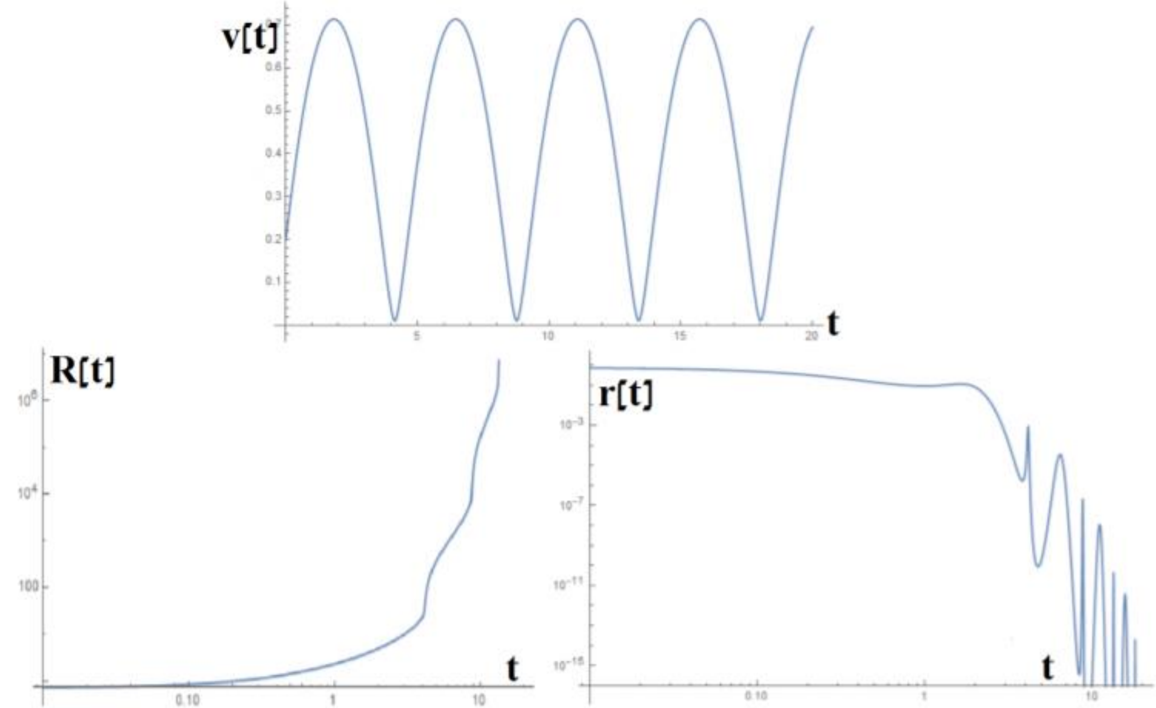


FIG. 1. The effective potential, for two cases, where $\Omega_\Lambda, \Omega_\kappa < 0$ and $\Omega_m > 0$.



Low “Energy”



High “Energy”

End of inflation compactification, using a step function potential

- For obtaining a partially analytic and more simple solution, lets use a step function:

$$V(\phi) = \frac{\Lambda_{+\infty} - \Lambda_{-\infty}}{2} \mathbf{Sign}(\phi) + \frac{\Lambda_{+\infty} + \Lambda_{-\infty}}{2}.$$

$\Lambda_{\pm}$ are the vales of the potential from right and left.

The variation with respect to the vector field: $\dot{\phi}^2 = \frac{2\Omega_m}{V}$

The variation with respect to the scalar: $\frac{2\Omega_m}{V^{\frac{3}{2}}} d(\xi V^{\frac{1}{2}}) = -dV(\phi).$

where $\xi = \dot{\chi}_0 - 1$

With the solution: $\xi(\phi < 0) = \frac{\kappa_-}{V^{\frac{1}{2}}_{(\phi=0)}}, \qquad \xi(\phi > 0) = \frac{\kappa_+}{V^{\frac{1}{2}}_{(\phi=0)}}.$

The discontinuity condition gives:

$$\kappa_+ - \kappa_- = -\frac{V^2_{(\phi=0)}}{2\Omega_m}(\Lambda_{+\infty} - \Lambda_{-\infty}).$$

The "energy equation" (EOM with respect to the metric):

$$E = \frac{1}{2}\dot{V}^2 - \frac{D+d}{D+d-1}V^2\left[\frac{2\Omega_m}{V}\left(\xi + \frac{1}{2}\right) + \frac{\Lambda_{+\infty} - \Lambda_{-\infty}}{2}\mathbf{Sign}(\phi) + \frac{\Lambda_{+\infty} + \Lambda_{-\infty}}{2}\right].$$

- **There is no jump** of the volume:

$$\frac{1}{2}\Delta\dot{V}^2 = \frac{D+d}{D+d-1}\left(\frac{2\Omega_m}{V_{(\phi=0)}}\Delta\xi + \Lambda_{+\infty} - \Lambda_{-\infty}\right)V^2_{(\phi=0)} = 0,$$

This fact gives a pure proof that there is a solution for any potential.

Final Remarks

Dynamical dark energy and dark matter:

- For understanding the nature of the coincidence problem
- Hubble constant tension?

The Dynamical space time Theories – both energy momentum tensor are conserved.

Valid after GW170817!

- Dynamical time has a correspondence to the cosmic time.

- Dynamical DE – DM from potentials - stable for low couplings values.

- With higher dimensions gives inflation.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Dark Energy & Dark Matter
EA7