Gravitational Anomalies, *axions* & *a string-inspired Running Vacuum Model in Cosmology*

KING'S College LONDON

QUANTUM · GRAVITY · MULTI · MESSENGER

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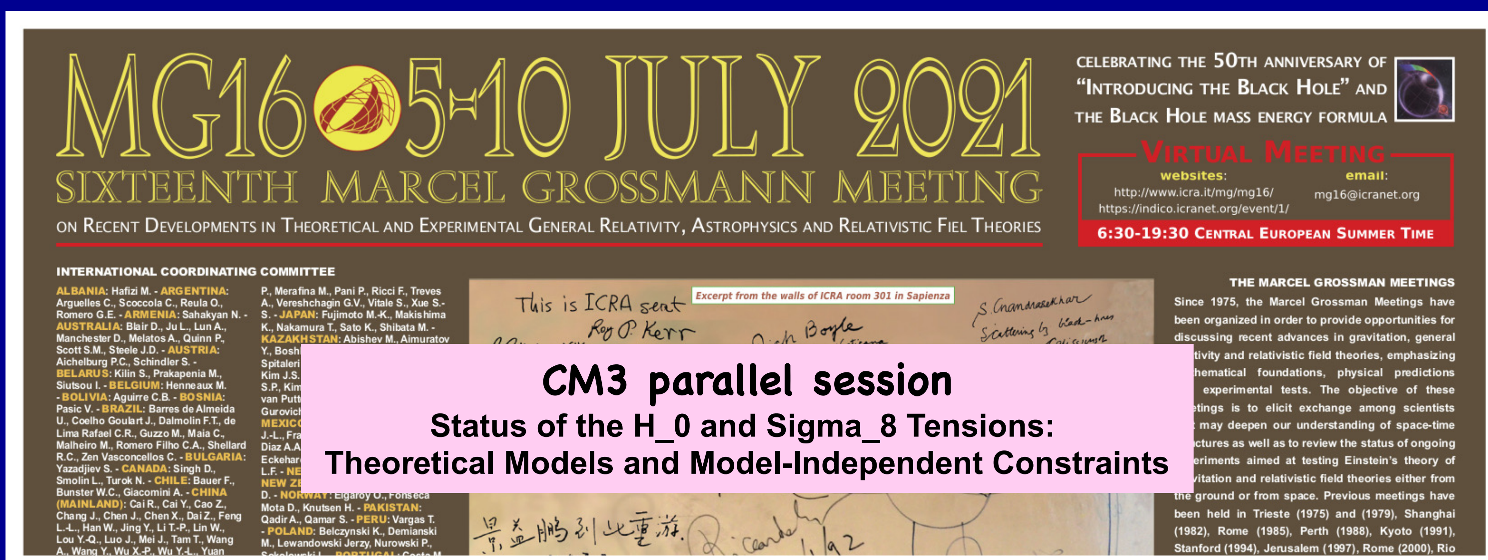
EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

CA18108 - Quantum gravity phenomenology in the multi-messenger approach



CM3 parallel session

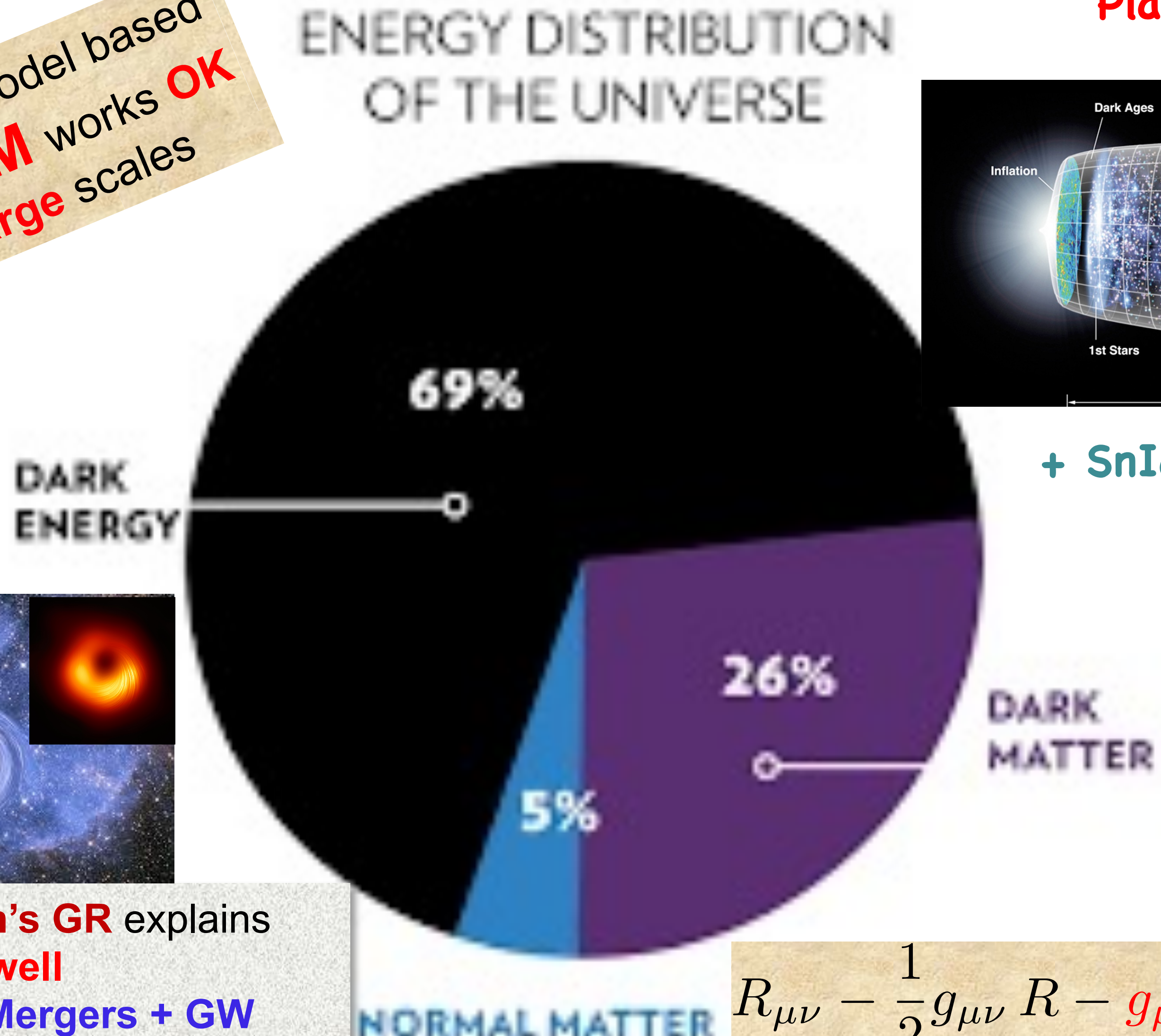
Status of the H_0 and Sigma_8 Tensions: Theoretical Models and Model-Independent Constraints

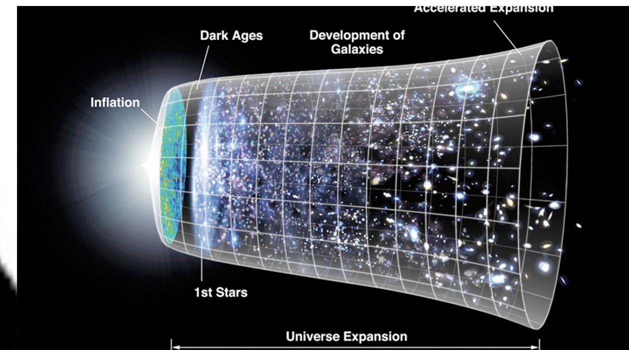
0. Motivation

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

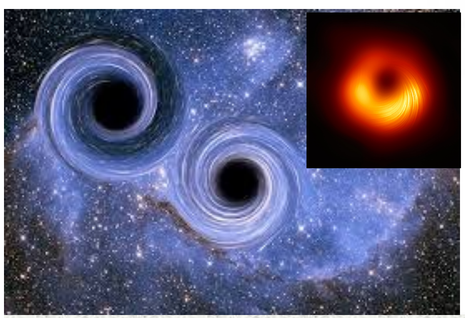
Simplest model based on **ΛCDM** works **OK** for **large** scales

Planck2018 data





+ SnIa, BaO, Lensing



Also **Einstein's GR** explains **sufficiently well**
Black-Hole Mergers + GW
(since 2015 LIGO),
Black-Hole `photographs' (EHT),...

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\,\Lambda = 8\pi G\, T_{\mu\nu}$$

$$T_{\mu\nu} \ni \text{Cold Dark Matter}$$

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

Planck 2013 data

Sim
on

But....

Need to go Beyond....

What still we do not know/did not observe:

Nature of Dark Energy

Nature of Dark matter

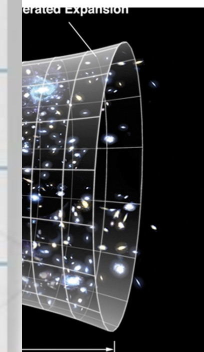
Primordial Gravitational Waves

(through detection of B-mode polarisation

in CMB from very early Universe)

Microscopic models of Inflation

(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)



Lensing

Also
suffic
Black
(since
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$8\pi G\, T_{\mu\nu}$

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Nature o

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(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)

⚠ **ΛCDM appears to be in tension with local measurements of present-era H_0 & also σ_8 galaxy-growth data ?**

This session talks

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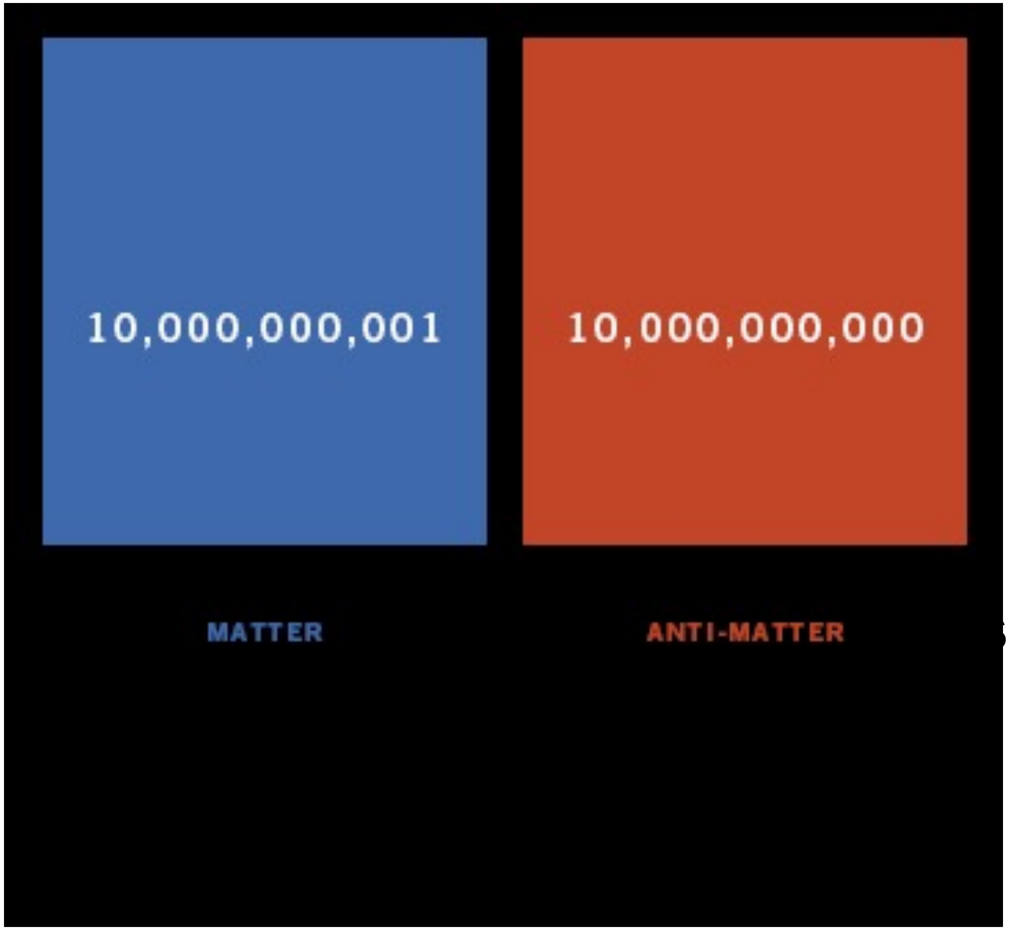
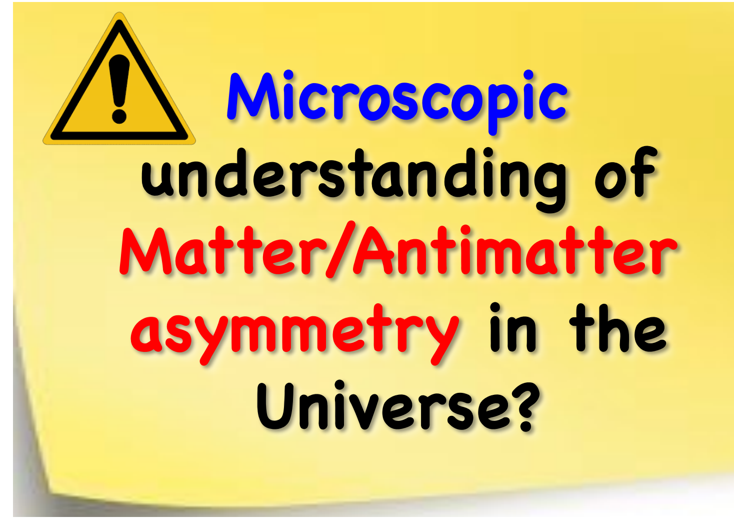
$8\pi G\, T_{\mu\nu}$





The Baryon Asymmetry

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

s = entropy density of Universe

Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

Baryon number violation

C-violation

and CP violation

Departure from thermodynamic equilibrium (non-stationary system)

CP |particle> = |anti-particle>

Need new physics beyond the SM → new sources of CP violation?

Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

Baryon number violation

C-violation

and CP violation

Departure from thermodynamic equilibrium (non-stationary system)

What if CPTV geometries in the early Universe ?

Need to go Beyond....

CP |particle> = |anti-particle>

Need new physics beyond the SM →
new sources of CP violation?

1. Overview

The Parts & the Whole

"There is a fundamental error in separating the parts from the whole, the mistake of atomizing what should not be atomized.

Unity and complementarity constitute reality"

Werner Karl Heisenberg
German Scientist & Nobel Prize
1901-1976

I will argue that:

Deviations from ΛCDM and alleviation of cosmological-data
Tensions in the current era

+

observed matter-antimatter asymmetry

Can be linked with

Microscopic string-inspired models of Cosmology with ANOMALIES,
primordial gravitational waves and induced spontaneous
(through gravitational anomaly condensates) Lorentz + CPT Violation

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geometric torsion interpretation of axion Dark matter

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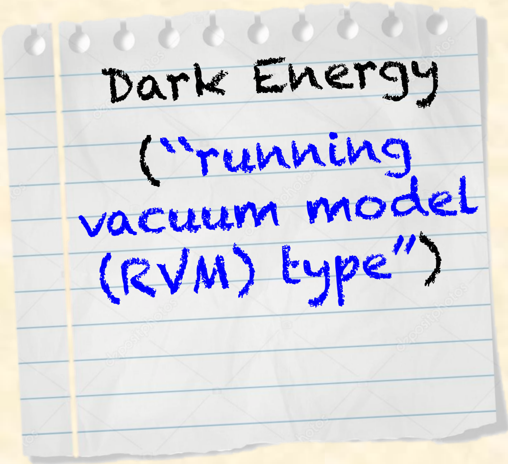

The Parts

Shapiro + Solà
Solà, ...

Dark Energy
("running vacuum model (RVM) type")

$$\rho_\Lambda^{\rm RVM} = \kappa^{-2}\,\Lambda + c_1 H^2 + c_2 H^4 + \dots \equiv \kappa^{-2}\,\Lambda(t)$$

$$\Lambda \equiv 3\,c_0 \qquad c_1 = 3\nu\kappa^{-2},\; c_2 = 3\alpha\kappa^{-2}\,H_I^{-2},$$
$$H_I \sim 10^{-5}\kappa^{-1}\ \text{(current pheno)}$$

Vacuum energy density assumed de Sitter like but with time-dependent Cosmological parameter Λ(t) :

$$\rho^{\Lambda}_{\rm RVM}(t) = \Lambda(t)/\kappa^2 \qquad \kappa = \sqrt{8\pi {\rm G}} = M_{\rm Pl}^{-1}$$

$$p(t)_{\rm RVM} = -\rho^{\Lambda(t)}_{\rm RVM}(t)$$

Renormalization-Group-like equation for the evolution of **vacuum energy density**
Hubble parameter H(t) ←→ RG scale *μ*

$$\frac{d\,\rho_\Lambda^{\rm RVM}(t)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_{i=F,B}\Big[a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}\Big(\frac{H^6}{M_i^2}\Big)\Big]$$

general covariance →
even powers of *H*

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$\omega = \rho_m / p_m$ *m* = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\dot{H} + \frac{3}{2}(1+\omega)H^2 \left(1 - \nu - \frac{c_0}{H^2} - \alpha \frac{H^2}{H_I^2}\right) = 0$$

Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$D > 0$

Early de Sitter (unstable) $\quad D a^{4(1-\nu)} \ll 1 \quad \Rightarrow \quad H^2 = (1-\nu) H_I^2/\alpha$

Radiation $\quad D a^{4(1-\nu)} \gg 1 \quad \Rightarrow \quad H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era $\quad H^2(a) = H_0^2 \left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

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Gómez-Valent, Solà

Could **Alleviate** Tensions in Data, e.g.

H_0 , σ_8 tensions

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2}\, c_0 \simeq 10^{-122}\, M_{\rm Pl}^4$$

$$\rho_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu_0\left(\frac{H_0}{M_{\rm Pl}}\right)^2 + \beta\frac{H_0^4}{M_{\rm Pl}^4}\right), \quad \beta > 0.$$

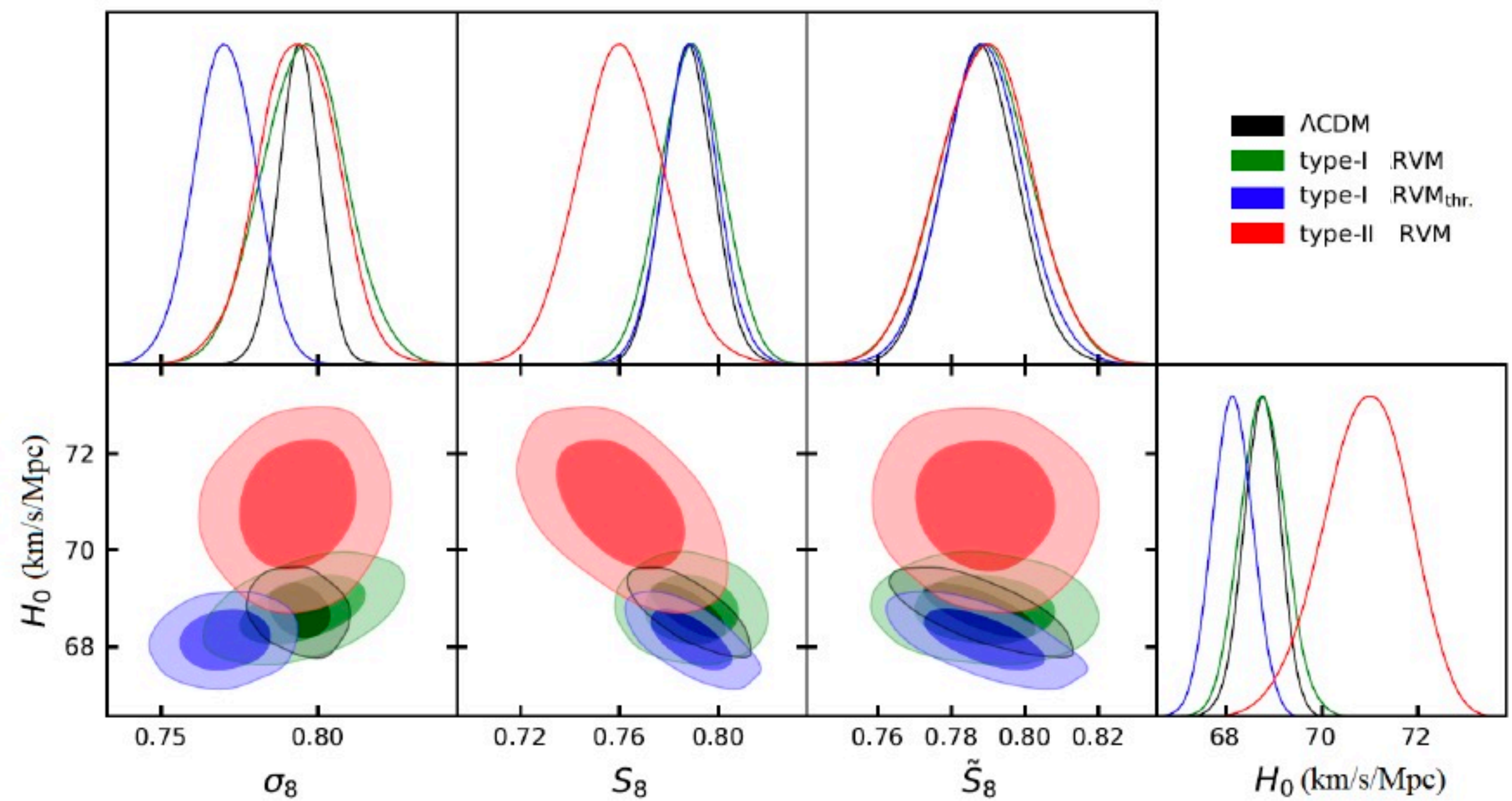
Not dominant today

Running RVM Dark Energy

Session talks

Solà, Gómez-Valent,
De Cruz Perez, Moreno-Pulido,
(Planck 2018 data)

Alleviation of the H_0 , σ_8 tension by RVM model



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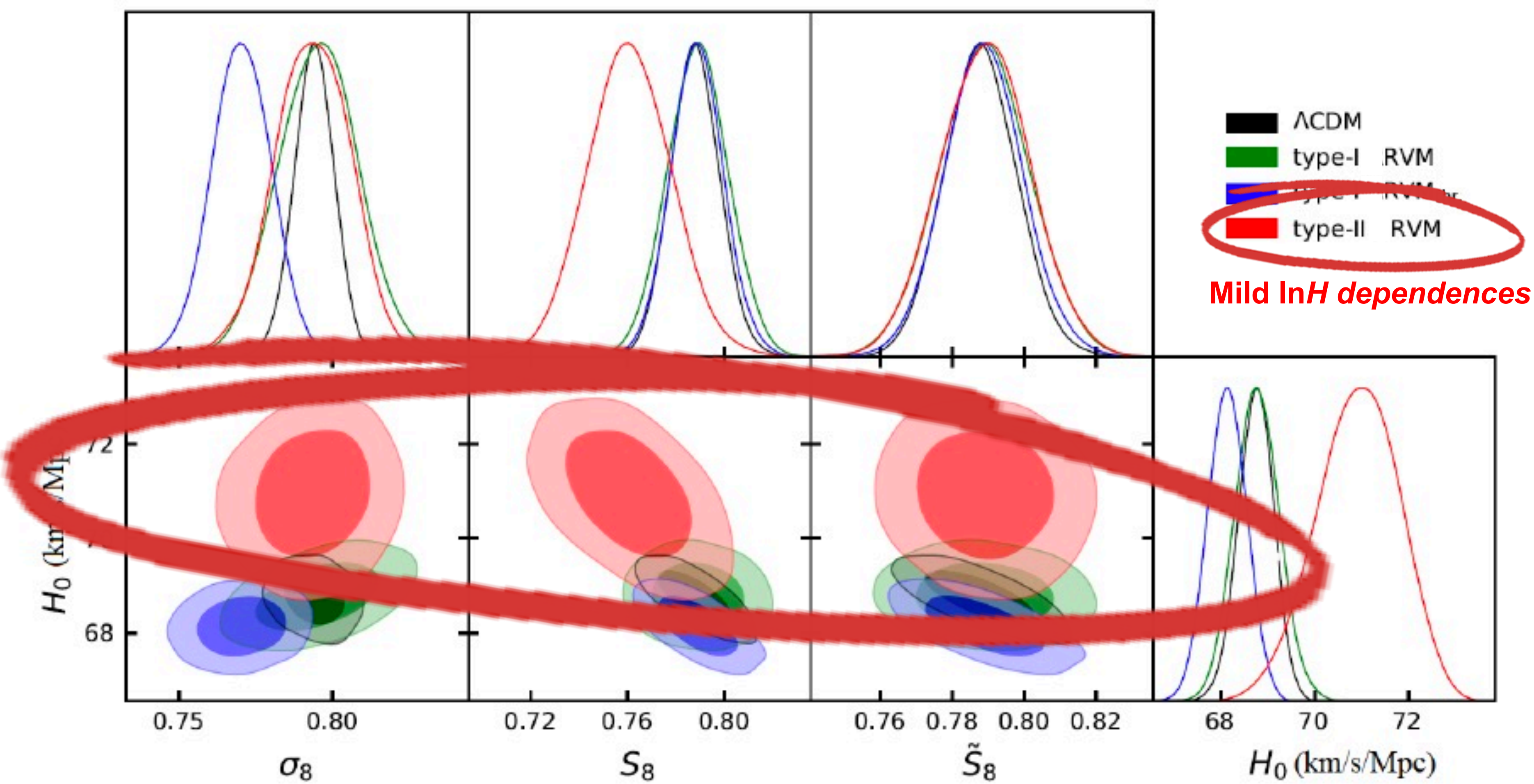

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Shapiro + Solà
Solà, ...

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("running vacuum model (RVM) type")

$$\rho_\Lambda^{\rm RVM} = \kappa^{-2}\,\Lambda \;\ldots\; + c_2 H^4 + \ldots$$
$$\equiv \kappa^{-2} \ldots$$

$\ldots^2,\; c_2 = 3\alpha\kappa^{-2}\,H_I^{-2},$

$\ldots^1$ (current pheno)

Vacuum energy de... ...t Cosmological parameter Λ(t) :

NEM, Solà + Basilakos

This talk:
We shall obtain an (type II) **RVM** from **Grav. Anomalies** in string-inspired Cosmology

Renormalization-Group-like ... **vacuum energy density**
... → **RG scale *μ***

$$\frac{d\,\rho_\Lambda^{\rm RVM}(t)}{d\,\ln H^2} = \frac{1}{(4\pi)^2}\sum_{i=F,B}\left[a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}\Big(\frac{H^6}{M_i^2}\Big)\right]$$

general covariance →
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Dark Energy

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Stringy gravitational Axions

+

torsion

Gross and Sloan, Metsaev and Tseytlin

Massless Gravitational multiplet of (closed) strings:

spin 0 scalar (dilaton Φ)

spin 2 traceless symmetric rank 2 tensor (graviton $g_{\mu\nu}$)

spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

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KALB-RAMOND FIELD

$$\mathrm{U(1)-symmetry}: B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

4-DIM action

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R+2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}e^{-4\Phi}H_{\lambda\mu\nu}H^{\lambda\mu\nu} + \dots\Big)$$

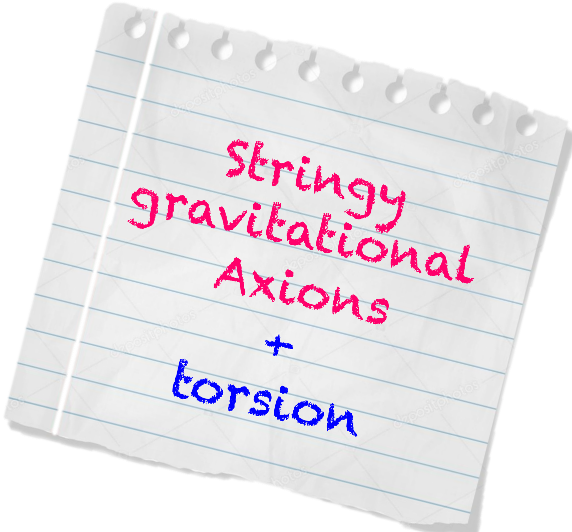
$\kappa^2 = 8\pi$ G

Green, Schwarz

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}\, B_{\nu\rho]} \quad \Rightarrow \quad \boldsymbol{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\Big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\Big)$$

$$\Omega_{3\mathrm{L}} = \omega^a_{\ c}\wedge \mathbf{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c}\wedge\omega^c_{\ d}\wedge\omega^d_{\ a}, \quad \Omega_{3\mathrm{Y}} = \mathbf{A}\wedge\mathbf{dA} + \mathbf{A}\wedge\mathbf{A}\wedge\mathbf{A},$$



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$$\overline{R}(\overline{\Gamma})$$

generalised curvature

Φ = constant throughout

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

Gross and Sloan, Metsaev and Tseytlin

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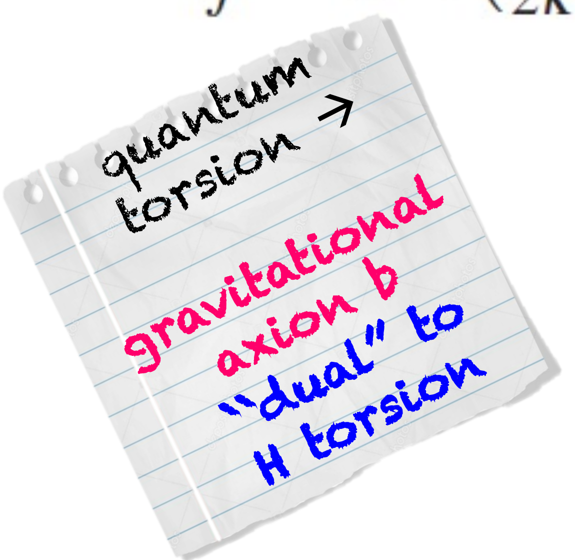

b(x) = Lagrange multiplier implementing **Bianchi identity constraint** for $H_{\mu\nu\rho}$:

$$d \star H \propto c_1 R \wedge \widetilde{R} - F \wedge \widetilde{F}$$

$H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$

torsion

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\partial_\mu b\right)J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}e_{b\lambda}\partial_a e^\lambda_c,$$ vielbeins

$$J^{5\mu} = \bar{\psi}_j\gamma^\mu\gamma^5\psi_j$$

Axial Current

All fermion species

KR-axion anomalous CP-Violating interaction

torsion

cf. classically in 4 dim:

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

Inclusion of Fermions

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ielbeins

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

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The Model

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All fermion species

The Model

Anomaly terms

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The Parts

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Stringy gravitational Axions

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Gravitational anomalies

NB: Anomalies: (CHIRAL)

Classically conserved current

AXIAL FERMION CURRENT $J^{\mu 5}$ CEASES to be conserved @ a quantum level

$$\nabla_\mu J^{\mu 5} \propto c_1 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad c_1 \in \mathbb{R}$$

CHIRAL FERMIONS IN LOOP :

$\frac{D}{2}+1$ graph

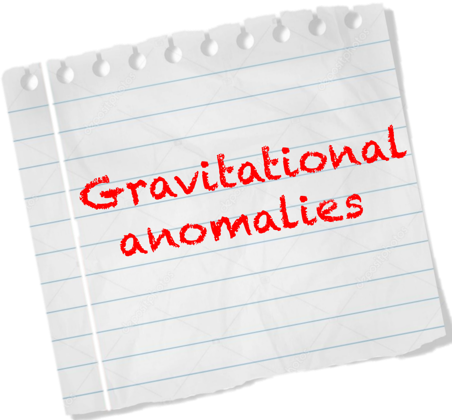
$J^{\mu 5} \equiv \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j$, $j = 1 \dots N$ SPECIES

chiral fermion $\gamma^5 \psi_j = \mp \psi_j$ (LEFT OR RIGHT HANDED)

$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$,

$\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}$

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \sqrt{-g}\, b(x) \Big(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \widetilde{F}^{\mu\nu} \Big)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x \sqrt{-g}\, b\, R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma}\Big] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu}\, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu}\, \delta g^{\mu\nu}$$

Cotton tensor

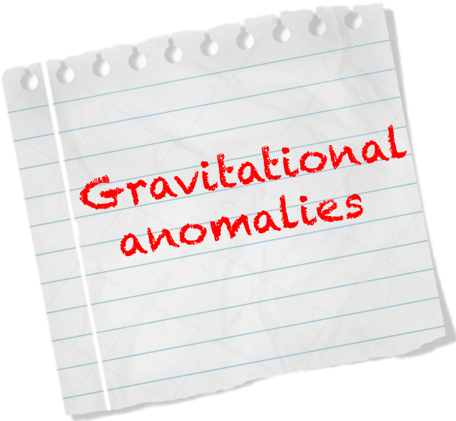
$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \Big(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha}\Big) + v_{\sigma\tau}\Big(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu}\Big)\Big] = -\frac{1}{2}\Big[\Big(v_\sigma \widetilde{R}^{\lambda\mu\sigma\nu}\Big)_{;\lambda} + (\mu \leftrightarrow \nu)\Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless $\quad g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance



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Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x \sqrt{-g}\, b\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \Big] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu}\, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu}\, \delta g^{\mu\nu}$$

Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \Big(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \Big) + v_{\sigma\tau} \Big(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \Big) \Big] = -\frac{1}{2}\Big[\Big(v_\sigma\, \widetilde{R}^{\lambda\mu\sigma\nu} \Big)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

not necessarily positive contributions to vacuum energy ⚠

Traceless $$g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$$

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\text{matter}}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{\text{matter}\,;\mu} = - \mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0$$

Diffeomoprphism invariance breaking by gravitational anomalies?

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

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$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{{\rm matter}\,;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0$$

No problem with diffeo

Conserved Modified stress-energy tensor

The Parts

Dark Energy
("running vacuum model (RVM) type")

Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

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NB:

absent before formation of GW

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

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No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

c.f. Zeldovich but for baryons in his model; cf. also Chavanis

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Basilakos, NEM, Solà (2019-20)

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Primordial Gravitational Waves
Potential Origins in pre-inflationary era?

NEM, Solà
EPJ-ST
(2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

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Primordial Gravitational Waves

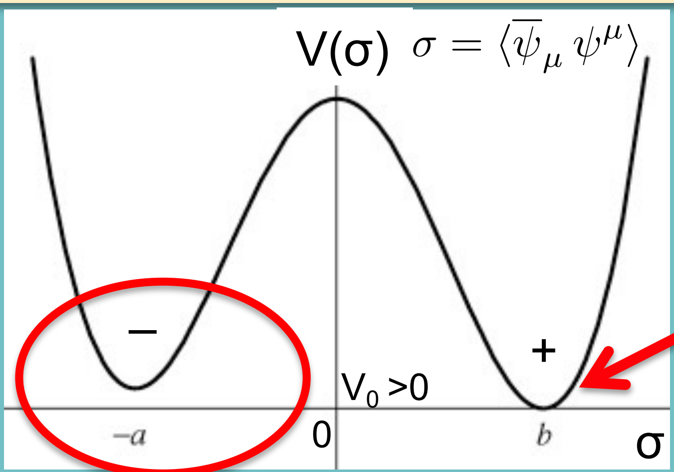
Potential Origins in pre-inflationary era?

Collapse/collisions of Domain walls formed in theories with (approxmate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM, Solà EPJ-ST (2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Statistical bias (percolation) in occupation probabilities of the +,- vacua

Lalak, Ovrut, Lola, G. Ross, Thomas

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

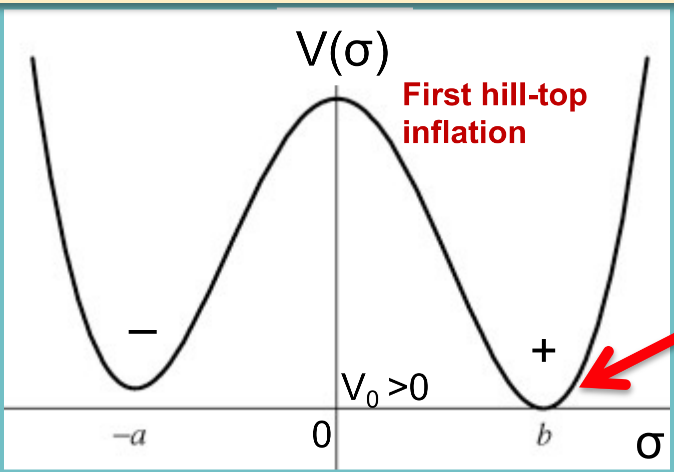
Collapse/collisions of Domain walls formed in theories with (approxmate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM,Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → **Imaginary parts** → **instabilities**

First Hill-top inflation = finite life –time → System **tunnels** to **RVM inflationary vacuum (GW condense)**

NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

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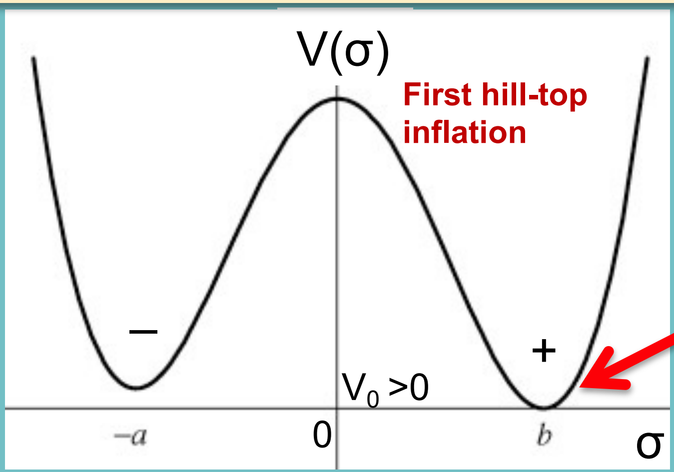

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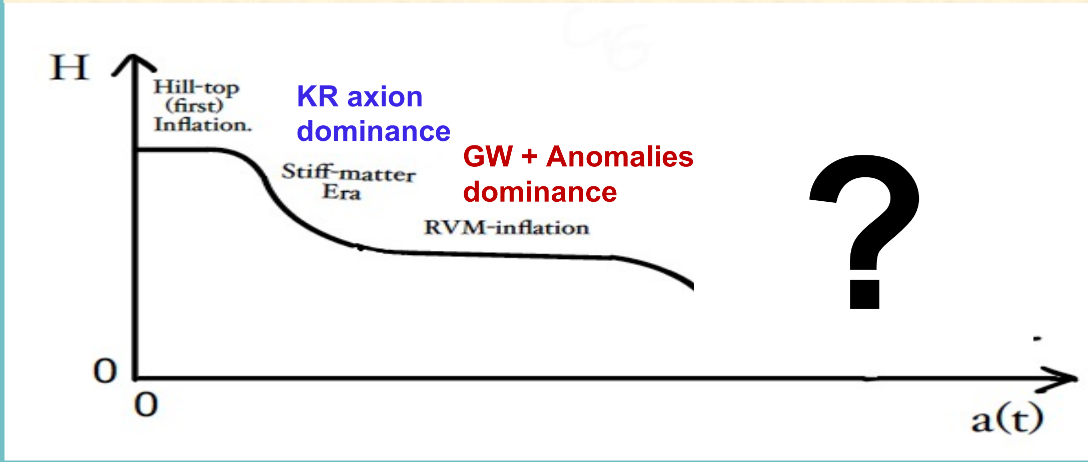
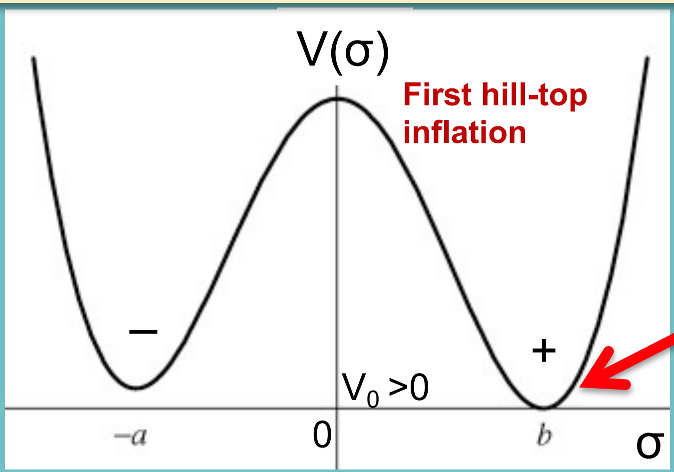


NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

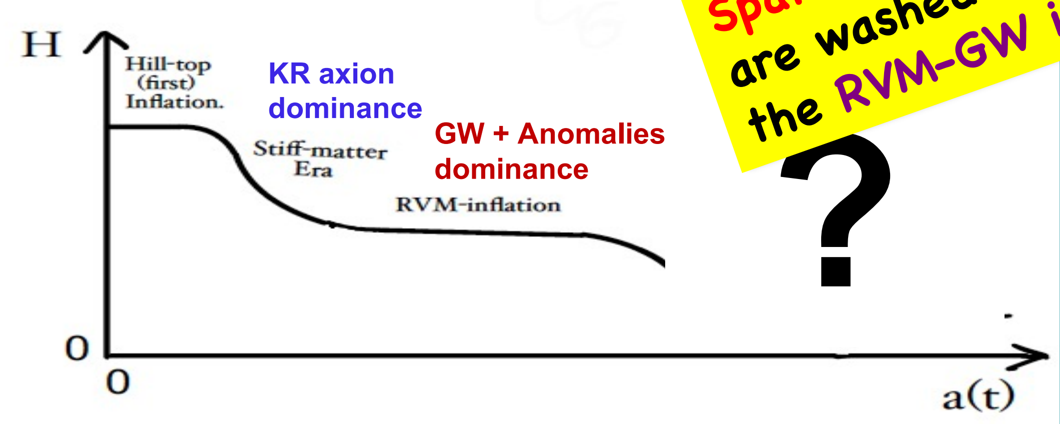


SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → Imaginary parts → instabilities

First Hill-top inflation = finite life –ti... System **tunnels** to RVM inflationary va...

First inflation ensures any Spatial inhomogeneities are washed out before the RVM-GW inflation



NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Parts


Dark Energy
("running vacuum model (RVM) type")
Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves
Spontaneous Lorentz + CPT Violation
from anomaly condensates

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Non-trivial if GW present

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$
$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big],$$

Primordial Gravitational Waves,
&
De Sitter space times &
Spontaneous Lorentz & CPT Violation

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$
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Primordial Gravitational Waves →
Condensate < ...> of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\, b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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$$+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Condensate < ...> of Gravitational Anomalies

Cosmological-Constant-like

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\, b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

Effective action contains **CP violating axion-like coupling**

$$\sqrt{-g}\,\mathcal{K}^\mu(\omega)_{;\mu}$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\Big) + \dots\Big]$$

(i) Assume de Sitter era, first, to discuss anomaly condensate in the presence of GW perturbation

(ii) deduce RVM vacuum behaviour

and

(iii) Inflation is obtained self consistently from RVM evolution

Effective action contains **CP violating axion-like coupling**

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\Big) + \dots\Big]$$

$$ds^2 = dt^2 - a^2(t)\Big[(1-h_+(t,z))\,dx^2 + (1+h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

b(x)=b(t)

Alexander, Peskin, Sheikh -Jabbari

μ = UV k-momentum Cut-off

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2\int^{\mu}\frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)$$

Homogeneity & Isotropy

$$\Theta = \sqrt{\frac{2}{3}}\,\frac{\kappa^3}{12}\,H\dot{b} \ll 1$$

$$\kappa = M_{\rm Pl}^{-1},\quad \dot{b} \equiv db/dt,\quad a(t) \sim e^{Ht}$$

H ≈ const. (inflation)

Solutions (backgrounds) to the Eqs of Motion

$\alpha' = M_s^{-2}$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0$$

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$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \propto \mathcal{K}^0$$

time evolution of Anomaly

μ = UV k-momentum Cut-off

$$\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\exp\left[-3Ht\left(1 - 0.73\times 10^{-4}\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]$$

$$\frac{\mu}{M_s} \simeq 15\left(\frac{M_{\rm Pl}}{H}\right)^{1/2} \quad \Rightarrow \quad \mathcal{K}^0 = \text{const.}$$

Planck Data $\quad H/M_{\rm Pl} < 10^{-4} \quad \Rightarrow$ **to ensure constant anomaly** $\mu / M_s = O(10^3)$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

$$\dot{\bar{b}} \propto \epsilon^{ijk} H_{ijk} = \text{constant}$$

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Spontaneous LV (+ CPTV) solution ⚠

Planck Data $\quad H/M_{\text{Pl}} < 10^{-4}$ ⇒ **to ensure constant anomaly $\mu / M_s = O(10^3)$**

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

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$$\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\text{begin}}(0)\exp\left[-3Ht\left(1 - 0.73\times 10^{-4}\left(\frac{H}{M_{\text{Pl}}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]$$

$$\frac{\mu}{M_s} \simeq 15\left(\frac{M_{\text{Pl}}}{H}\right)^{1/2} \quad \Rightarrow \quad \mathcal{K}^0 = \text{const.}$$

No transplanckian modes !

Planck Data $H/M_{\text{Pl}} < 10^{-4}$ ⇒ **to ensure constant anomaly**

$\mu = O(10^3)\, M_s \leq M_{planck}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

$$\dot{\bar{b}} \sim \varepsilon_{ijk} H^{ijk}$$

≈ constant torsion

Using **slow-roll assumption** *b*

$$\varepsilon = \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

Using **slow-roll assumption** *b*

2.6 x 10^{-5} M_{Pl} < M_s ≤ 10^{-4} M_{Pl}

NEM + Solà (2021)

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Constant anomaly during inflation, no transplanckian modes !

NB:

$$\Theta \equiv \sqrt{\frac{2}{3}}\frac{\alpha'\kappa}{12} H \dot{\bar{b}} \ll 1$$

$$\dot{\bar{b}} \ll H/\kappa$$

$$H/M_s \ll 3.83\,,\; H \simeq (10^{-5} - 10^{-4}) M_{\rm Pl}$$

$$\frac{M_{\rm Pl}}{M_s} \ll 3.83 \times (10^4 - 10^5).$$

$$M_s \le 10^{-4} M_{\rm Pl}$$

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("running vacuum model (RVM) type")

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+
torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz + CPT Violation
from anomaly condensates

Dynamical Inflation of RVM type
without external inflatons

Solutions (backgrounds) to the Eqs of Motion

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$$\varepsilon = \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\bar{b}}^2 \sim 10^{-2}$$ **Planck Data**

$$\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

@ end of Inflationary era

$$b_{\rm end} \sim b_{\rm initial} + 0.14\,M_{\rm Pl}\,H_{\rm infl}\,t_{\rm end},$$

$$t_{\rm end}H_{\rm infl} \sim \mathcal{N} = \text{e} - \text{foldings}$$

~ 55-70

Fix $b_{initial}$ to arrange approx. constant condensate during appropriate time period (inflation)

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

e-foldings

Positive Cosmological Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \, (\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

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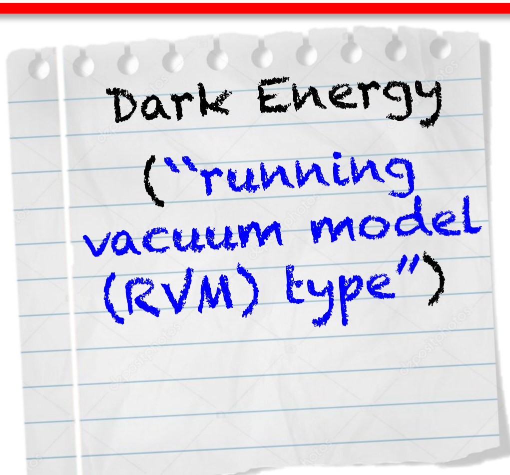
Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^{7} \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$



Gravitational Anomaly Condensates → Dynamical Inflation

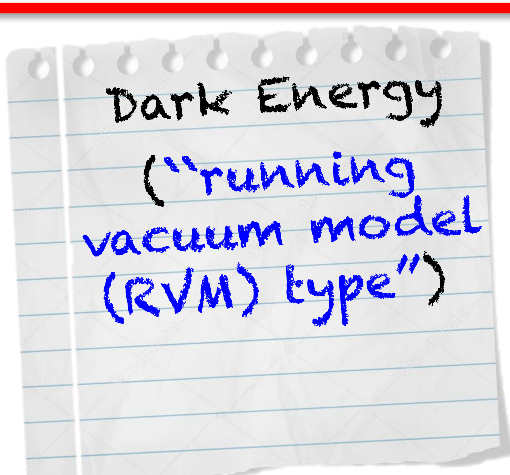
NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3 M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$



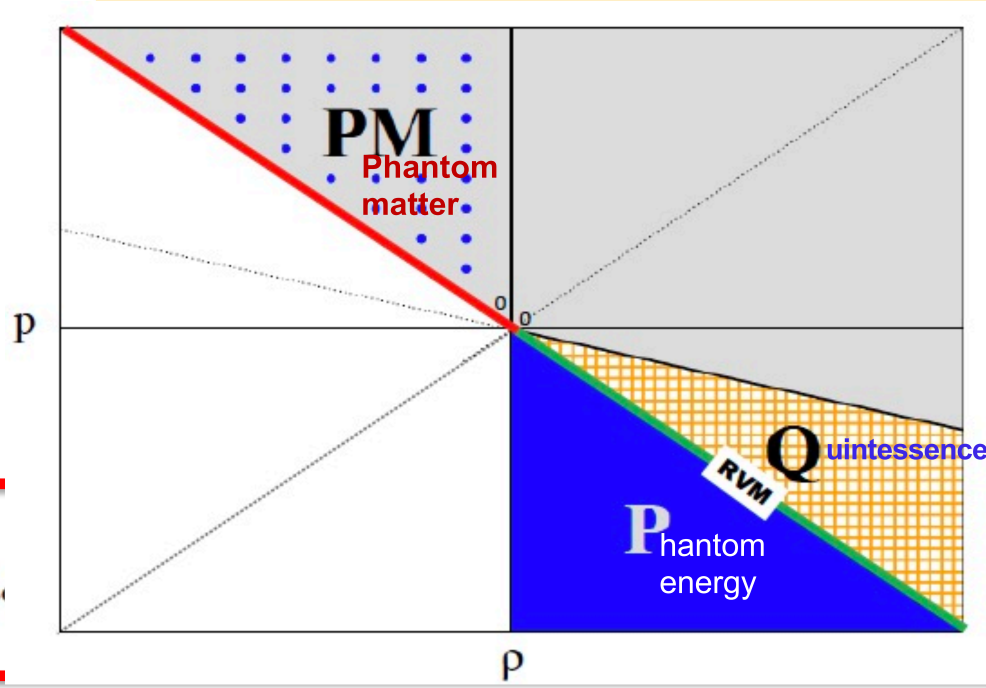
Equation of state :

$0 > \rho_b + \rho_{gCS} = -(p_b + p_{gCS})$ cf. phantom "matter"

$0 < \rho_\Lambda = -p_\Lambda$ → dominates →

$0 < \rho_b + \rho_{gCS} + \rho_\Lambda = -(p_b + p_{gCS} + p_\Lambda)$ true RVM vacuum

Gravitational Anomaly Condensates → Dynamical Inflation

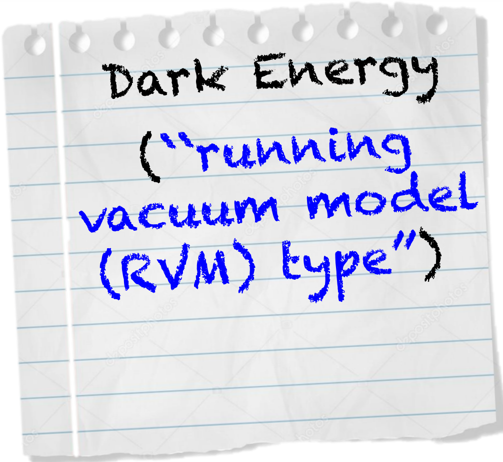


NEM, Solà

$$10^7 \, \epsilon \mathcal{N} \, H^4 > 0$$

Positive Cosmological Constant-like

ρ_{t} … $)^2 + \big(1.17 - 1.37\big) \times 10^7 \, (\frac{H}{M_{\rm Pl}})^4 \big] > 0$



Equation of state :

$0 > \rho_b + \rho_{gcs} = -\,(p_b + p_{gcs})$ cf. phantom "matter"

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Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

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RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

Dark Energy ("running vacuum model (RVM) type")

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\overline{b}}^2 \sim 10^{-2}$

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$\omega = \rho_m / p_m$ *m* = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\dot{H} + \frac{3}{2}(1+\omega)H^2\left(1 - \nu - \frac{c_0}{H^2} - \alpha\frac{H^2}{H_I^2}\right) = 0$$

Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$D > 0$

Early de Sitter (unstable)

$D a^{4(1-\nu)} \ll 1 \Rightarrow H^2 = (1-\nu) H_I^2/\alpha$

Radiation

$D a^{4(1-\nu)} \gg 1 \Rightarrow H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era

$$H^2(a) = H_0^2\left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$$

$\tilde{\Omega}_{\Lambda 0}$ dominant

Gravitational Anomaly Condensates → Dynamical Inflation

NEM, Solà

Cannot obtain such terms
in **ordinary Quantum Field Theories**
You need the **condensate of**
the gravitational anomalies
which have **CP-violating couplings**
with the **gravitational axions**

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7\times 10^{-3}\Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \big(1.17 - 1.37\big)\times 10^7 \,\big(\frac{H}{M_{\rm Pl}}\big)^4\Big] > 0$$

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in **ordinary Quantum Field Theories**
You need the **condensate of the gravitational anomalies**
which have **CP-violating couplings**
with the **gravitational axions**

Another important role of CP-violation in Early Universe

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7\times 10^{-3}\Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big)\times 10^7\,\Big(\frac{H}{M_{\rm Pl}}\Big)^4\Big] > 0$$

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Basilakos, NEM, Solà

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Negative coefficient ν < 0
due to CS anomaly
in early Universe, unlike
late-era RVM

RVM-like terms
drive inflation
contain scalar d.o.f.
from the anomaly
condensate

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2} \frac{1}{(H M_{\rm Pl})^2} \dot{b}^2 \sim 10^{-2}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

Undiluted KR axion background at the end of Inflation

@ end of Inflationary era

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha \left[\sqrt{-g} \left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\,\kappa} \mathcal{K}^\alpha(t) \right) \right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\,\kappa} \mathcal{K}^0 \sim \text{constant}$$

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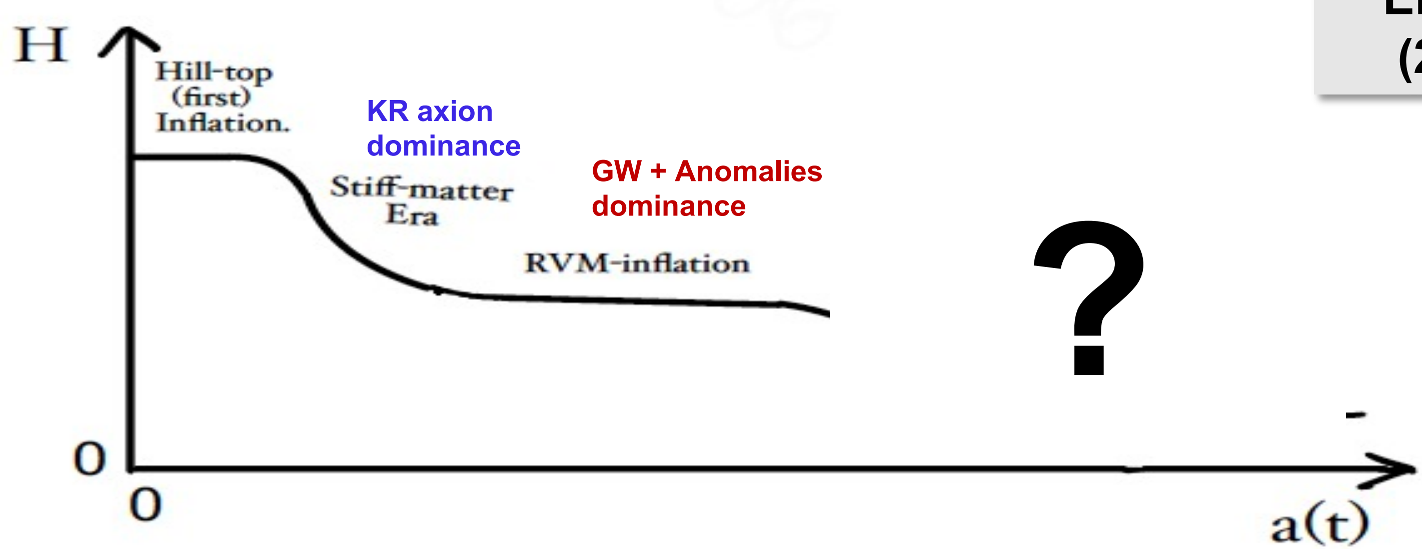
Important for Leptogenesis @ radiation era

Post-RVM-Inflation Eras & Evolution



NEM,Solà
EPJ-ST
(2020)

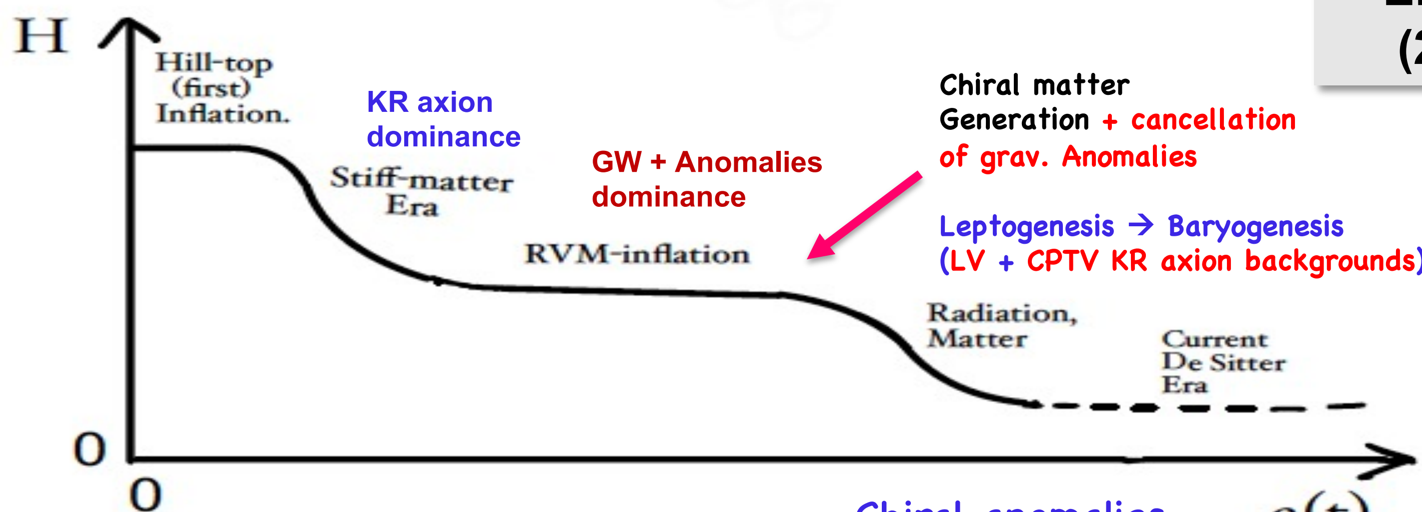
Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM, Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

NEM, Solà EPJ-ST (2020)



NEM, Sarkar + De Cesare, Bossingham

Chiral anomalies Remain in matter era

→ KR axion mass generation through QCD instantons (Dark Matter)

3. The Whole

Stringy-RVM Cosmological Evolution

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

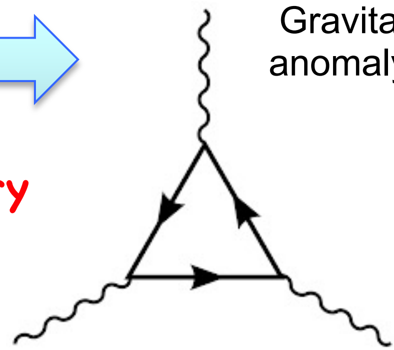
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves →



Gravitational anomaly (GA) →

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

Cancellation of GA

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

NEM, Sarkar + De Cesare, Bossingham

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

ΔL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

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Cosmic Time

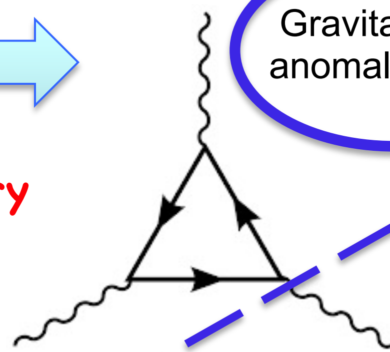
Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

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chiral matter generation @ inflation exit

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Cosmic Time

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Basilakos, NEM, Solà

forward direction

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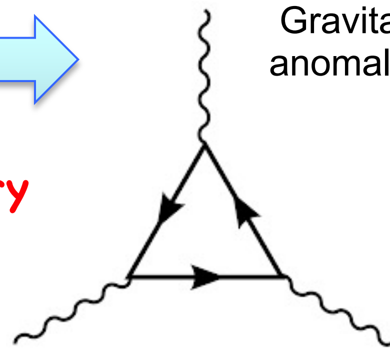
Primordial Gravitational Waves → Gravitational anomaly (GA) →



From a pre-inflationary era after Big-Bang

Undiluted constant KR axial backround

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chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$N_I \;\to\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$ AL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

B-L conserving sphaleron processes → **Baryongenesis**

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Chiral anomalies @ QCD era (instantons)

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

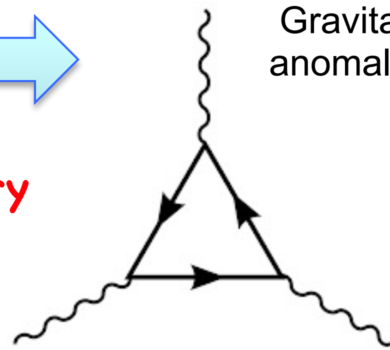
Basilakos, NEM, Solà

Big-Bang, pre-inflationary phase

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\, M_{\rm Pl}\, H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$ Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

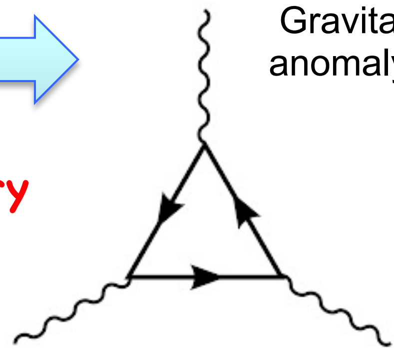
forward direction

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) →



From a pre-inflationary era after Big-Bang

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chiral matter generation @ inflation exit

Cancellation of GA

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$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era Possible potential (mass) generation f b – → axion Dark matter

Modern de-Sitter Era GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

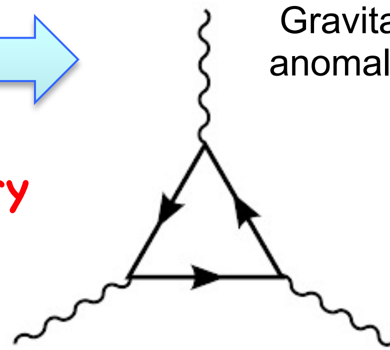
forward direction

Basilakos, NEM, Solà

Big-Bang, pre-inflationary phase

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

From a pre-inflationary era after Big-Bang

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$$

$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

Radiation Era

$$B_0 \propto \dot{\overline{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

$$B_0\Big|_{\rm today} \sim 2.435 \times 10^{-34}\,\text{eV}$$

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era

Possible potential (mass) generation f... b – → axion Dark matter

Modern de-Sitter Era

GA resurfacing

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Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Basilakos, NEM, Solà

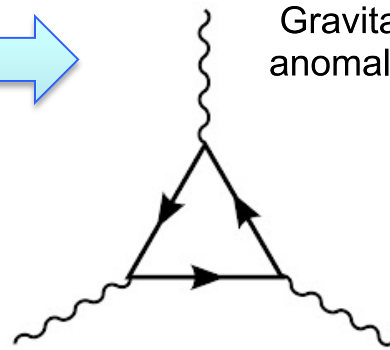
Cosmic Time

Big-Bang, pre-inflationary phase

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphal

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Need to understand Modern Era better

Matter Era

Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

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Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Big-Bang, pre-inflationary phase

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gr Wa

Gravitational

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$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar{b}}\,\delta_{\mu 0}$$

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chiral matter generation

lation exit

Distinguishing feature from ΛCDM

Alleviate data tensions

$$\rho_{\rm RVM}^{\rm today}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu_0\left(\frac{H_0}{M_{\rm Pl}}\right)^2\right)$$

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\frac{3}{\kappa^2}\,c_0 \simeq 10^{-122}\,M_{\rm Pl}^4$$

Gómez-Valent Solà

Ra

B_0

Lep

RH

N_I

B-L

Ma

ion matter

session talks

$$\varepsilon' M_{\rm Pl} H_0$$

Modern de-Sitter Era

GA resurfacing

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

RVM-type Running Dark Energy

Gómez-Valent, Solà

Could **Alleviate** Tensions in Data, e.g.
H_0 , σ_8 tensions

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

$$\frac{3}{\nu_0^2}\, c_0 \simeq 10^{-122}\, M_{\rm Pl}^4$$

$$\rho_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu_0\left(\frac{H_0}{M_{\rm Pl}}\right)^2 + \beta\,\frac{H_0^4}{M_{\rm Pl}^4}\right), \quad \beta > 0.$$

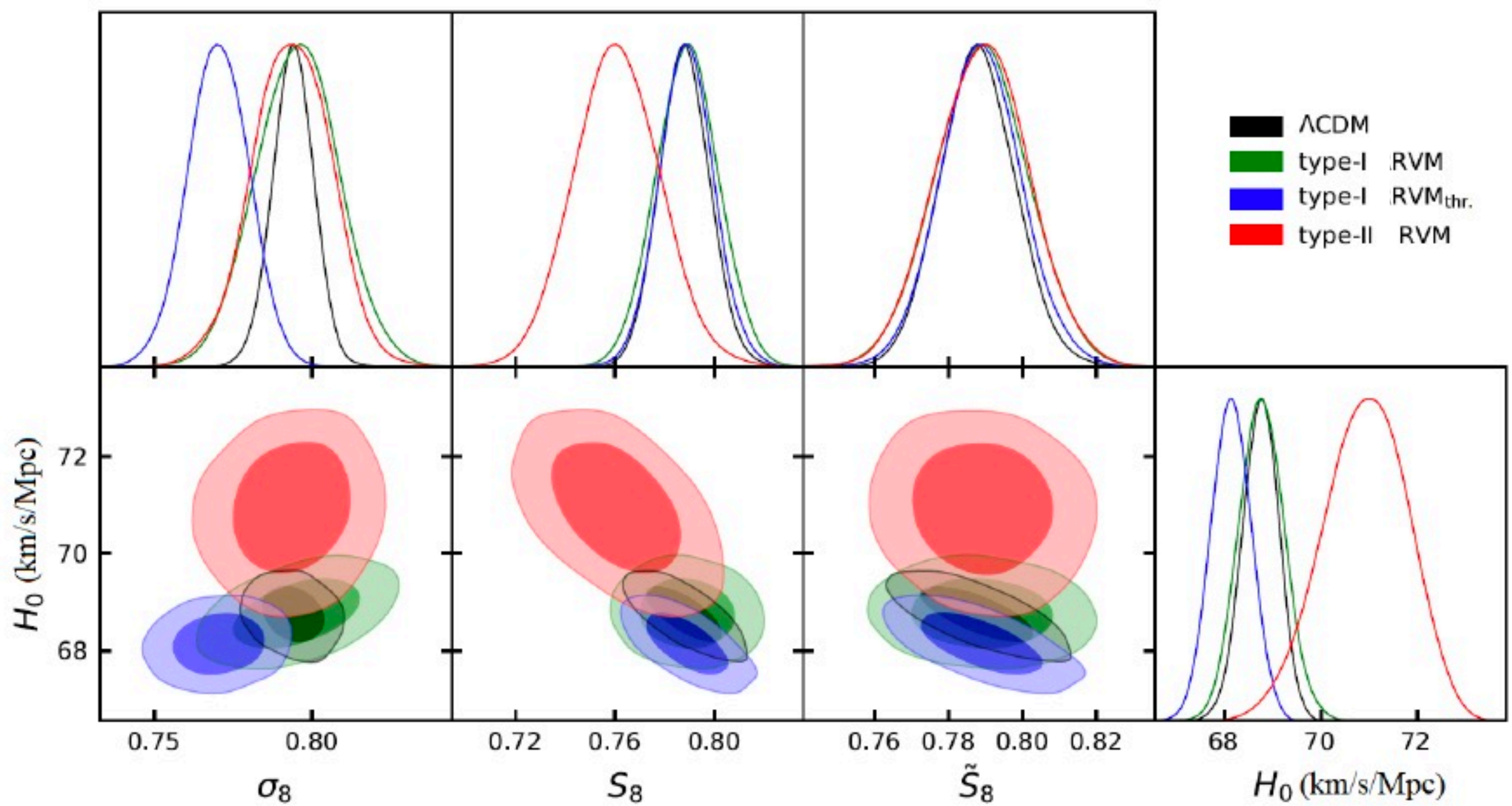
Not dominant today

Running RVM Dark Energy

Session talks

Solà, Gómez-Valent,
De Cruz Perez, Moreno-Pulido,
(Planck 2018 data)

Alleviation of the H_0, σ_8 tension by RVM model

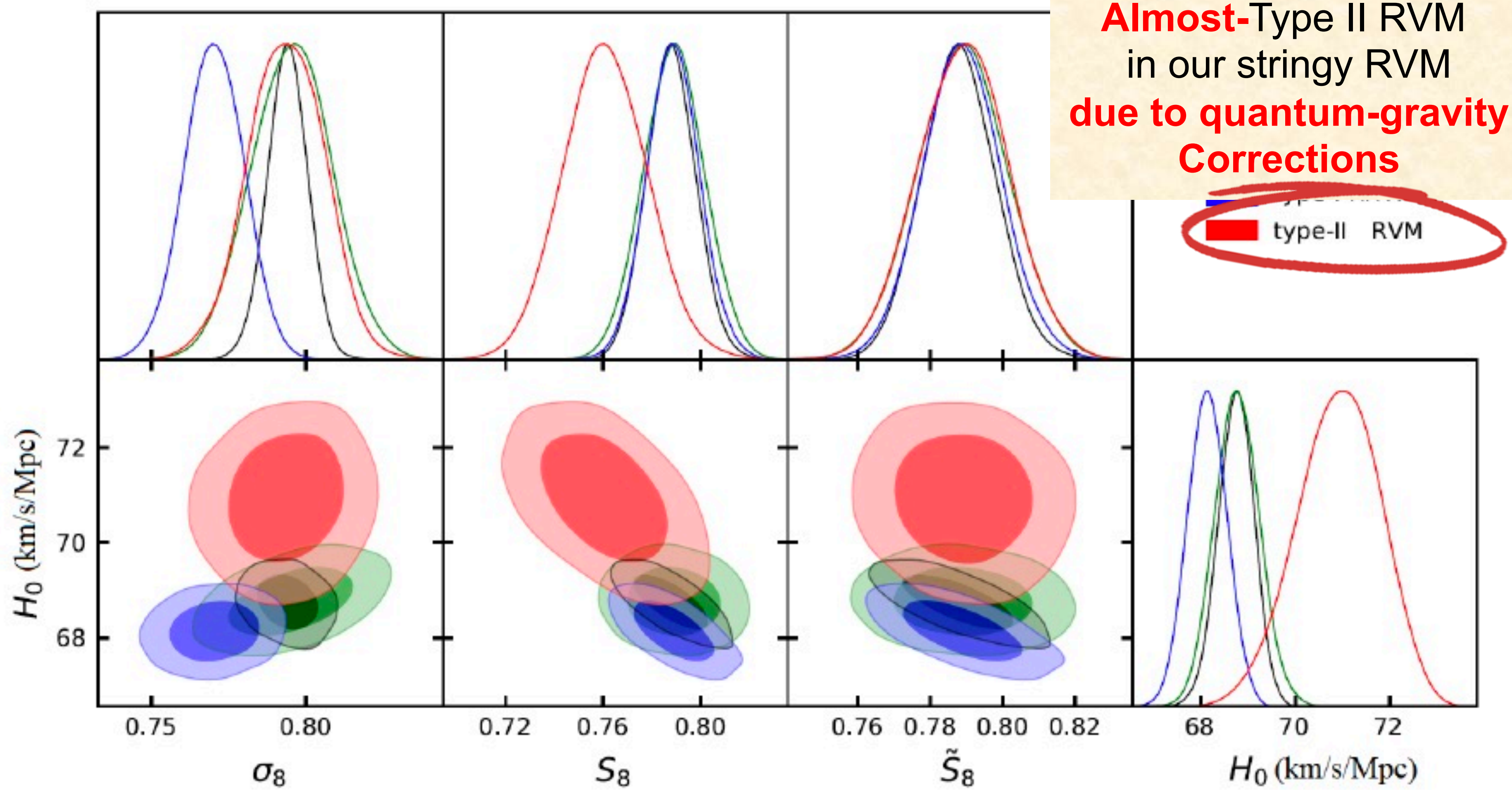


NEM, Solà (2021)

Integrating out graviton flcts

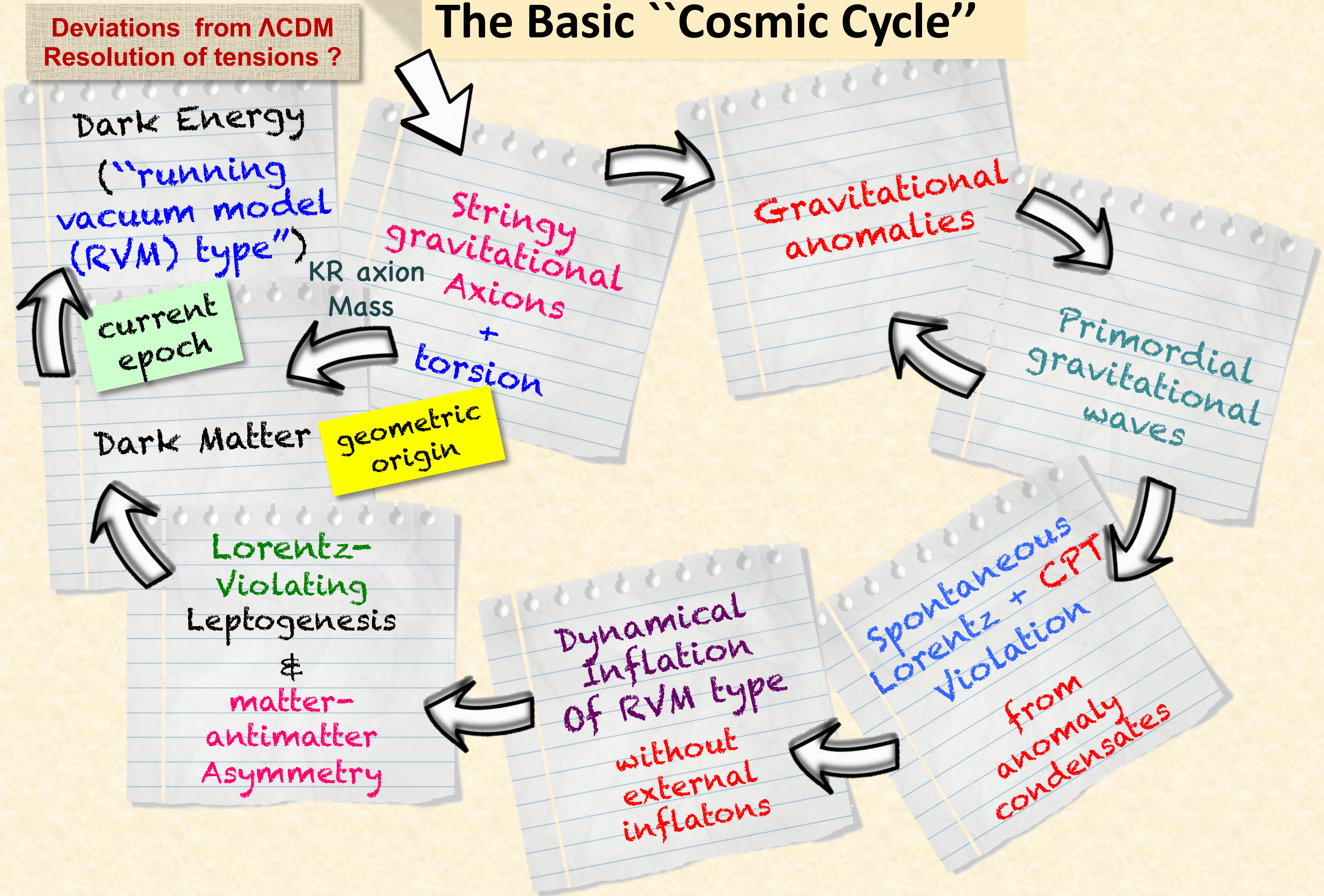
$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (c_3 + c_4 \ln H)\, H^4 + \Lambda$$

Almost-Type II RVM
in our stringy RVM
due to quantum-gravity Corrections



4. Conclusions & Outlook

The Basic ``Cosmic Cycle''



The Parts/the Whole

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

We exist because of Anomalies !

Paraphrasing C. Sagan: we are anomalously made of star stuff !

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

$$1.17 \times 10^{-5}\ \text{eV} \gtrsim m_b \gtrsim 1.17 \times 10^{-8}\ \text{eV}$$

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

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We exist because of Anomalies !

OUTLOOK: (i) Incorporate other model-dependent stringy axions → Axiverse
Interesting Cosmology (*eg* Marsh 2015)
could be ultralight → AION etc

Leptogenesis induced by RHN (tree-level) decays

Spontaneous

$$1.17\times10^{-5}\ \mathrm{eV} \gtrsim m_b \gtrsim 1.17\times10^{-8}\ \mathrm{eV}$$

Matter Era

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malies !

OUTLOOK: (ii) Look for imprints of the LV & CPTV KR axial background in CMB in early eras.

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

$1.17 \times 10^{-5}\ \mathrm{eV} \gtrsim m_b \gtrsim 1.17 \times 10$ eV

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

Pre RVM-Inflationary era

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

OUTLOOK: (iii) Can we also get evidence of v < 0 coefficient of H^2 during RVM inflation?

$$\rho_{\rm RVM}^{\rm string} \simeq 3\,M_{\rm Pl}^4\Big[-1.7\times 10^{-3}\Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \mathcal{O}(10^7)\Big(\frac{H}{M_{\rm Pl}}\Big)^4\Big]$$

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

$$1.17\times 10^{-5}\ {\rm eV} \gtrsim m_b \gtrsim 1.17\times 10^{-8}\ {\rm eV}$$

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

References:

Thank you!

a microscopic (string-inspired) model for RVM Universe....

Links with :
spontaneous Lorentz violation
(via (gravitational axion)
backgrounds)
and
Matter-Antimatter Asymmetry
in theories with
Right-Handed Neutrinos

Basilakos, NEM, Solà
(i)JCAP 12 (2019) 025
(ii) IJMD28 (2019) 1944002
(iii) Phys.Rev.D 101 (2020) 045001
(iv) Phys.Lett.B 803 (2020) 135342
(v) Universe 2020, 6(11), 218
NEM, Solà
(vi) arXiv: 2012.07971 (2020) EPJst
(vii) arXiv: 2105.02659 (2021)

(i) NEM & Sarben Sarkar, EPJC 73 (2013), 2359
(ii) John Ellis, NEM & Sarkar, PLB 725 (2013), 407
(iii) De Cesare, NEM & Sarkar, EPJC 75 (2015), 514
(iv) Bossingham, NEM & Sarkar, EPJC 78 (2018), 113; 79 (2019), 50
(v) NEM & Sarben Sarkar, EPJC 80 (2020), 558

SPARES

Post-RVM-Inflationary Era

Cancellation of Gravitational Anomalies

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\alpha'}{\kappa}b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots,$$

$$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j$$

Chiral current, including RHN

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM, Solà (2019-20)

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$$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j;$$

Chiral current, including RHN

$$\partial_\mu\Big[\sqrt{-g}\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\Big(\frac{\alpha_{\rm EM}}{2\pi}\sqrt{-g}\,F^{\mu\nu}\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\sqrt{-g}\,G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}\Big)$$

chiral U(1) Gluon QCD

Cancellation of Gravitational Anomalies in Radiation Era by:

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$$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j;$$

Chiral current, including RHN

$$\partial_\mu\Big[\sqrt{-g}\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\frac{1}{96}K^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\Big(\frac{\alpha_{\rm EM}}{2\pi}\sqrt{-g}\,F^{\mu\nu}\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\sqrt{-g}\,G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}\Big)$$

chiral U(1)   Gluon QCD

instanton generated potential for KR axion b-field during matter dominance → axion Dark Matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

forward direction

Basilakos, NEM, Solà

$$2.6 \times 10^{-5}\, M_{Pl} < M_s \leq 10^{-4}\, M_{Pl}$$

$$V_b^{\rm QCD} \simeq \Lambda_{\rm QCD}^4 \Big(1 - \cos(\frac{b}{f_b})\Big), \quad f_b \equiv \sqrt{\frac{8}{3}}\frac{\kappa}{\alpha'} = \sqrt{\frac{8}{3}}\Big(\frac{M_s}{M_{\rm Pl}}\Big)^2 M_{\rm Pl}$$

$$\Lambda_{\rm QCD} \sim 218 \ \text{MeV}$$

Remaining chiral anomalies

@ QCD Era

$$S_b^{\rm eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\, \partial^\mu b - \frac{\alpha'}{\kappa}\sqrt{\frac{3}{8}}\frac{\alpha_s}{8\pi} b(x)\, G^a_{\mu\nu}\, \widetilde{G}^{a\mu\nu}\Big]$$

T~ 200 MeV

$$1.17 \times 10^{-5}\,\text{eV} > m_b > 1.17 \times 10^{-8}\,\text{eV}$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM, Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Scale factor ***a(t) ~ T*** $^{-1}$

Possibly also QCD

$$\dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

sufficiently slowly varying during leptogenesis (brief) epoch → qualitatively similar to approximately const. background

Bossingham, NEM, Sarkar

Lorentz- & CPT-Violating

Leptogenesis →

→ Baryogenesis

in models with Massive Right-handed Neutrinos

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

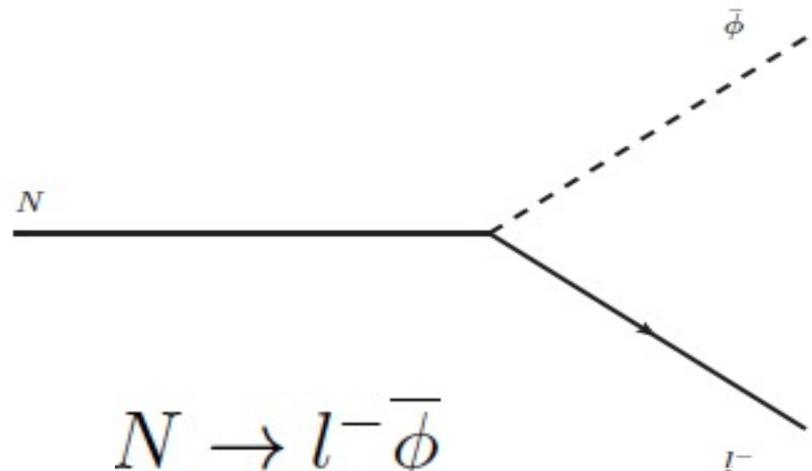
Early Universe
$T \gg T_{EW}$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

Heavy Right-Handed-Neutrinos (*N*) interact with **axial (approx.) constant background** with only temporal component $B_0 \propto \dot{b} \neq 0$

Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to Lorentz/CPTV Background



$$N \rightarrow l^+ \phi \qquad\qquad N \rightarrow l^- \overline{\phi}$$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega + B_0}{\Omega - B_0} \neq \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV & LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

$$\mathcal{L} = i\overline{N}\not\partial N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not B\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

(approx.) Constant B_0 Background

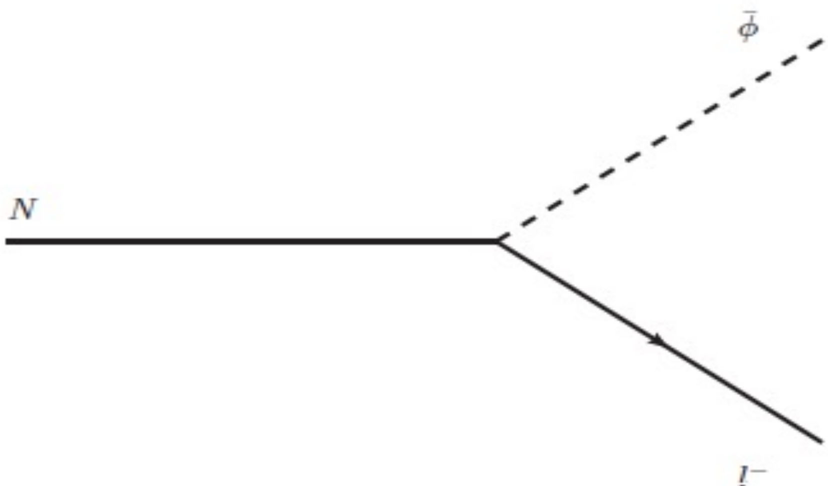
CPT Violation

Early Universe
$T \gg T_{EW}$

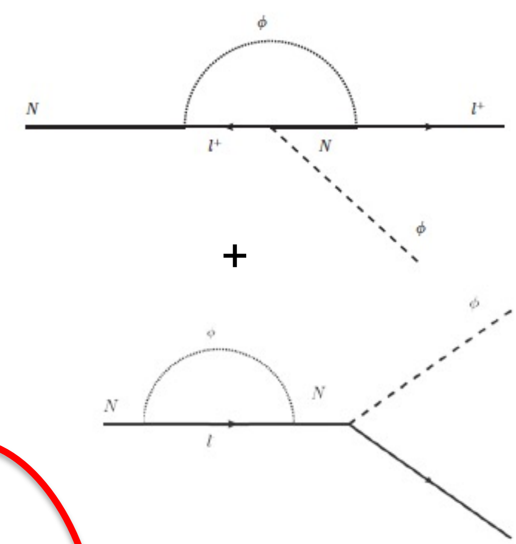
Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\;\phi\,\overline{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

CPTV Thermal L

$$\mathcal{L} = i\overline{N}\not\partial N - \frac{m}{2}\left(\overline{N^c}N + \overline{N}N^c\right) - \overline{N}\not B\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10}, \quad \rightarrow \quad \frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Solving system of Boltzmann eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}$$

$$Y_k \sim 10^{-5}$$
$$m \geq 100\text{TeV} \rightarrow$$
$$B^0 \sim 1\text{MeV}$$
$$T_D \simeq m \sim 100\ \text{TeV}$$

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal L

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

CPT Violation

Early Universe
T > 10^5 GeV

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Equilibrated electroweak
B+L violating sphaleron interactions

B-L conserved

Environmental Conditions Dependent

$$L = \frac{2}{M} l_L l_L \phi\phi + \mathrm{H.c.}$$

where

$$l_L = \begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$$

Observed Baryon Asymmetry In the Universe (BAU)

Fukugita, Yanagida,

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV