Fornax globular cluster timing problem as a test of dark matter properties

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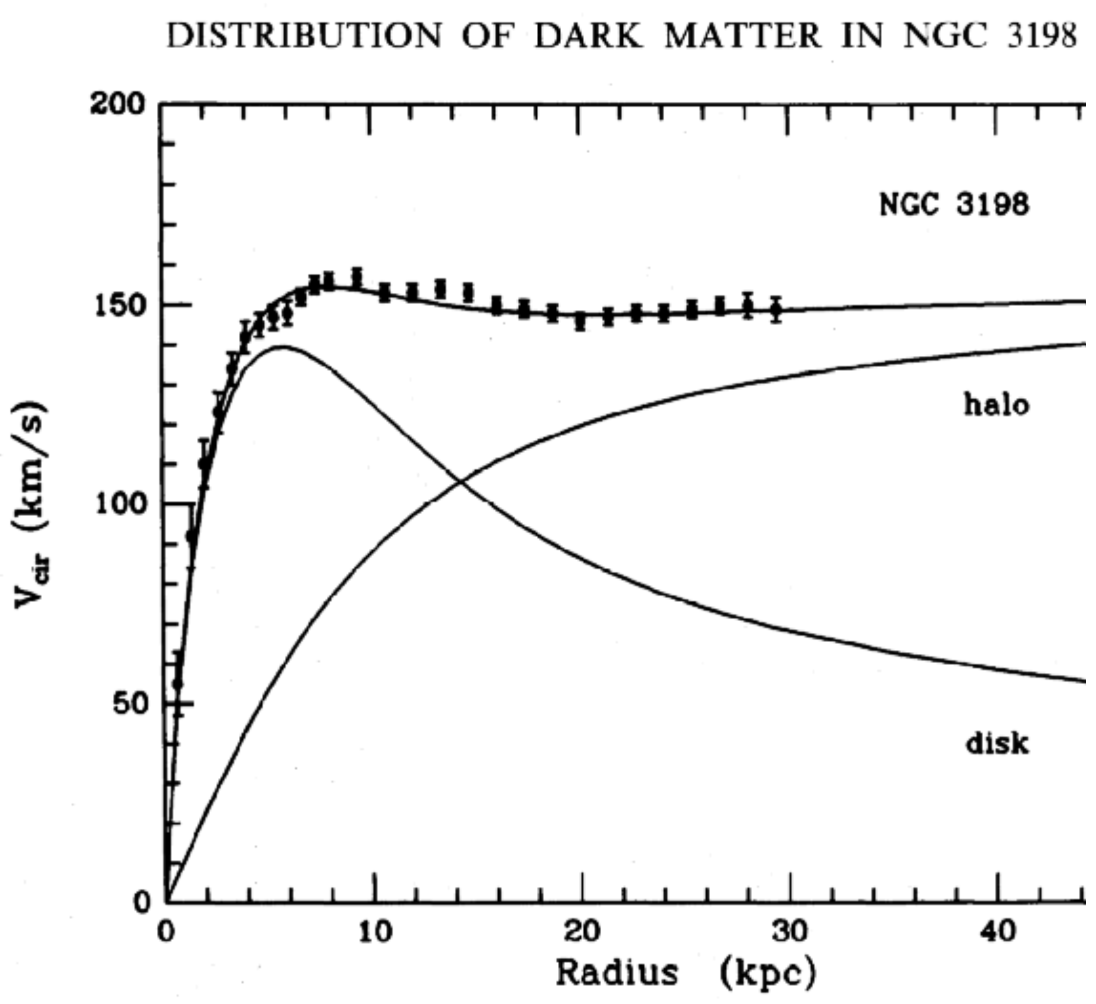
IFAE

with Nitsan Bar, Kfir Blum and Hyungjin Kim, arXiv:2102:11522

Beyond rotation curves: effects of DM on galactic scales

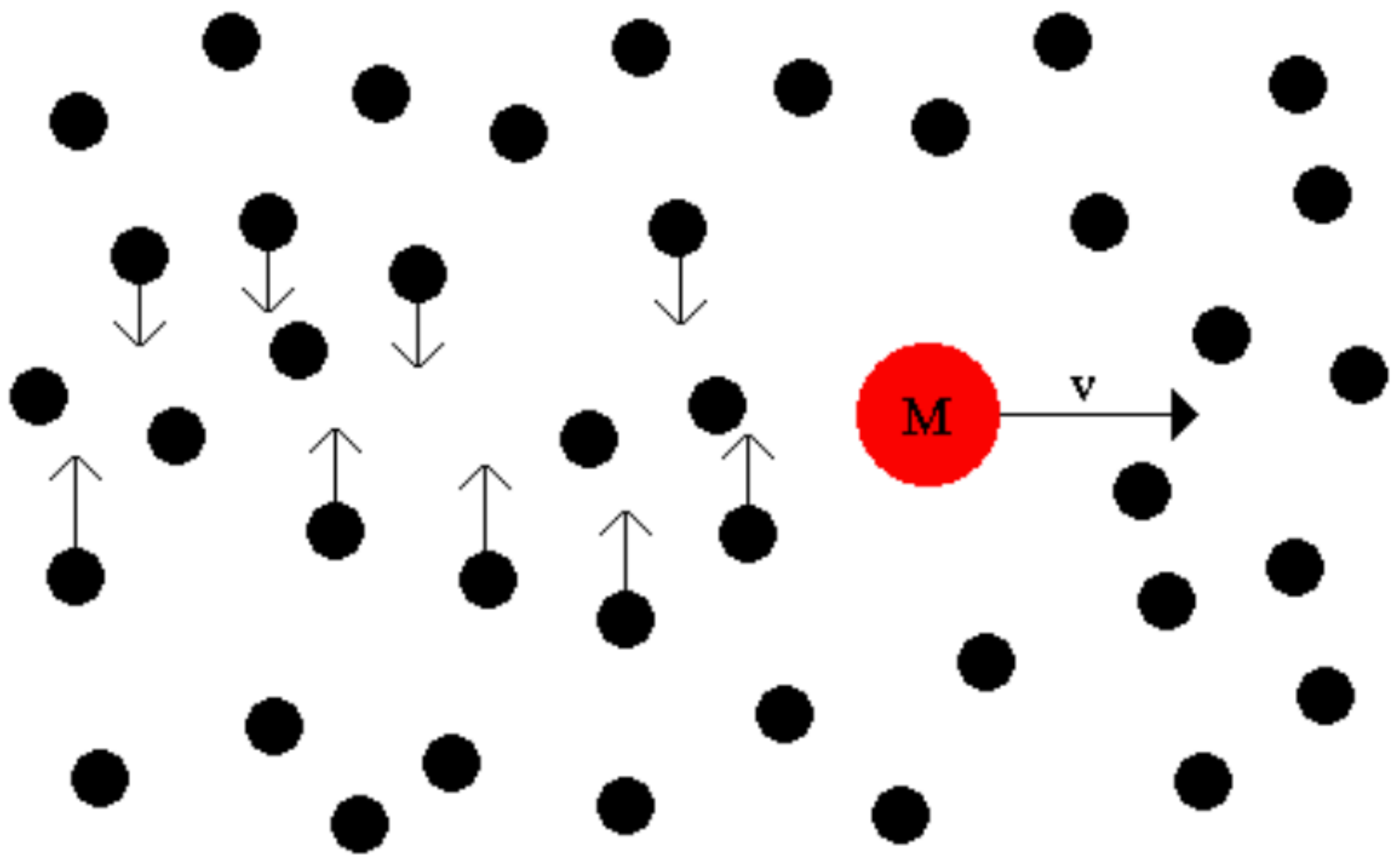


Galactic rotation curves probe the gravitational potential Φ → presence of dark matter

The presence of a ‘medium’ also has **dynamical effects:**
tidal disruption, dynamical friction, dynamical heating…

They depend on the nature of DM!!

Dynamical friction - general intuition



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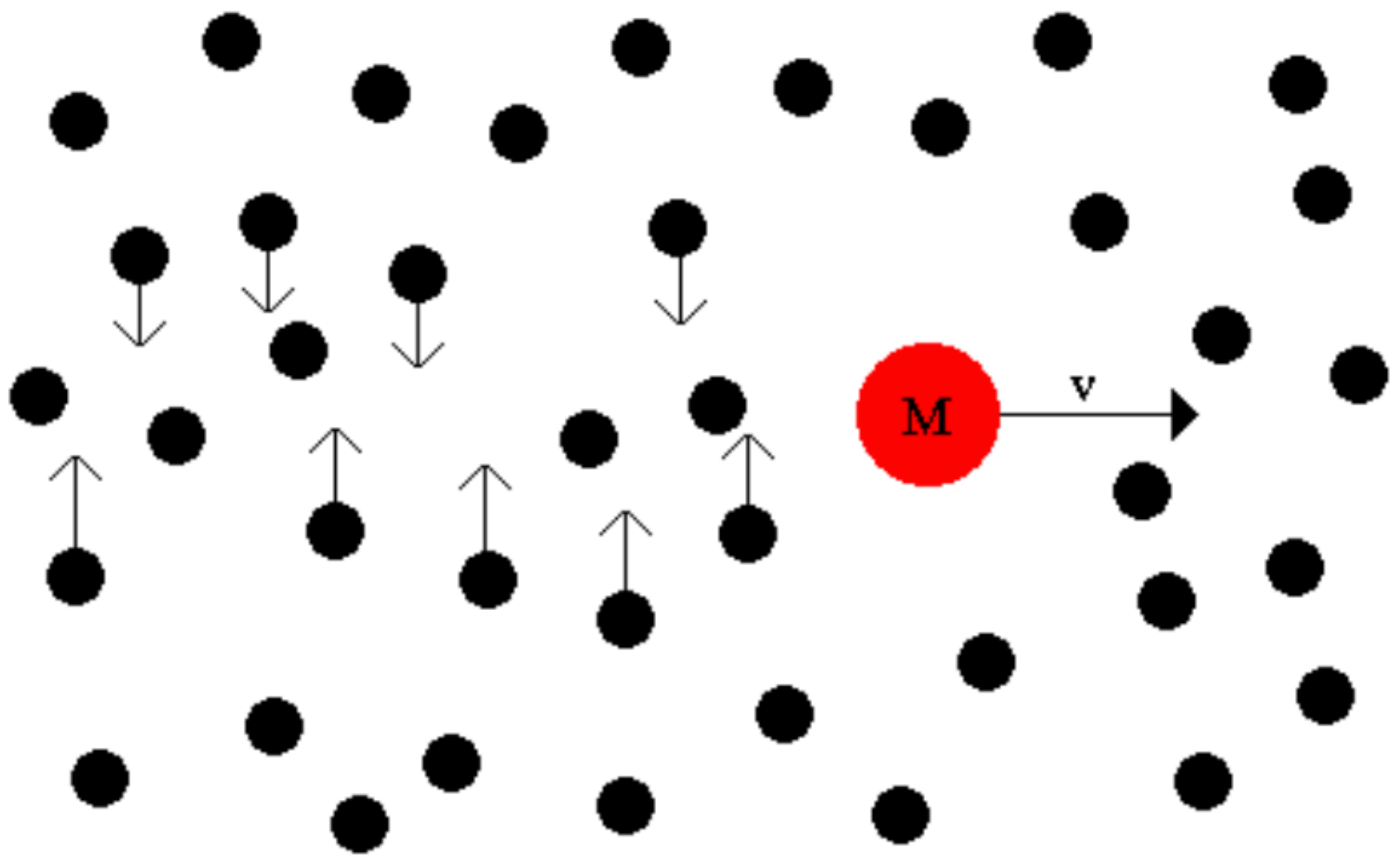
A mass *M* **moving** in a medium of particles with mass *m* interchanging momentum gravitationally:



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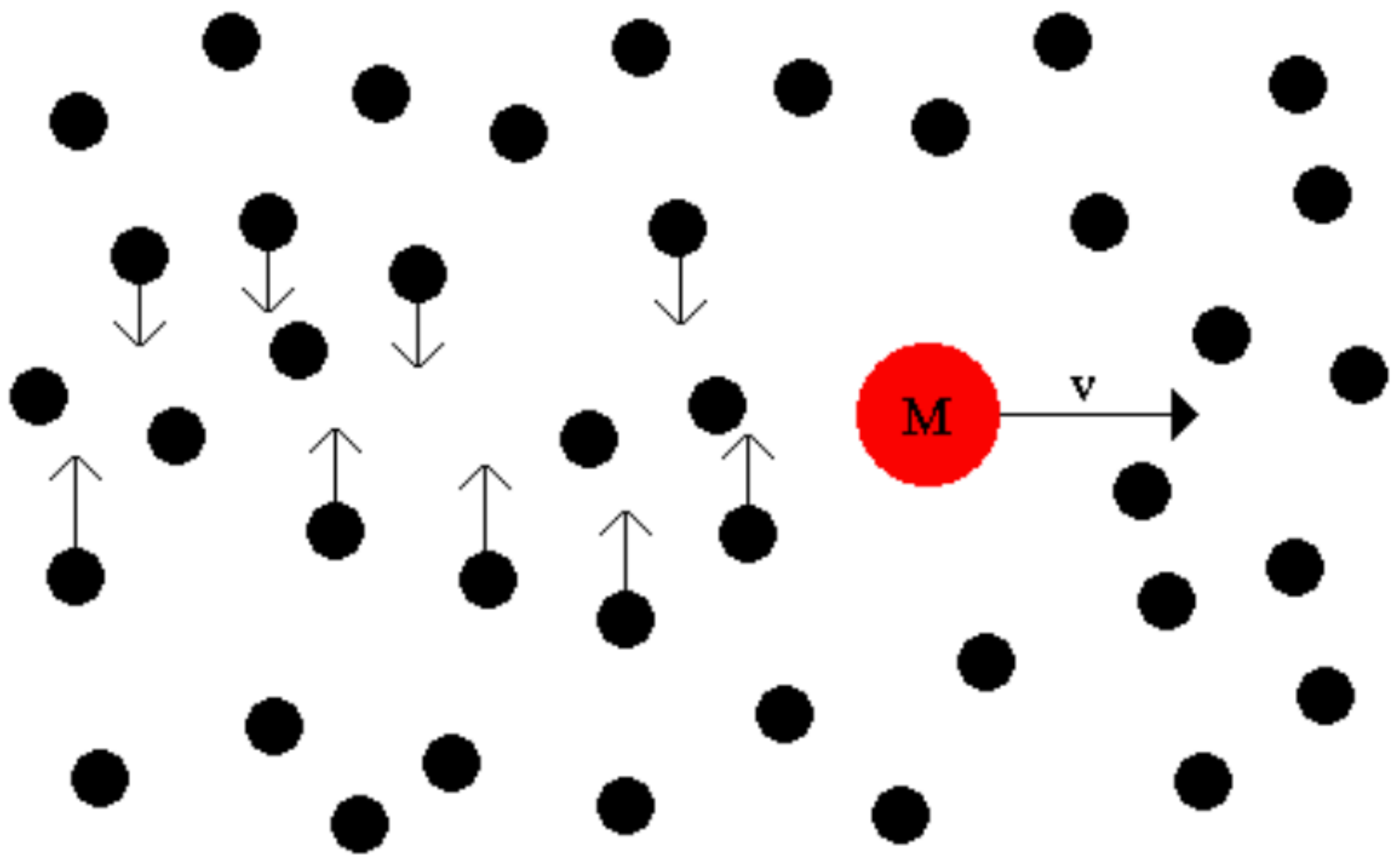

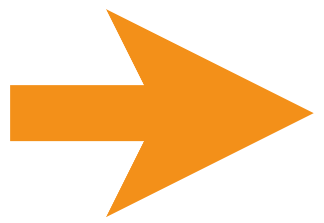

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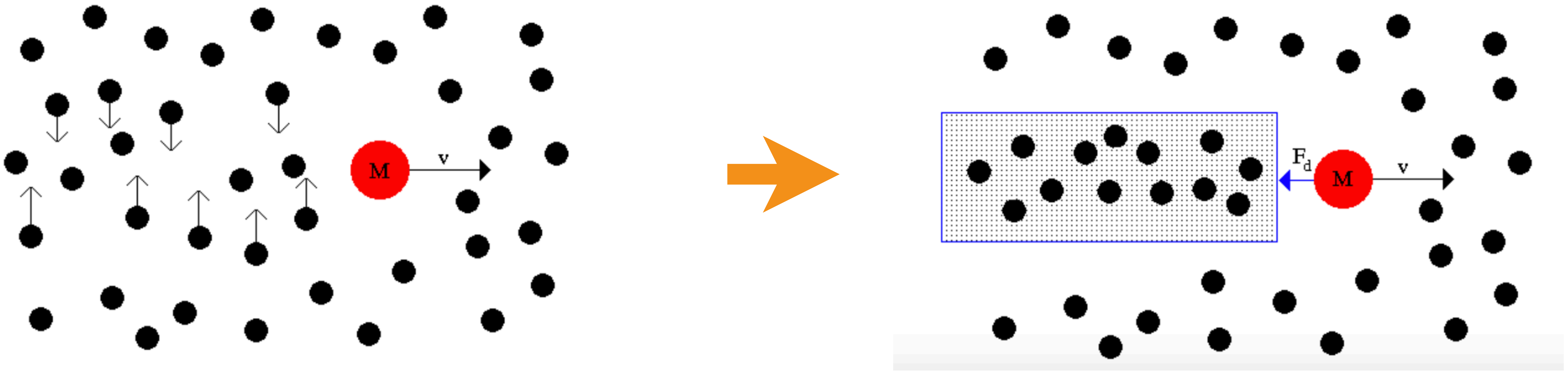

Figure by J. Schombert

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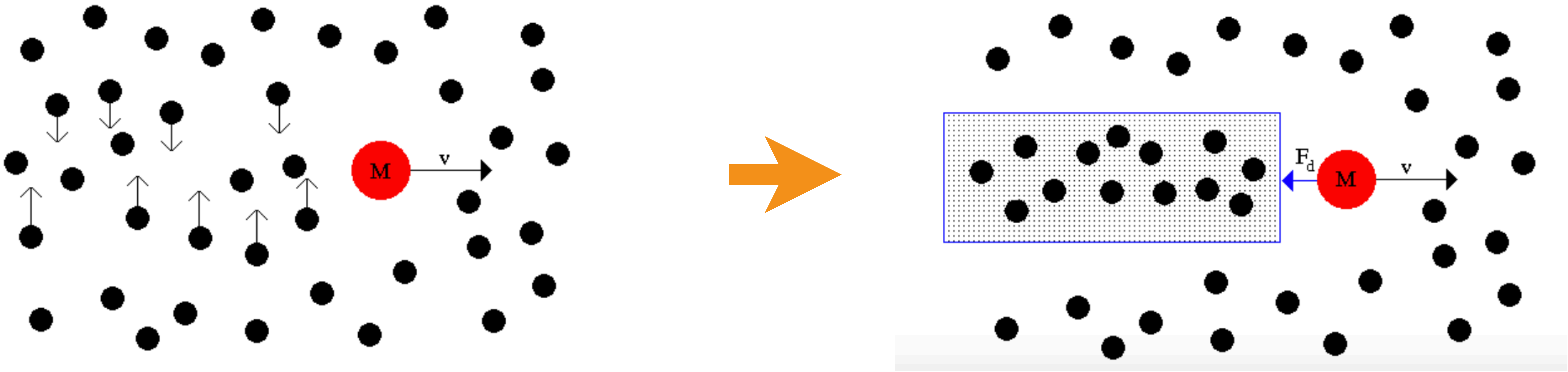


if *M* orbiting: lost of *K* makes it fall to the center

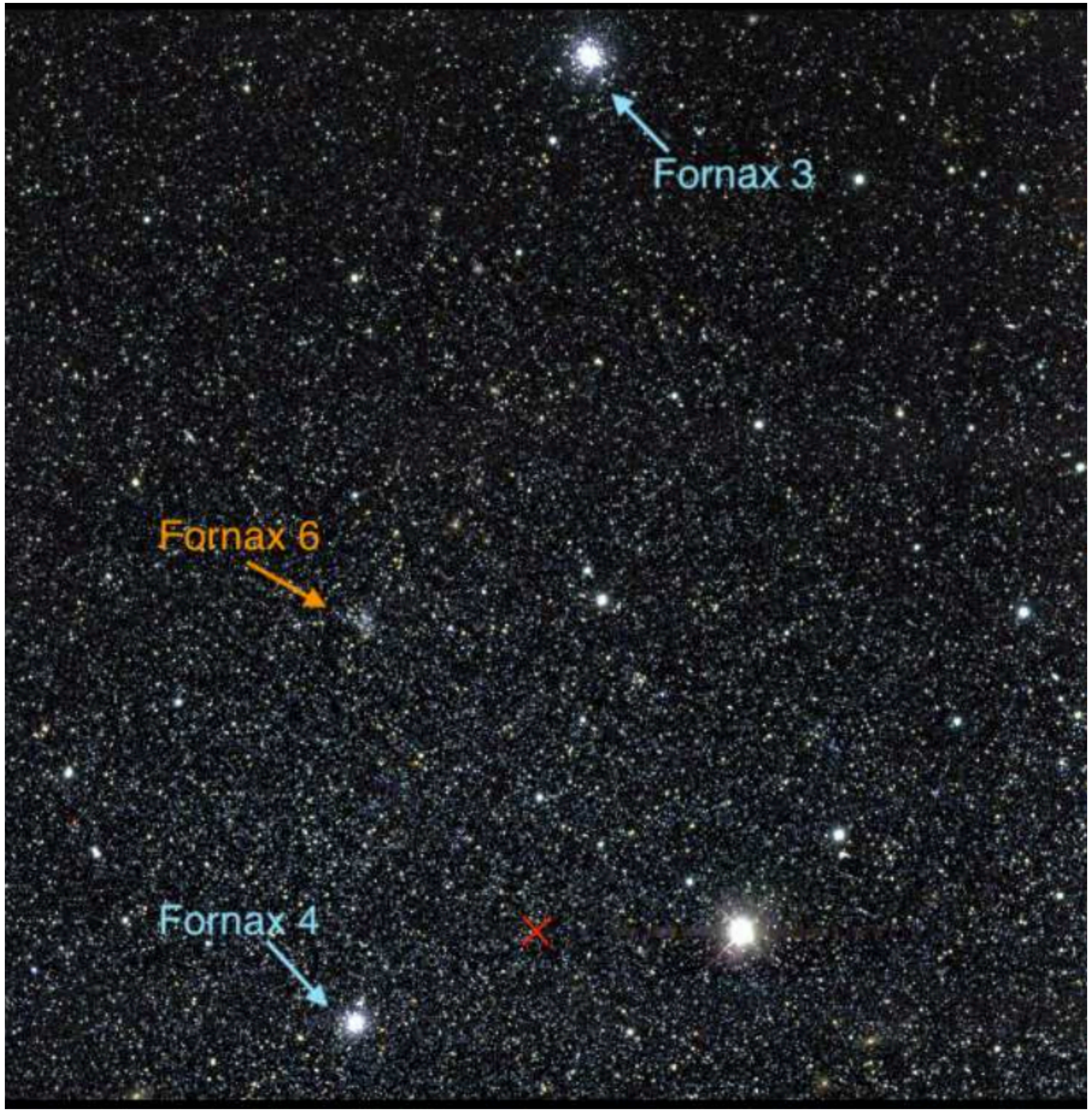
Figure by J. Schombert

Fornax timing problem in a glimpse

Wang+2019 arXiv:1902.04589

About 800 pc X 800 pc

Red Cross: luminous center



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Hui, Tremaine, Ostriker & Witten 2016

	projected radius	cluster mass	CDM
n	$r_\perp$ (kpc)	$m_{\rm cl}$ ($M_\odot$)	Dynamical Friction time τ (Gyr)
1	1.6	3.7×10^4	112
2	1.05	1.82×10^5	9.7
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- Seems tuned.
  How tuned? An explanation?

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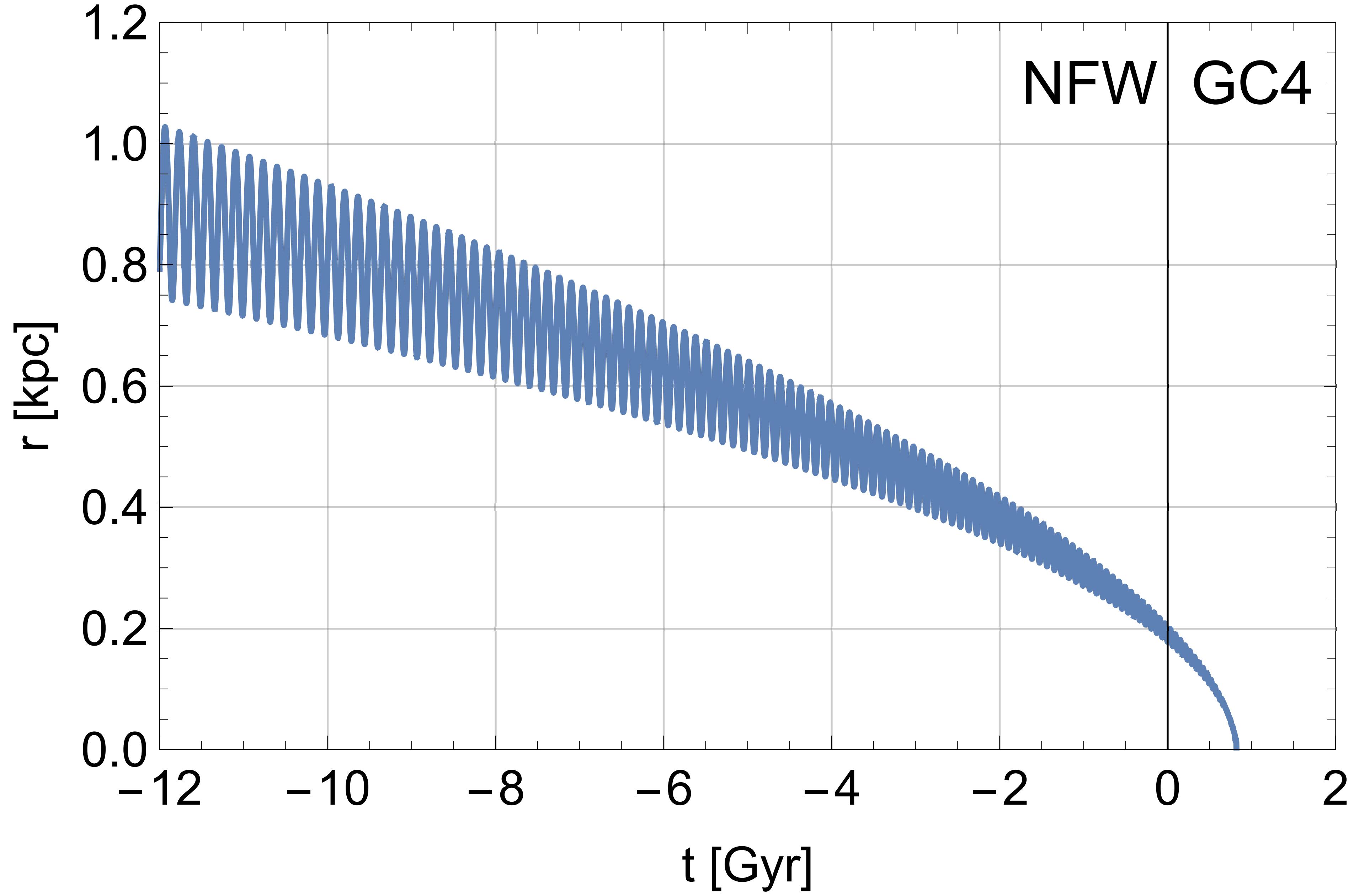

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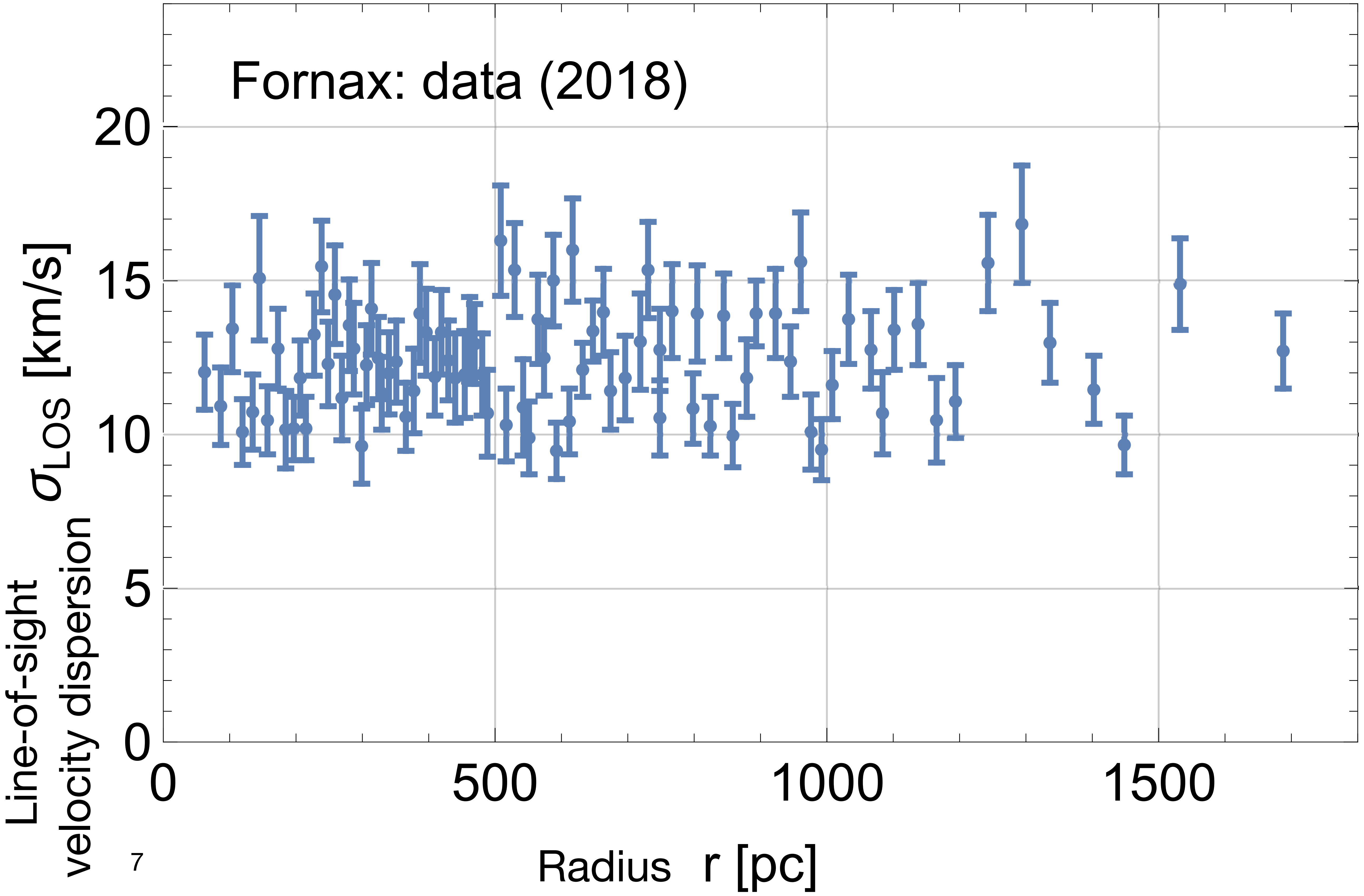
- Robust cosmological free-streaming bound on fermions, regardless of production mechanism (not in this talk).
- New analytical results of dynamical friction in galaxies.
- State of the art of the Fornax globular cluster timing problem (is it really tuned?).

Many mass models fit kinematic data

Kinematic data: line-of-sight velocities of ~2500 stars. No proper motion. Many models fit OK; “velocity anisotropy degeneracy”.



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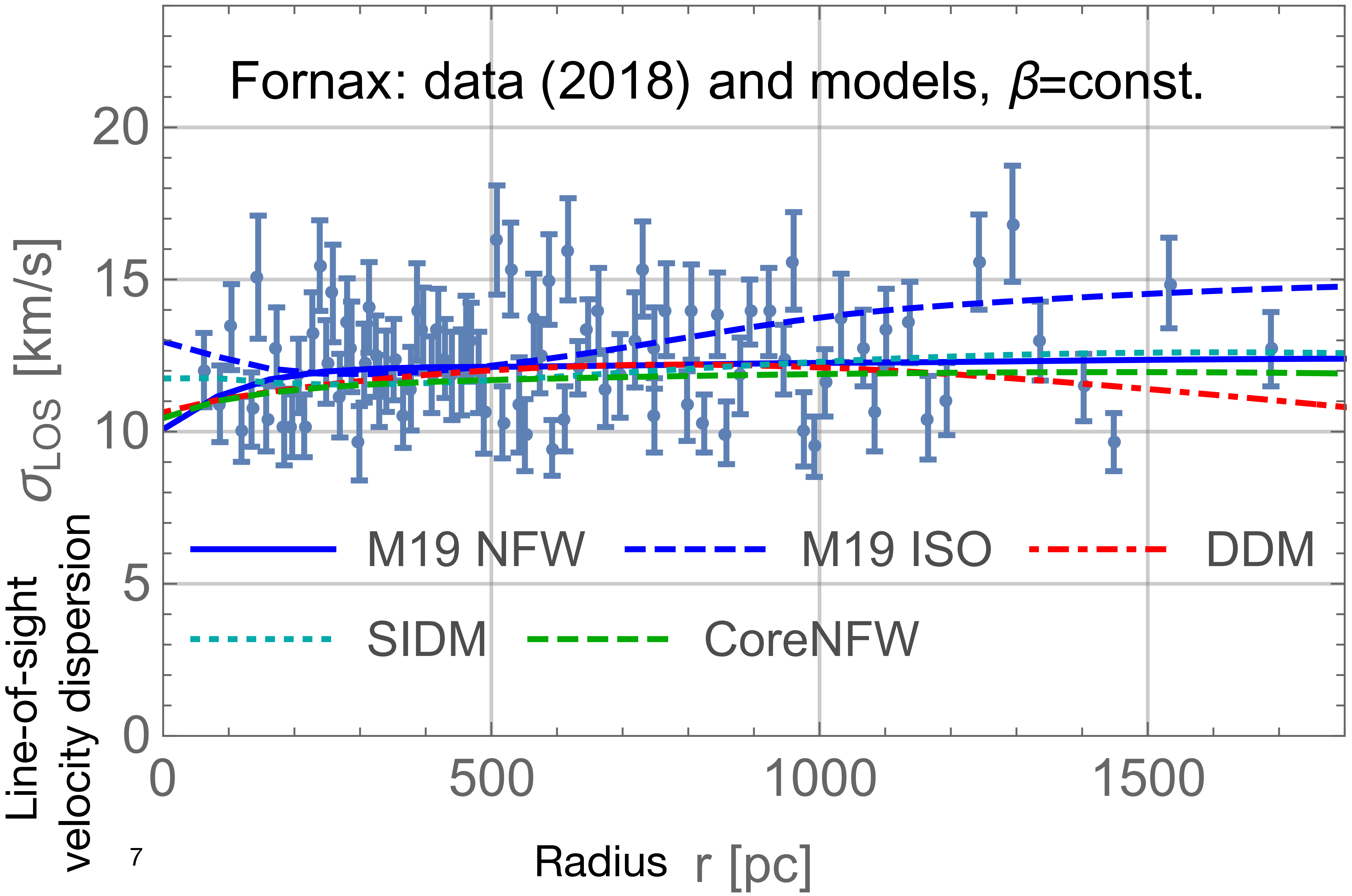

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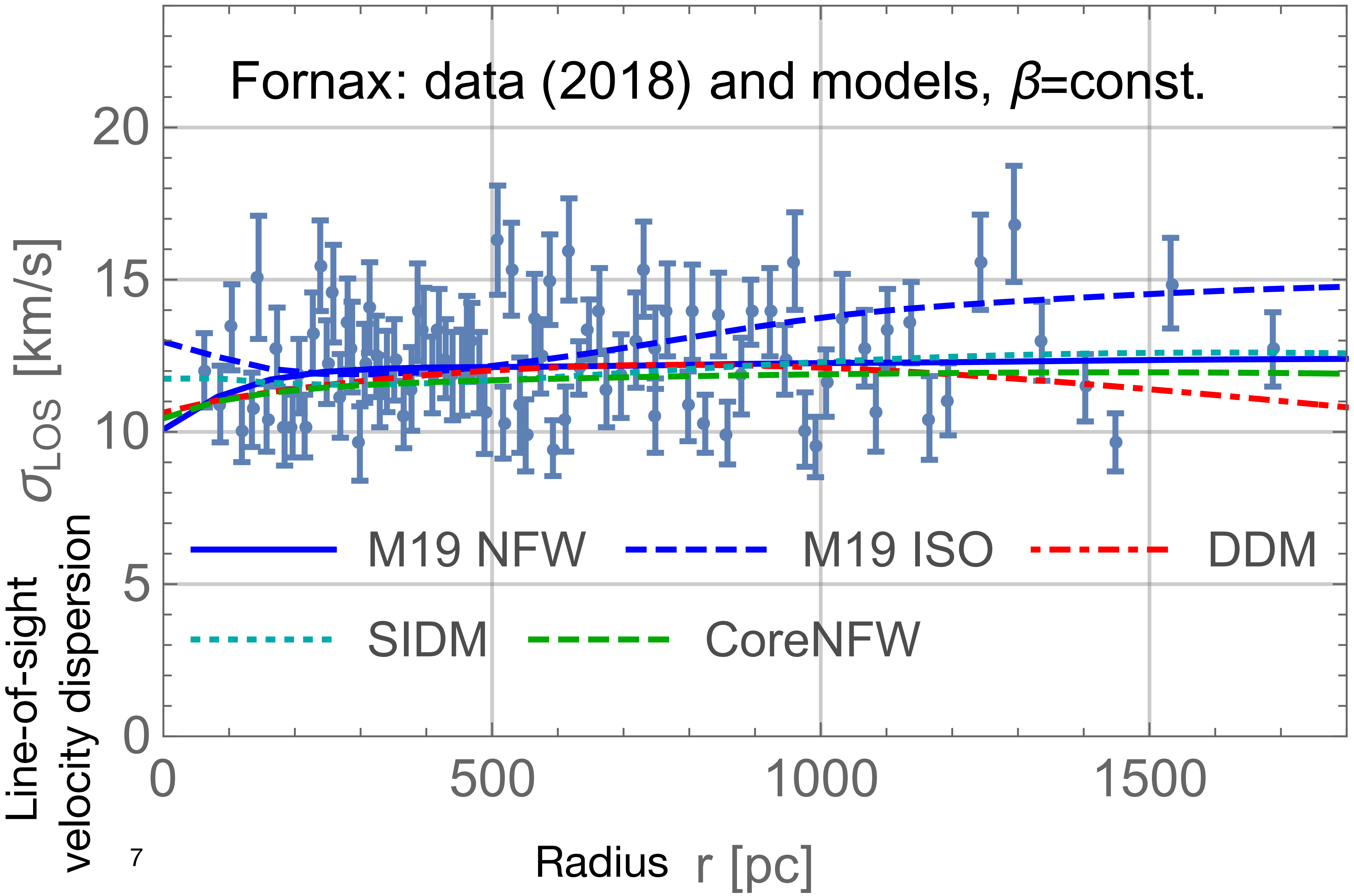

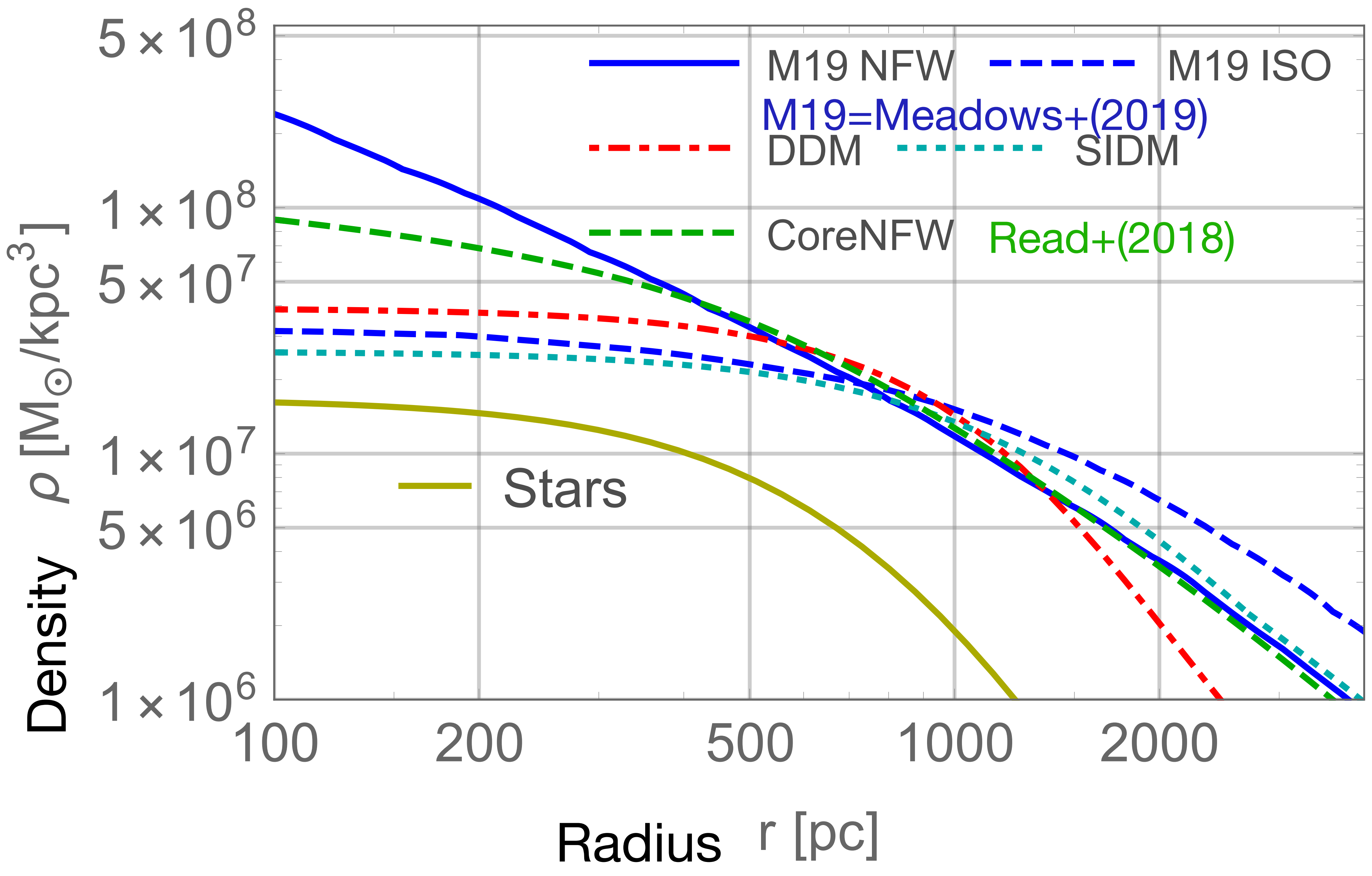

Orbital deceleration (Chandrasekhar's formula)

$$\frac{d\mathbf{V}}{dt} = \frac{D}{M}\hat{V} = -\frac{4\pi G^2 \rho M \ln\Lambda}{V^3} \underbrace{C(V/\sigma)}_{O(1)} \mathbf{V}$$

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G^2 effect

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Perturber mass

Coulomb logarithm

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For a given halo, it depends on radius and velocity of object.

Assuming a circular orbit: Effective function of radius.

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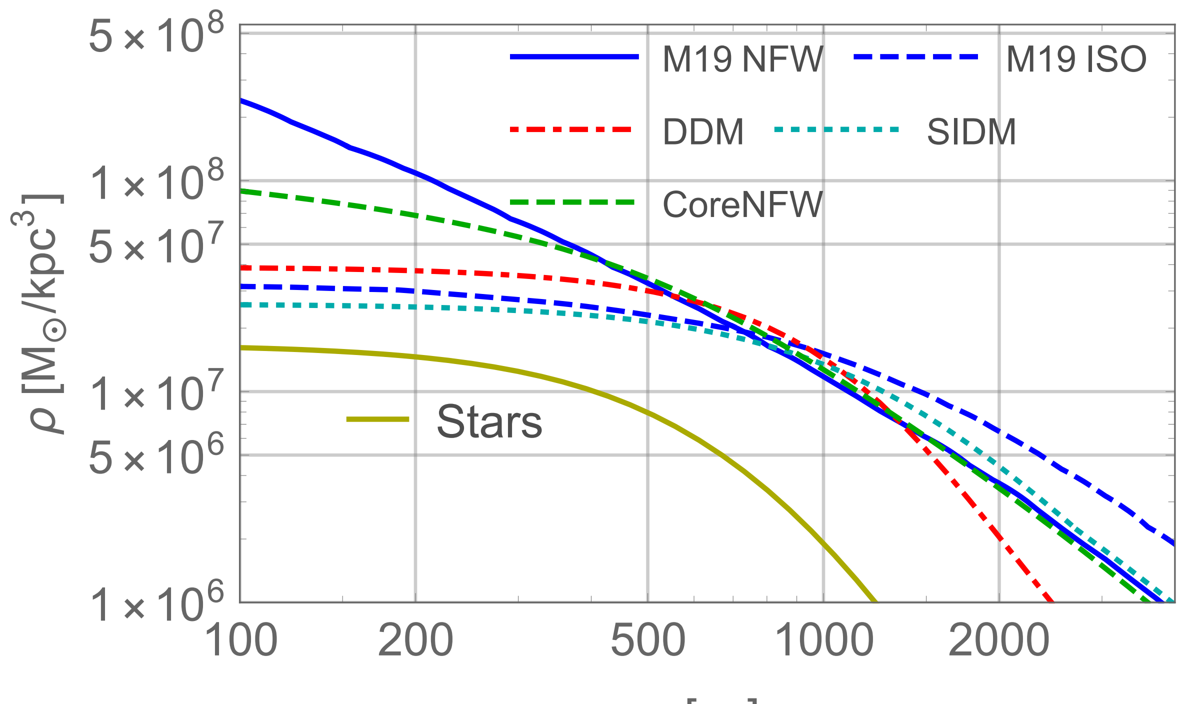

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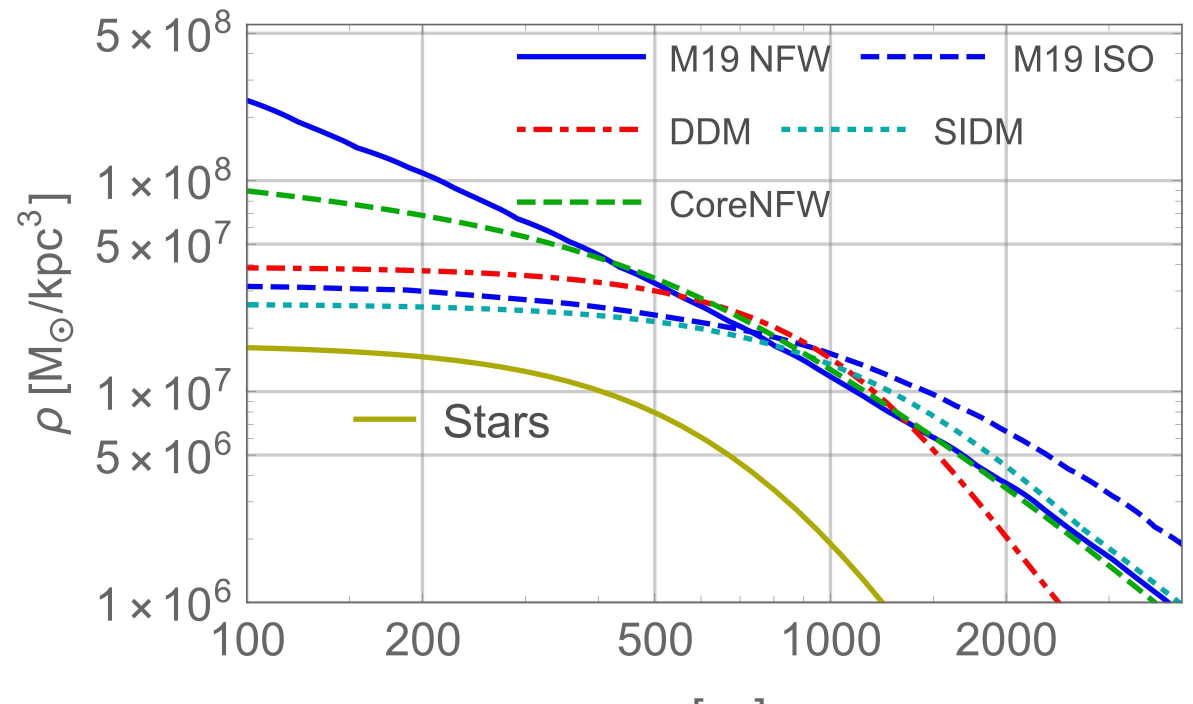
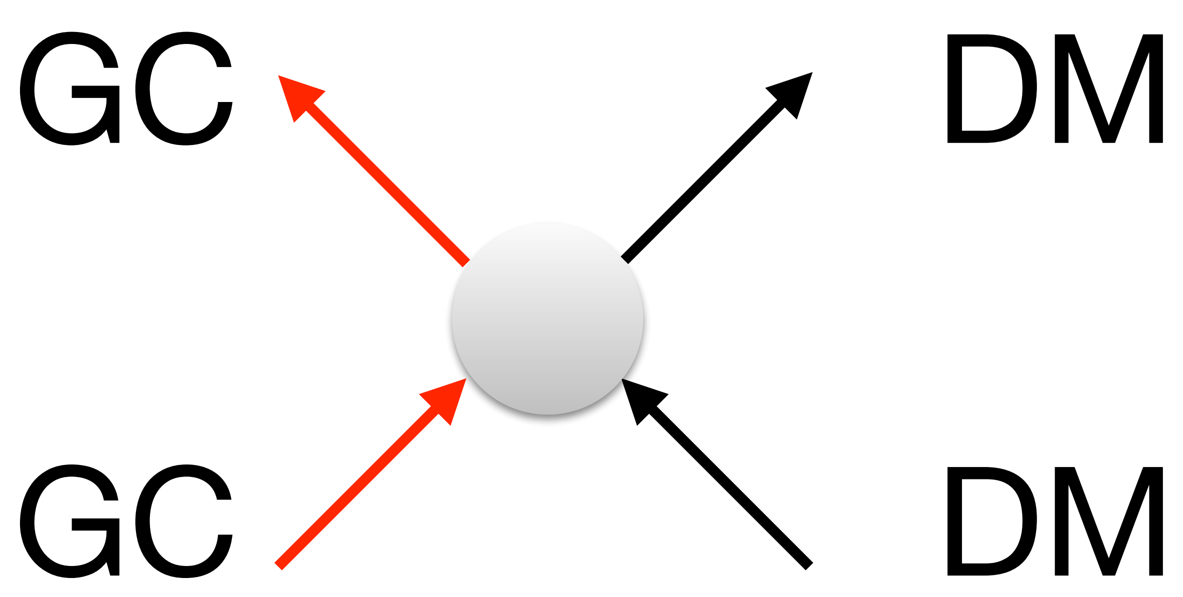
For classical Maxwellian $C_{\mathrm{Max}} \to \ln\Lambda \begin{cases} 1 & V \gg \sigma \\ \frac{\sqrt{2}}{3\sqrt{\pi}}\frac{V^3}{\sigma^3} & V \ll \sigma \end{cases},$



Dynamical friction derived from Fokker-Planck



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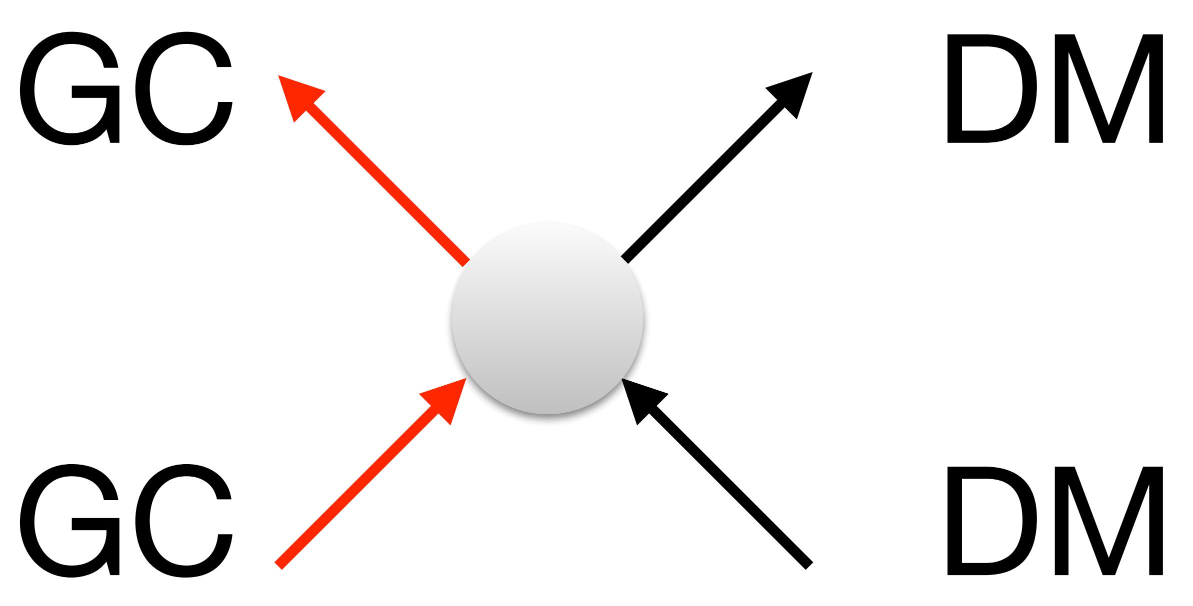
Fokker-Planck: dynamical friction as diffusion in momentum space.



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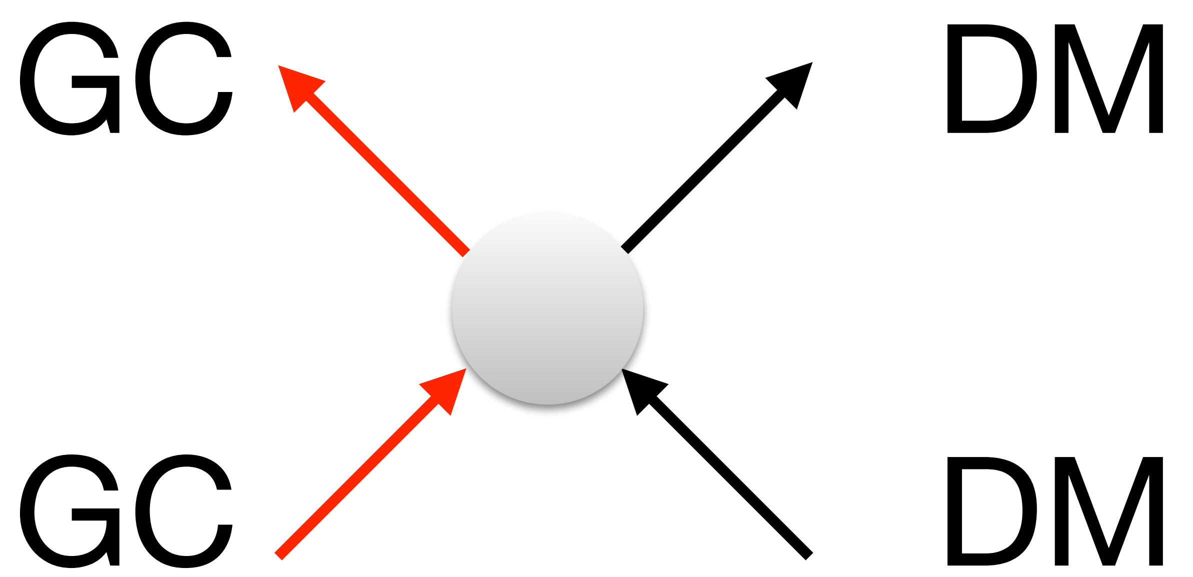
From Boltzmann equation with collision term between perturbers (GCs) and DM

$$\frac{df_1}{dt} = C[f_1],$$



$|\overline{\mathcal{M}}|$

$$C[f_1] = \frac{(2\pi)^4}{2E_p} \int d\Pi_{k,p',k'}\, \delta^{(4)}(p + k - p' - k')\, |\overline{\mathcal{M}}|^2 \left[f_1(p')f_2(k')(1 \pm f_1(p))(1 \pm f_2(k)) - f_1(p)f_2(k)(1 \pm f_1(p'))(1 \pm f_2(k')) \right]$$

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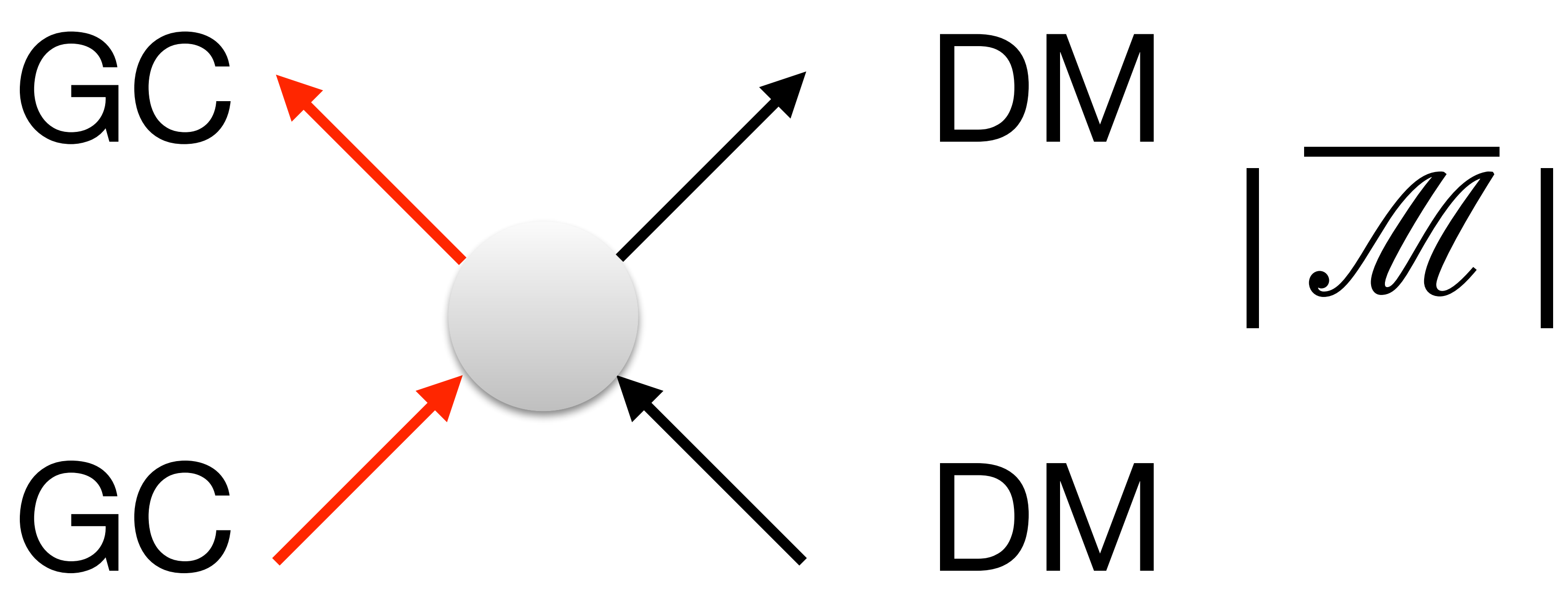

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Microphysics: Bose enhancement or Fermi-blocking

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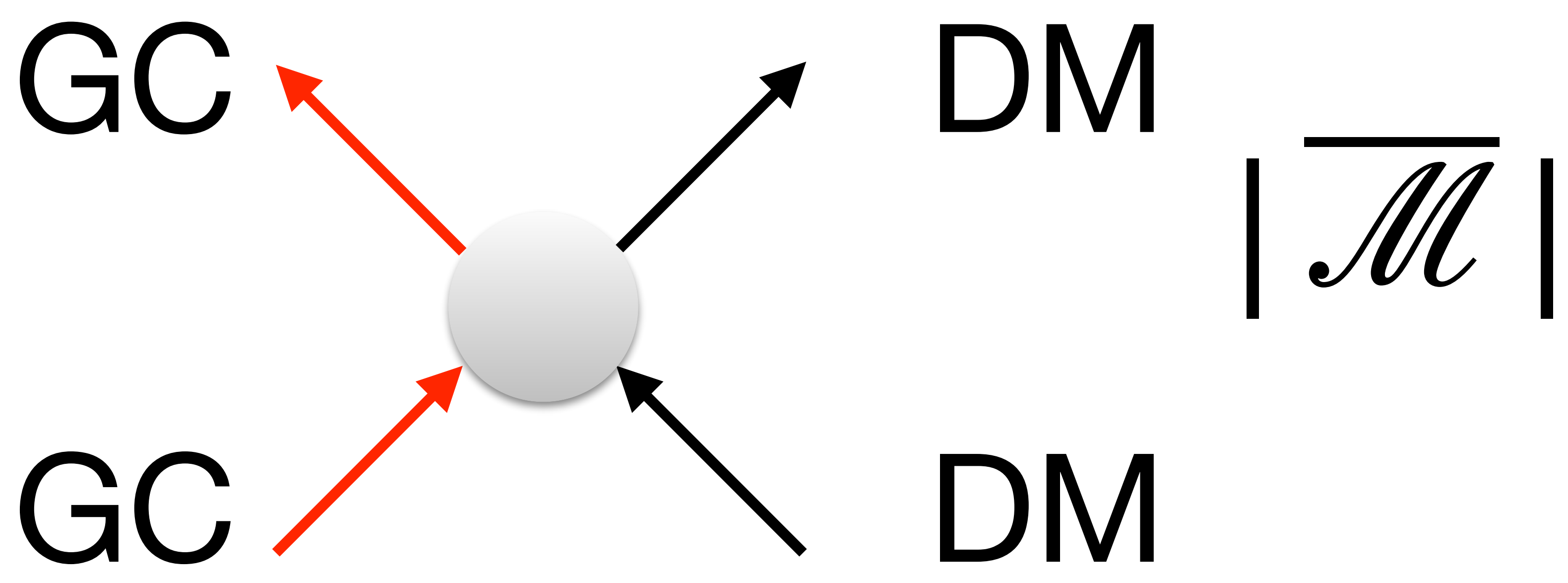

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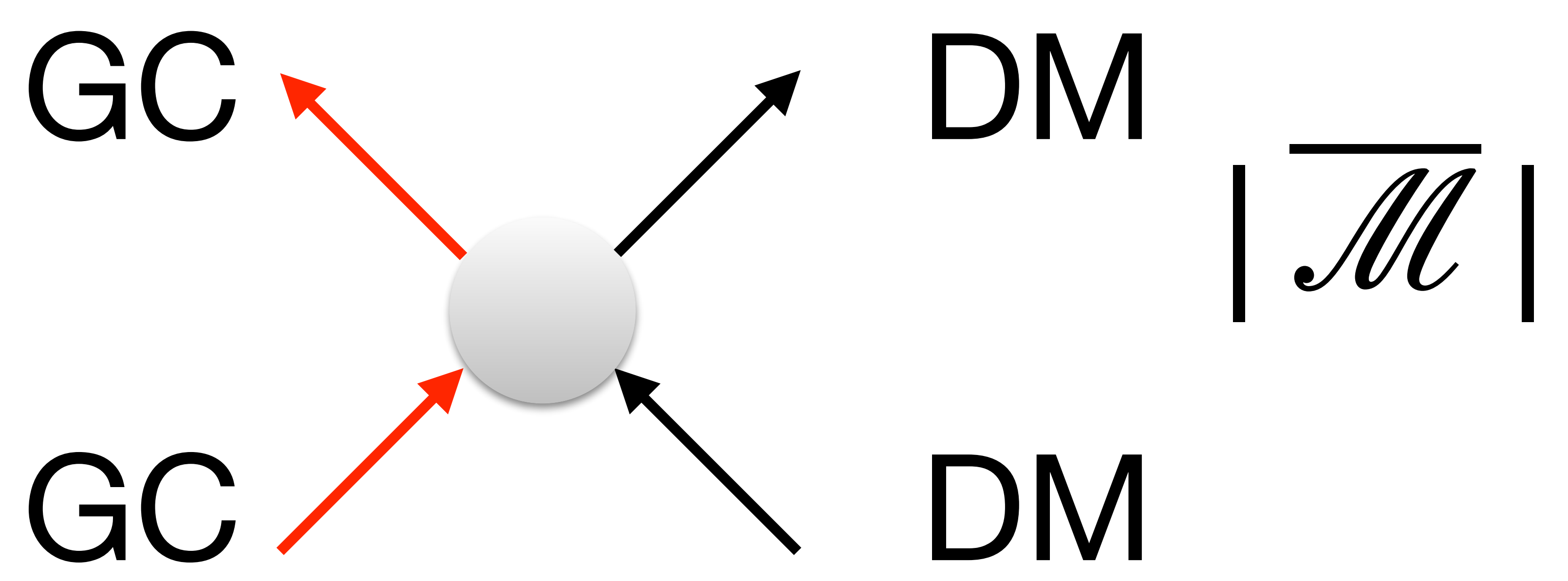

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$m_{DM} \lesssim 10^{-22}\,\mathrm{eV}$ may work , but in tension with other constraints

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Large core (~1 kpc)

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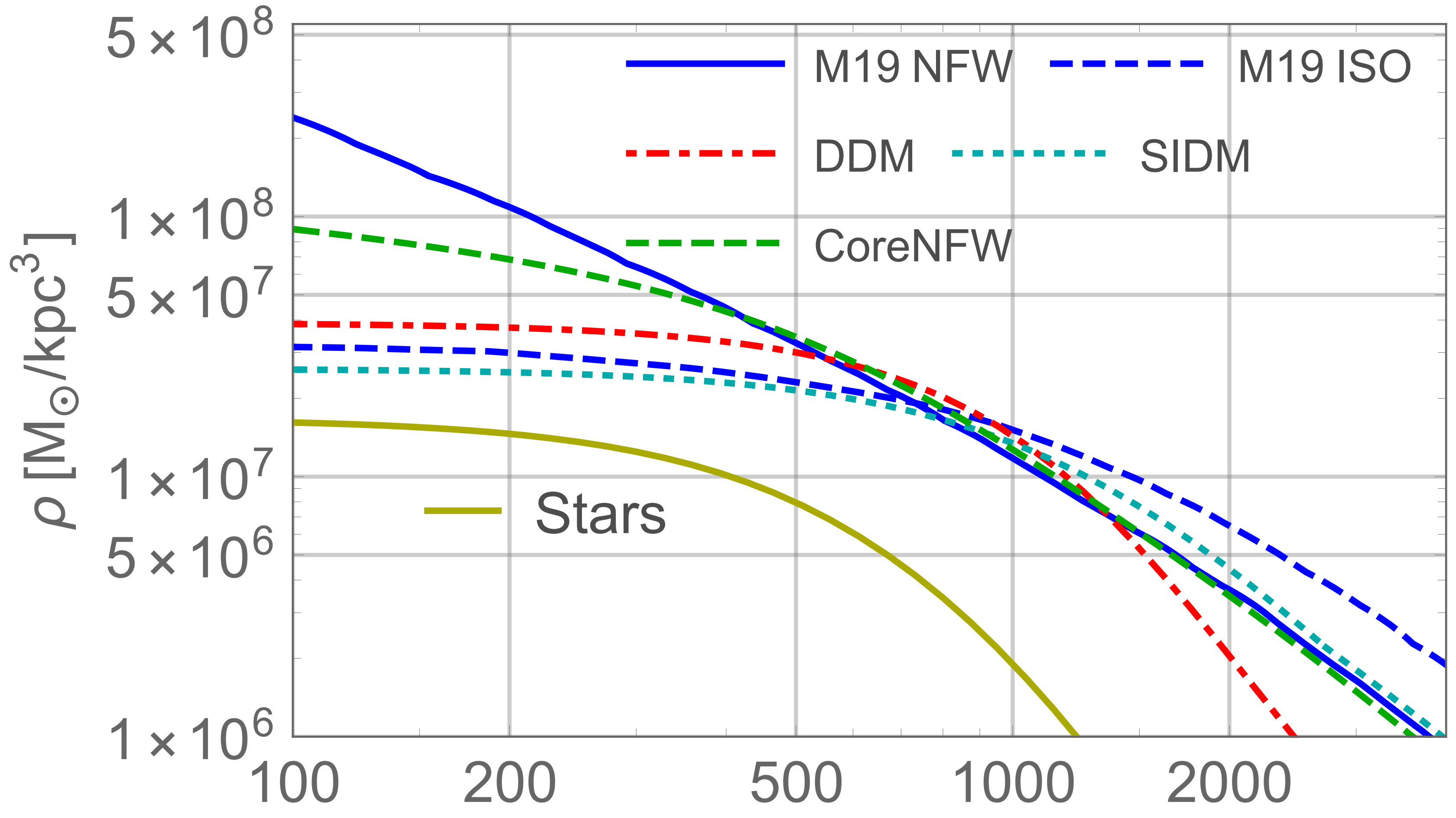

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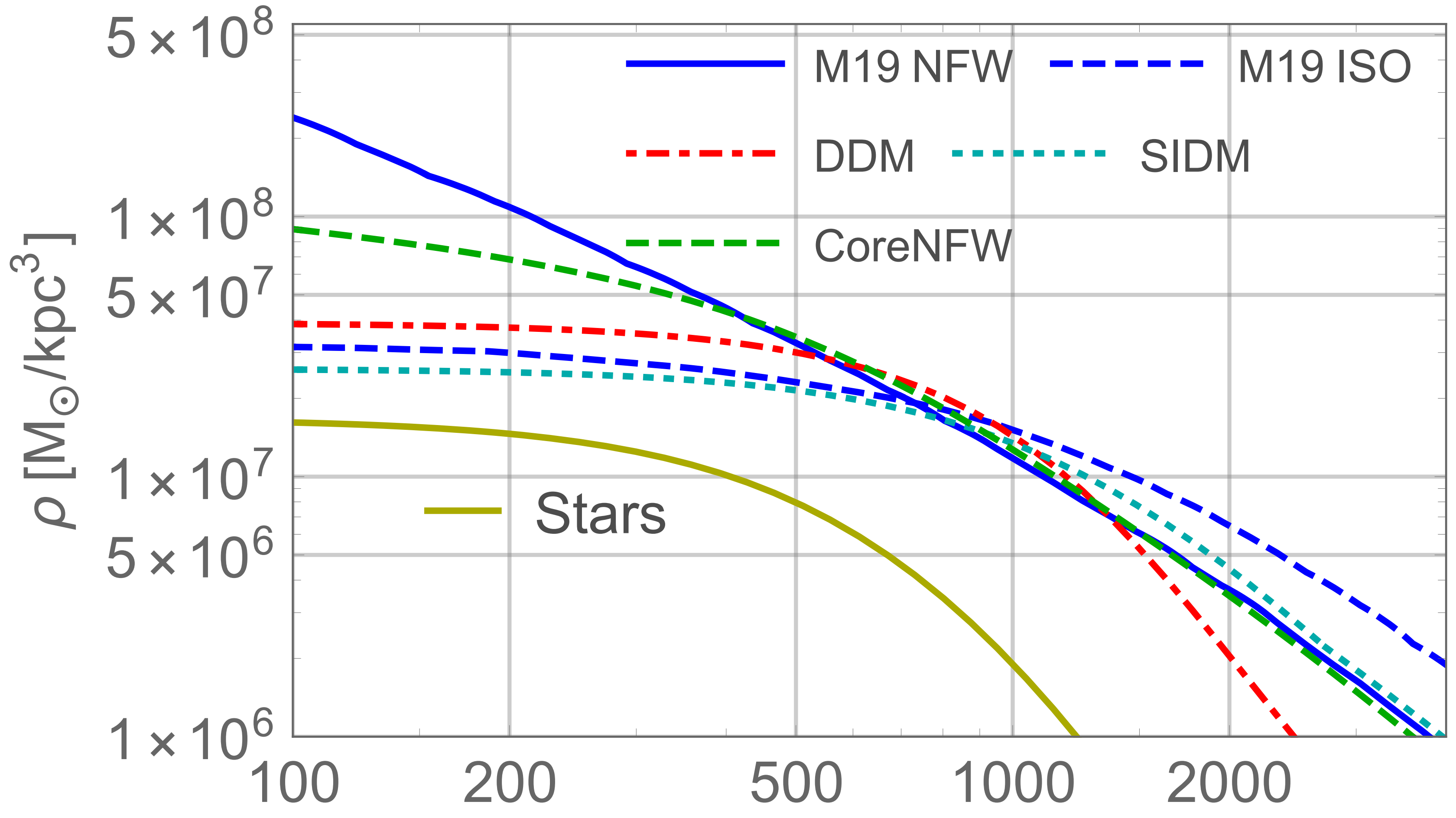
- SIDM $r_c \sim \frac{m}{\rho\sigma} = 48 \frac{10^8 M_\odot/\mathrm{kpc}^3}{\rho} \frac{1\ \mathrm{cm^2/gr}}{\sigma/m} \mathrm{kpc}$



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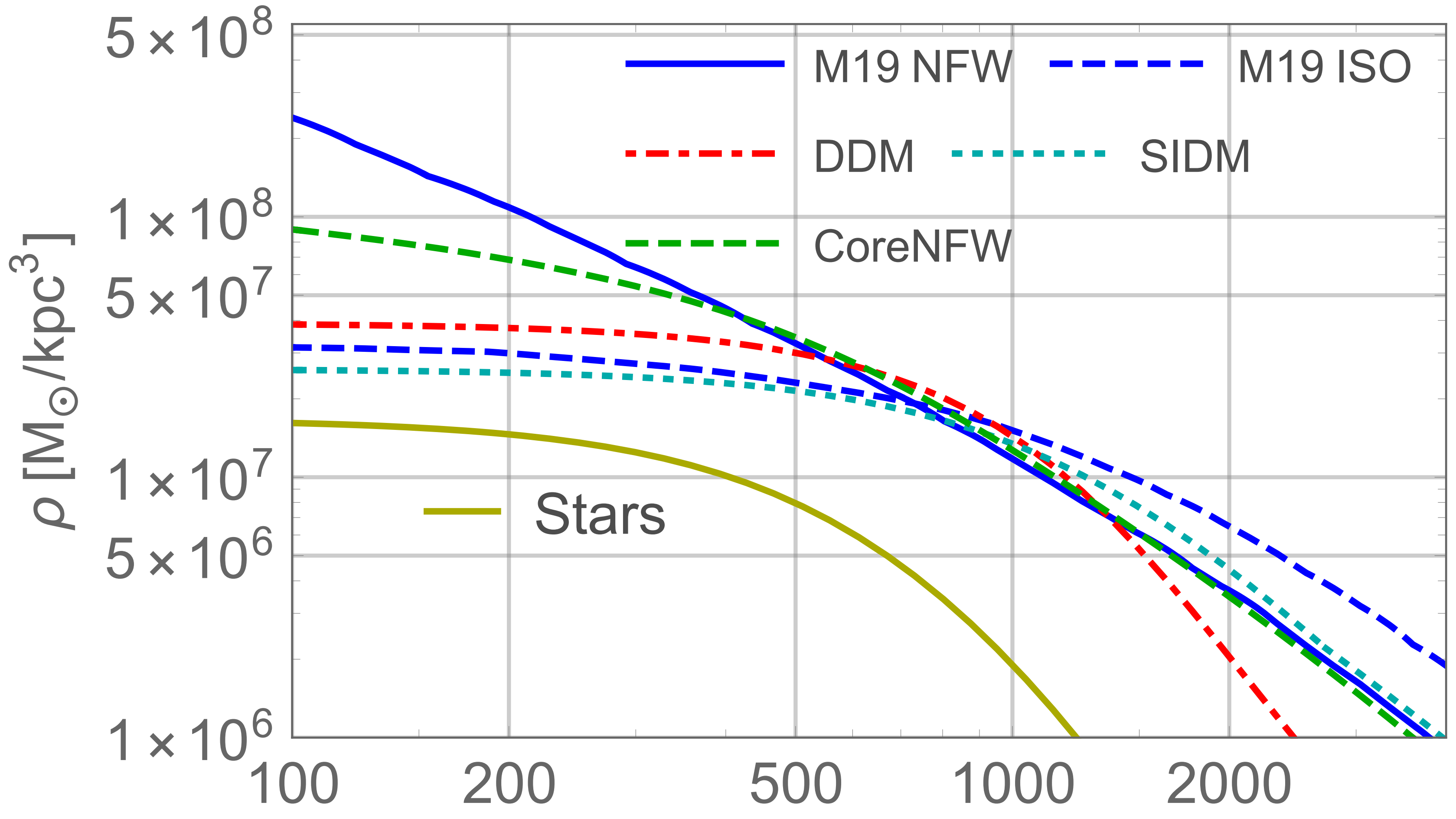
Ly-a?

- SIDM $r_c \sim \frac{m}{\rho\sigma} = 48 \frac{10^8 M_\odot/\mathrm{kpc}^3}{\rho} \frac{1 \ \mathrm{cm}^2/\mathrm{gr}}{\sigma/m} \mathrm{kpc}$

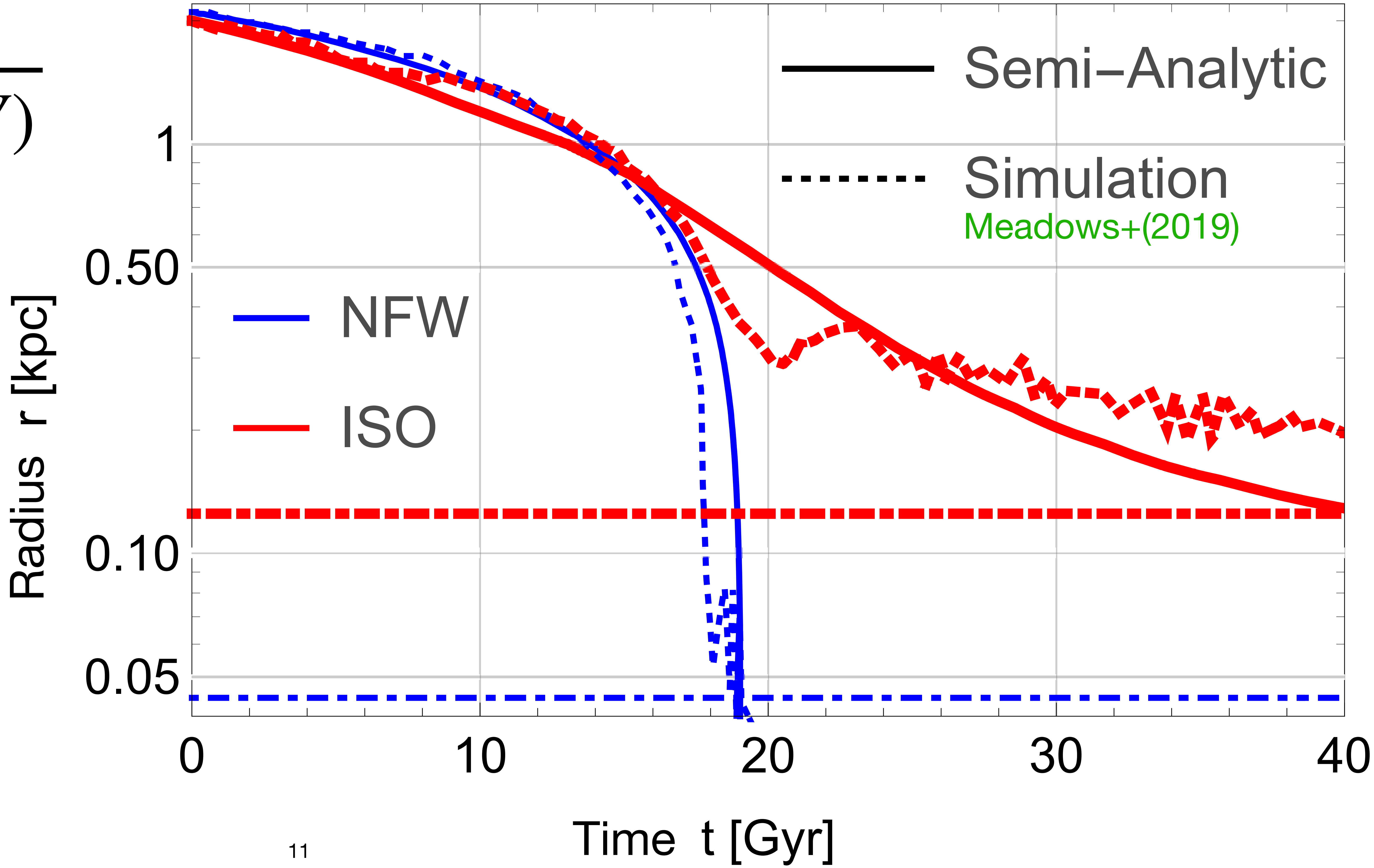


Analytical treatment can roughly reproduce N-body simulations

Semi-analytic integration:

$$\frac{d\mathbf{V}}{dt} = -\frac{GM(r)}{r^2}\hat{r} - \frac{\mathbf{V}}{\tau(r, V)}$$

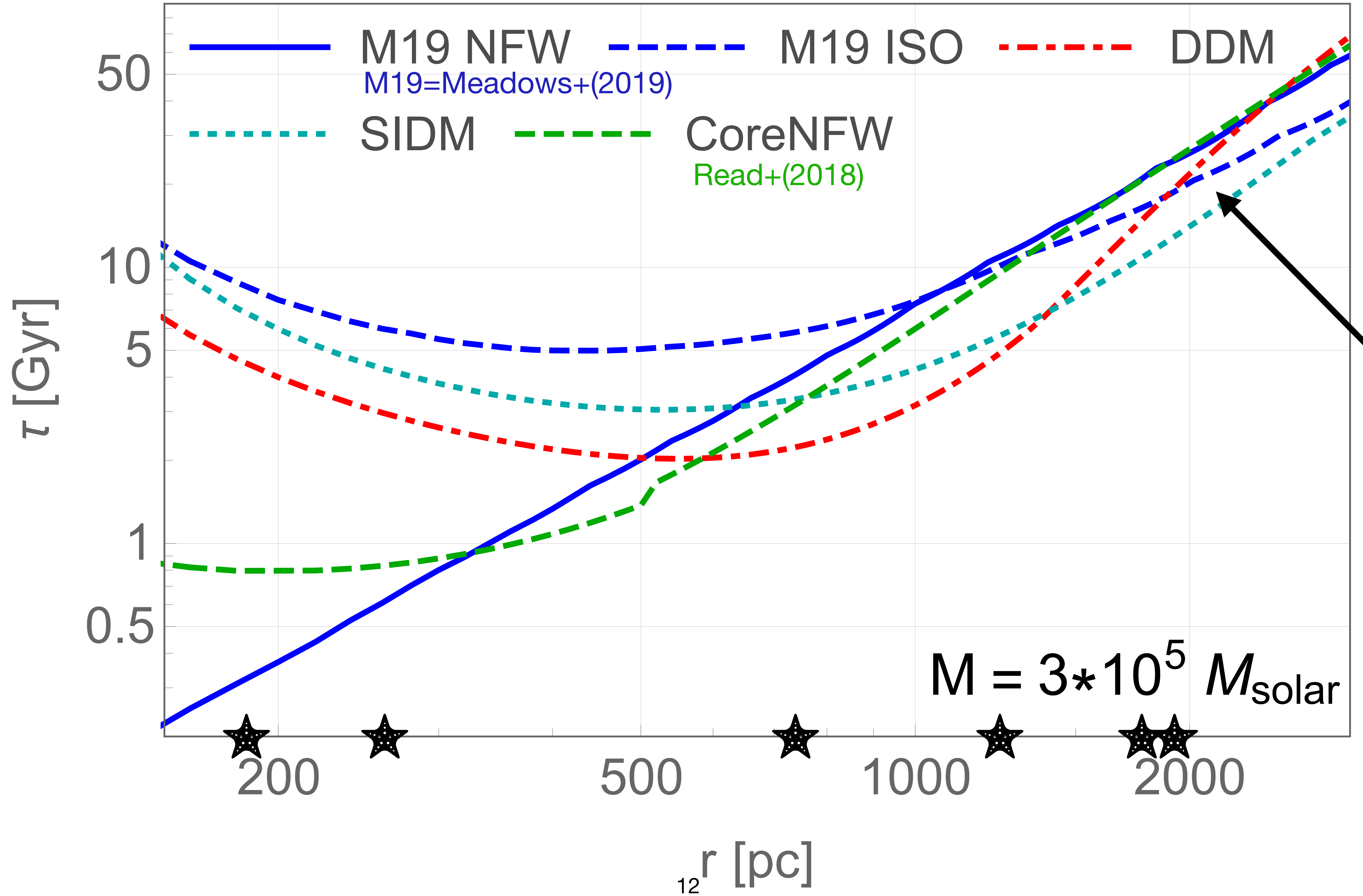
Coulomb logarithm needs to be calibrated.



Dynamical friction time demonstrates core stalling

Radially-dependent dynamical friction time, assuming circular orbits, i.e.

$$\tau(r,V) \equiv |\mathbf{V}|/|d\mathbf{V}/dt|_{\text{dyn.fric.}} \rightarrow \tau(r, V_{\text{circ}}(r)) = \tau(r)$$



Agreement
On halo outskirts

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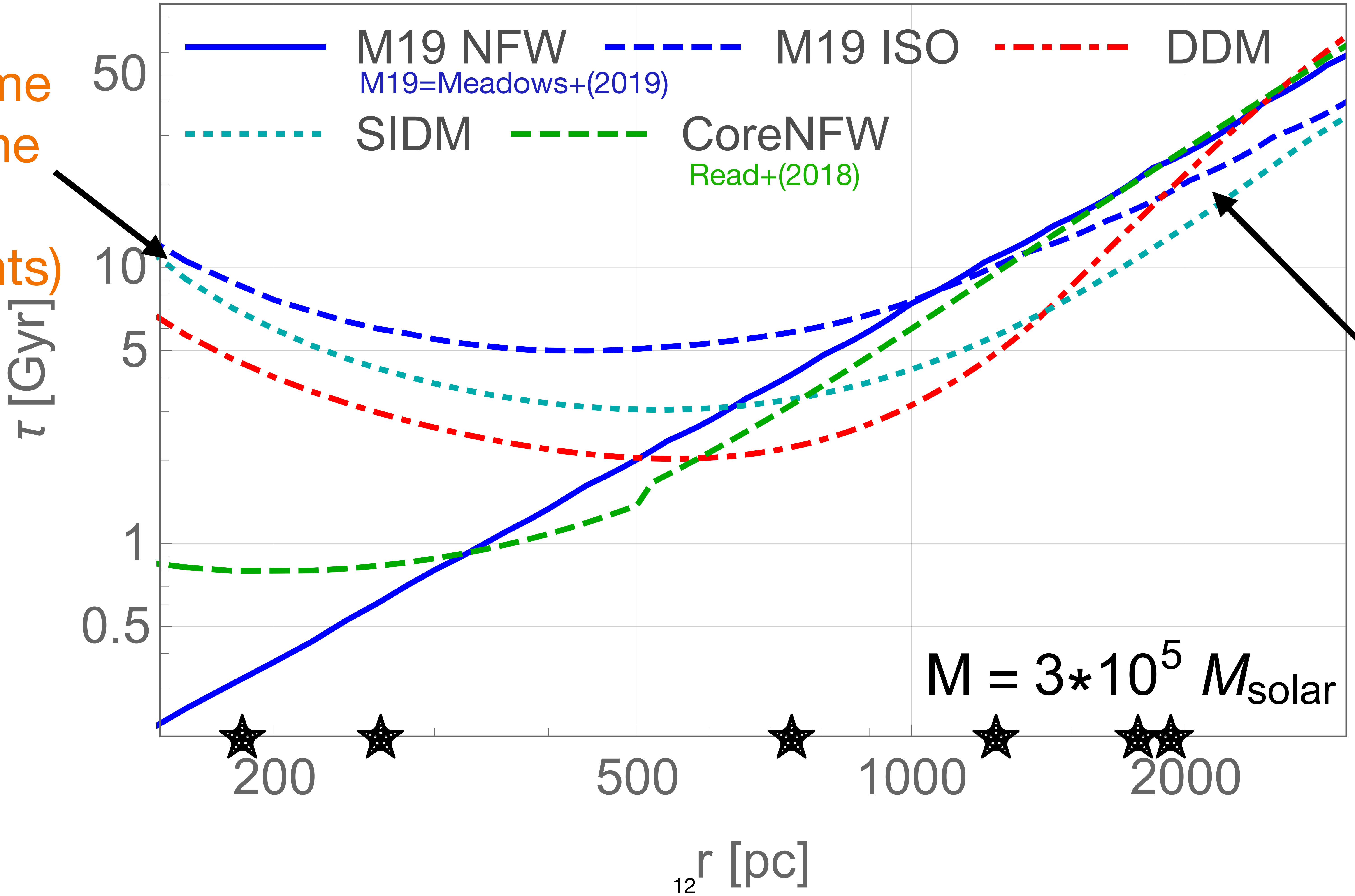
~1 kpc cores lengthen inspiral time to cosmological time (problem for DDM from Ly-a constraints)



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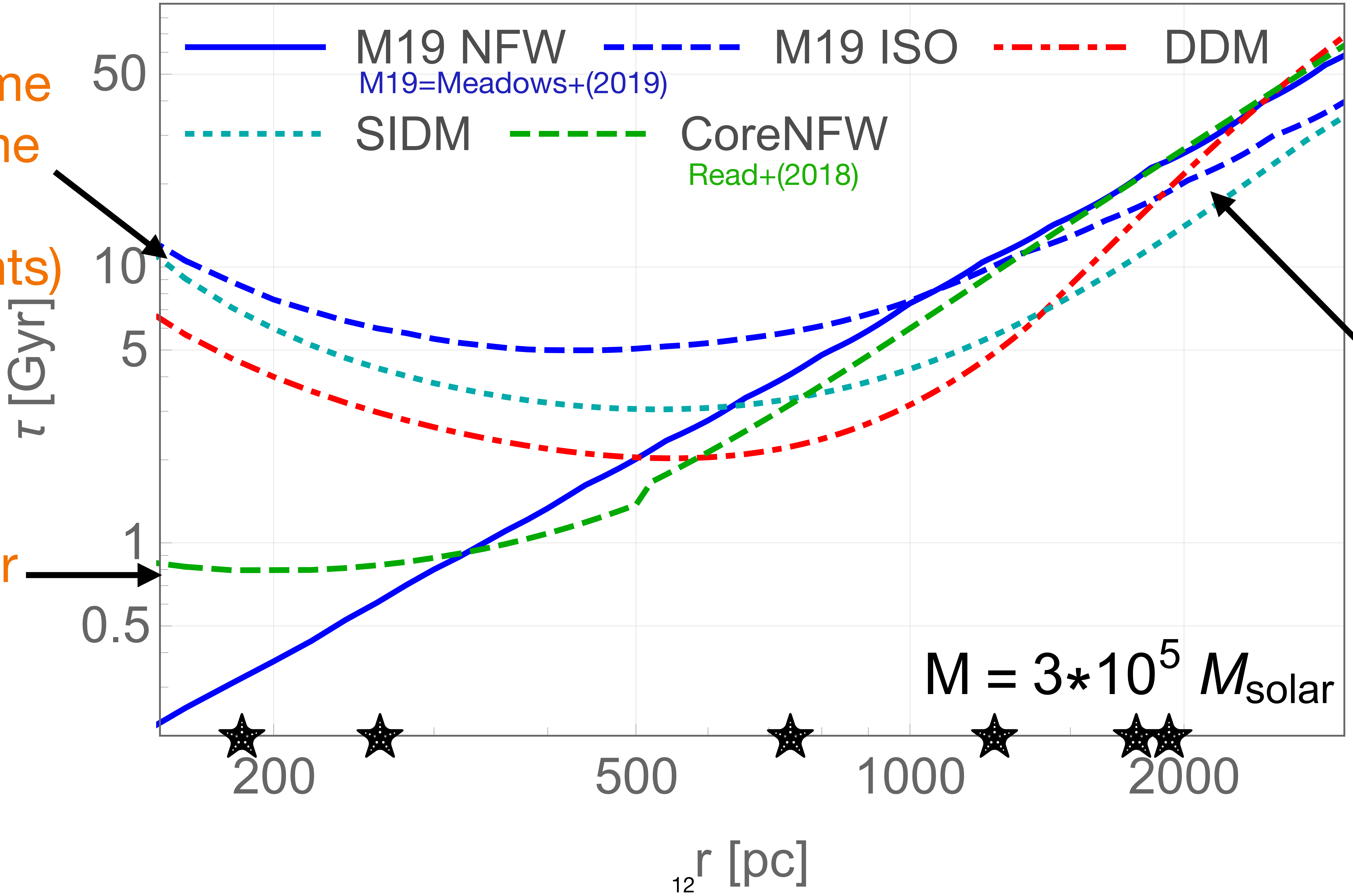
Smaller core stalls inner GC to few Gyr



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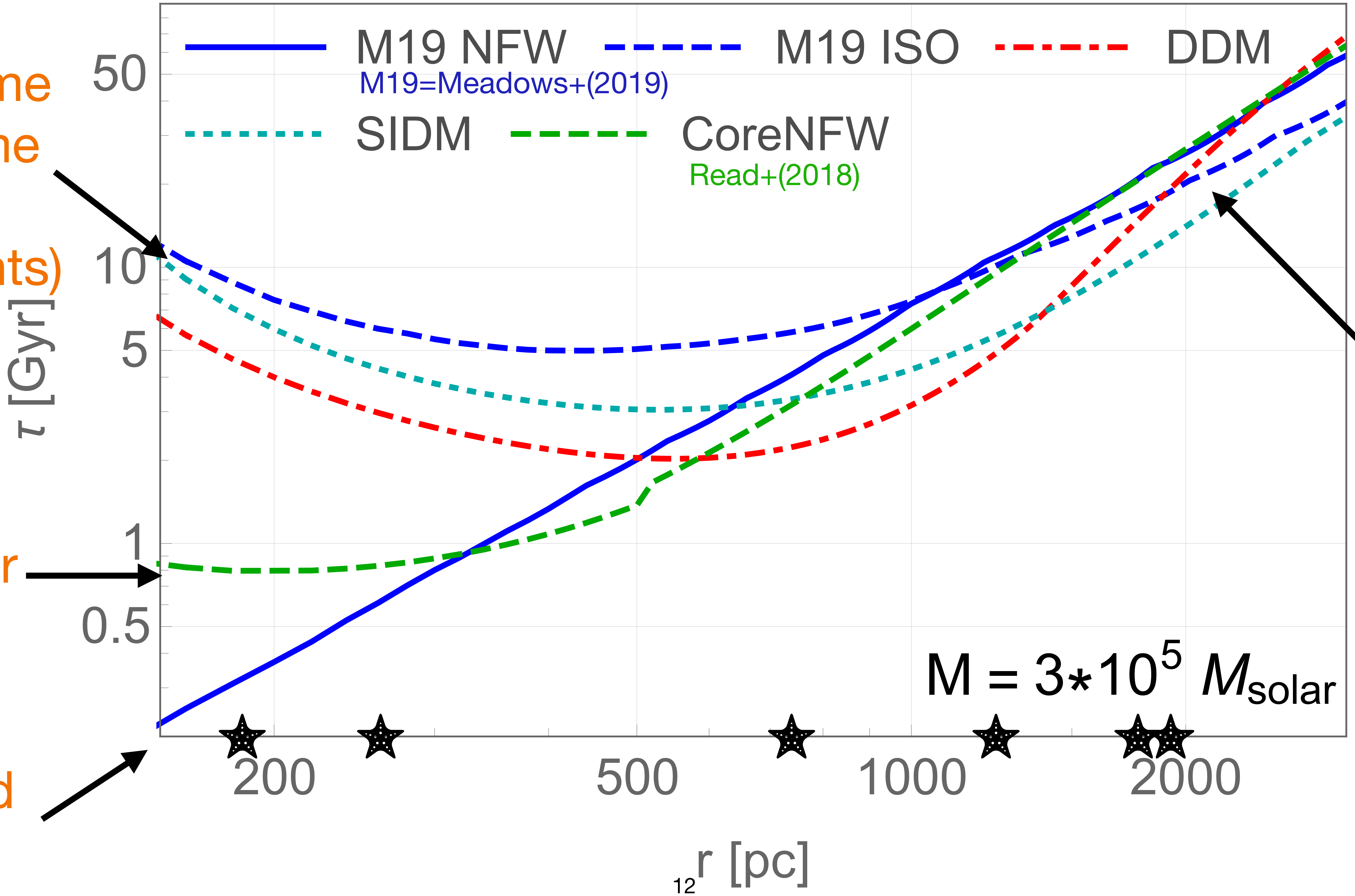
Cusp predicts rapid infall of inner GCs



Agreement On halo outskirts

Quantifying distribution of GCs

First, we find that the time it takes a GC on circular orbit to inspiral from r_0 to r_f is

$$\Delta t(r_i; r_f) = \int_{r_f}^{r_0} \frac{dr}{2r} \left(1 + \frac{d\ln M}{d\ln r}\right) \tau(r, v_{\rm circ}(r))$$

Can easily be evaluated for const. or power law τ.

Then, given initial distribution $n_{GC}(r, t=0)$, one can find final distribution

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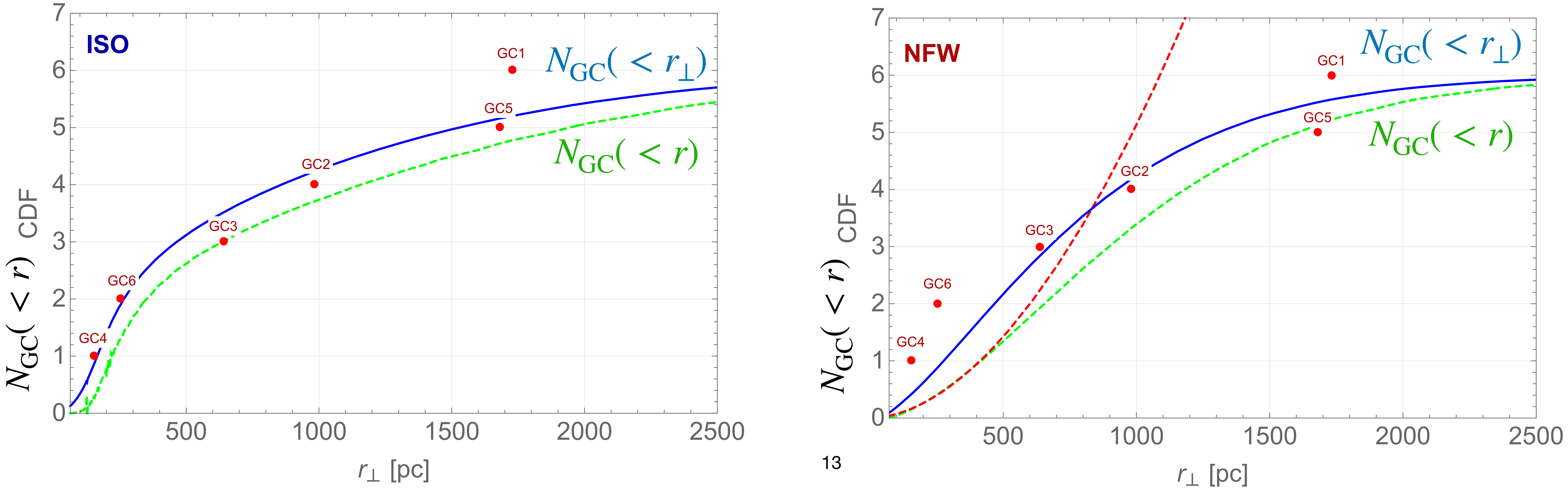

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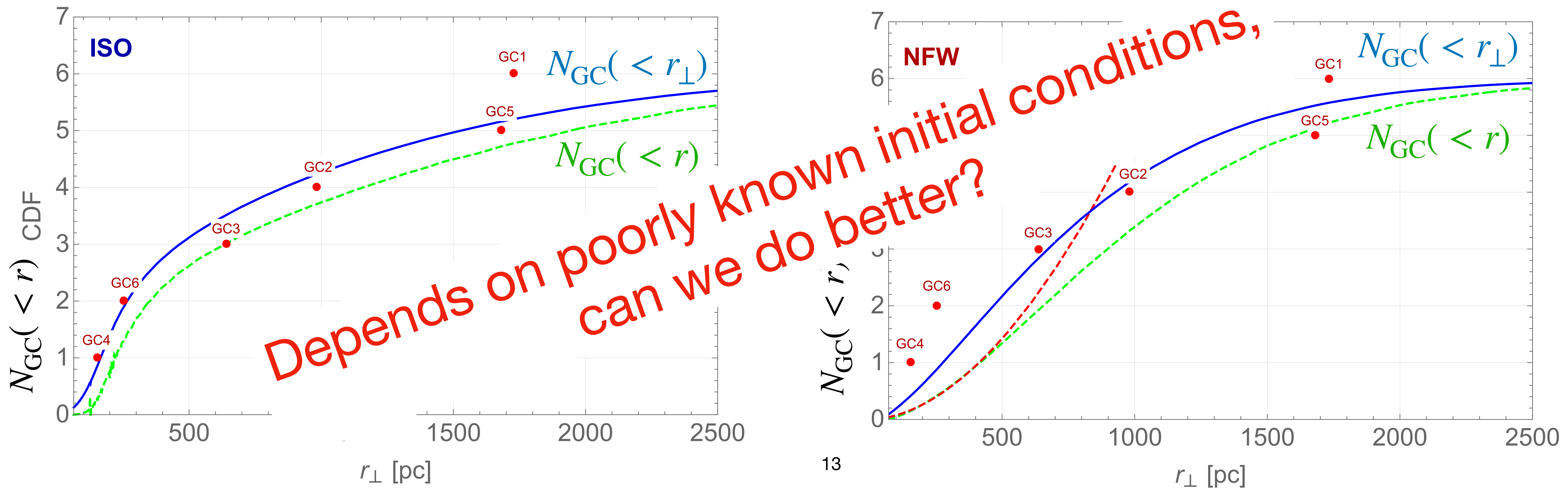

Is this enough? Where are the GCs that fell to the center?

In taking an initial distribution that roughly reproduces the GC distribution in Fornax, some $30\% - 50\ \%$ of GCs should fall to the center by today's time.

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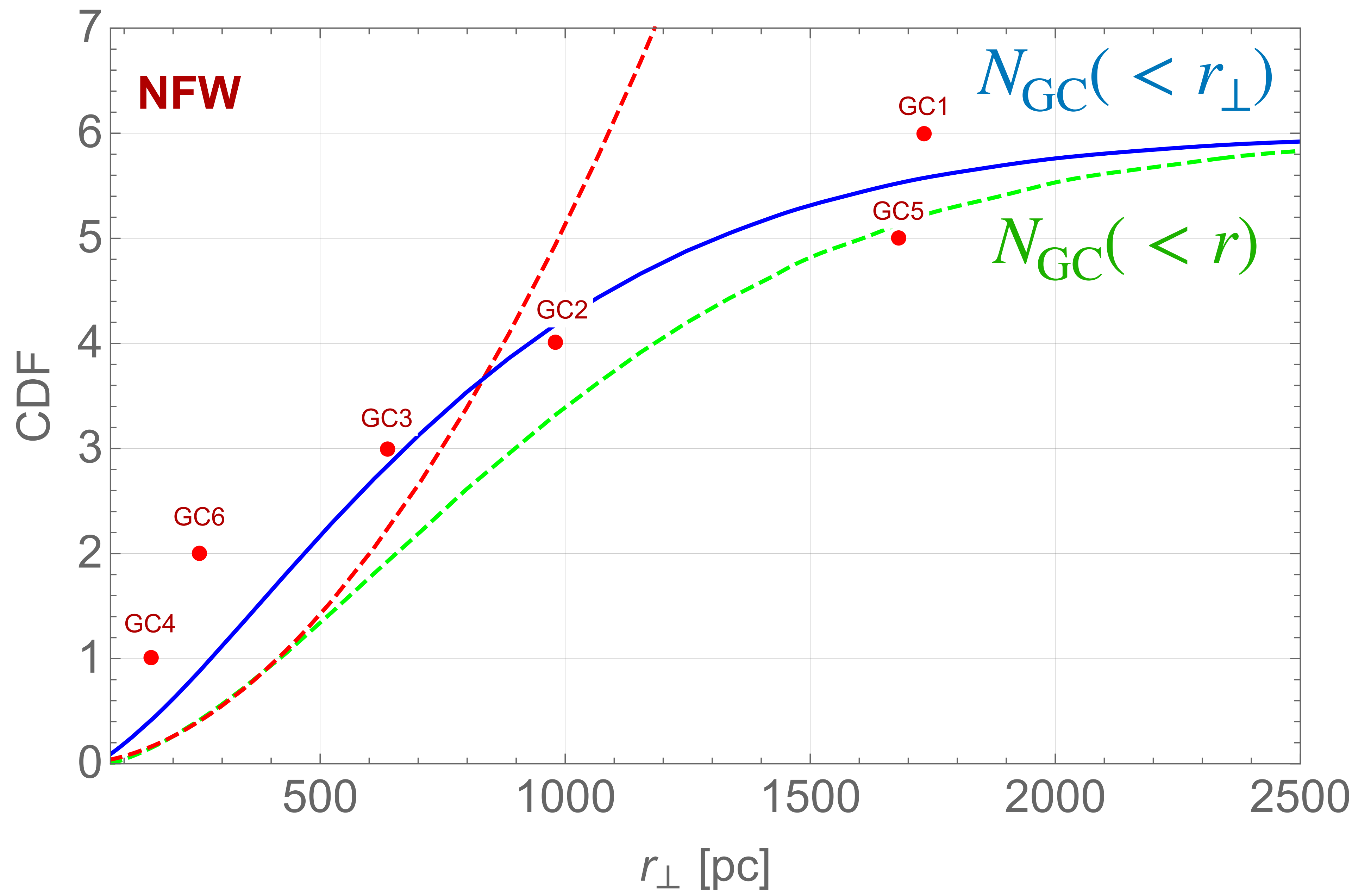

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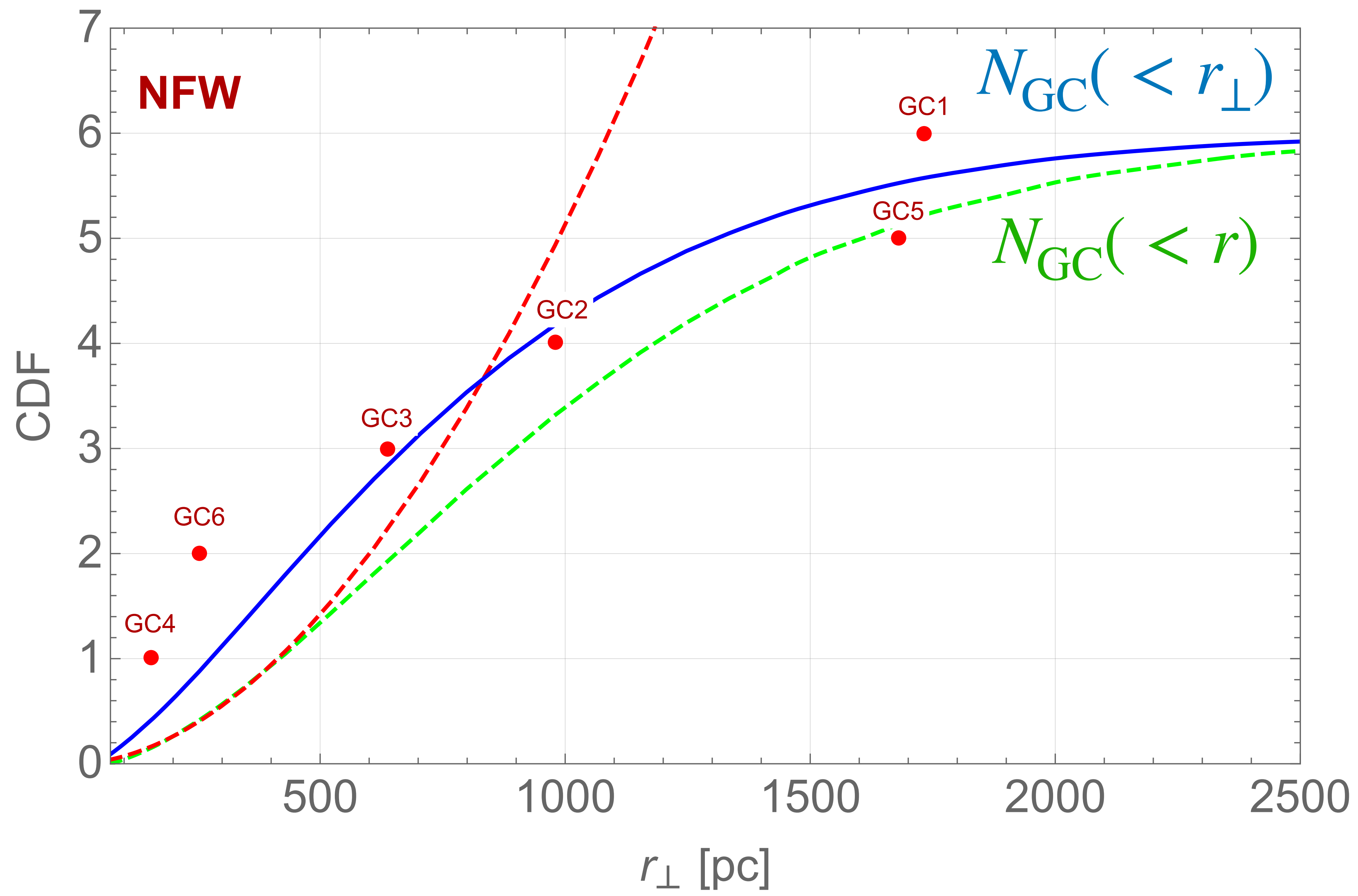

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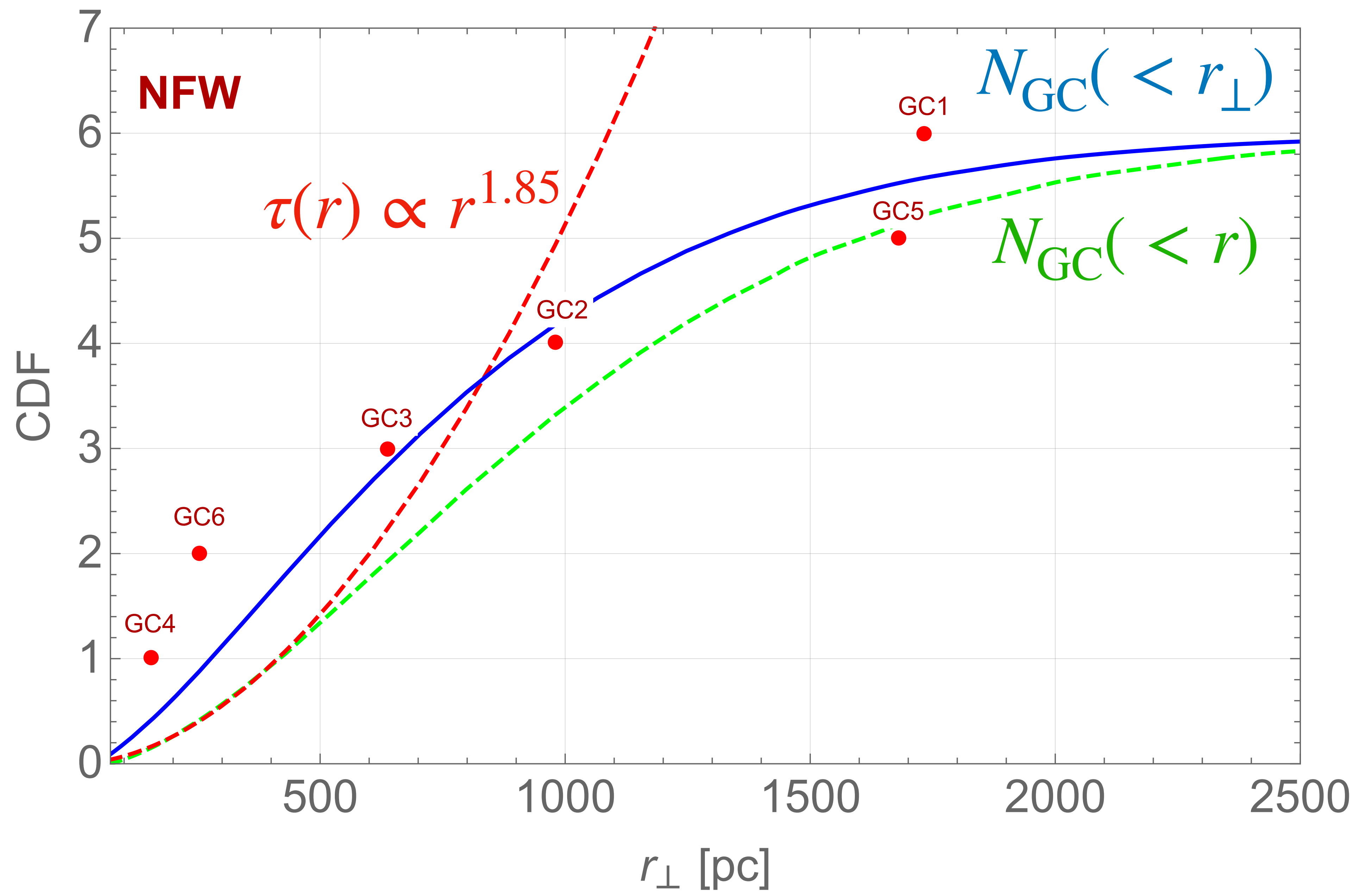


THE FORMATION OF THE NUCLEI OF GALAXIES. II. THE LOCAL GROUP

SCOTT D. TREMAINE
Joseph Henry Laboratories, Physics Department, Princeton University

Received 1975 March 12; revised 1975 April 28

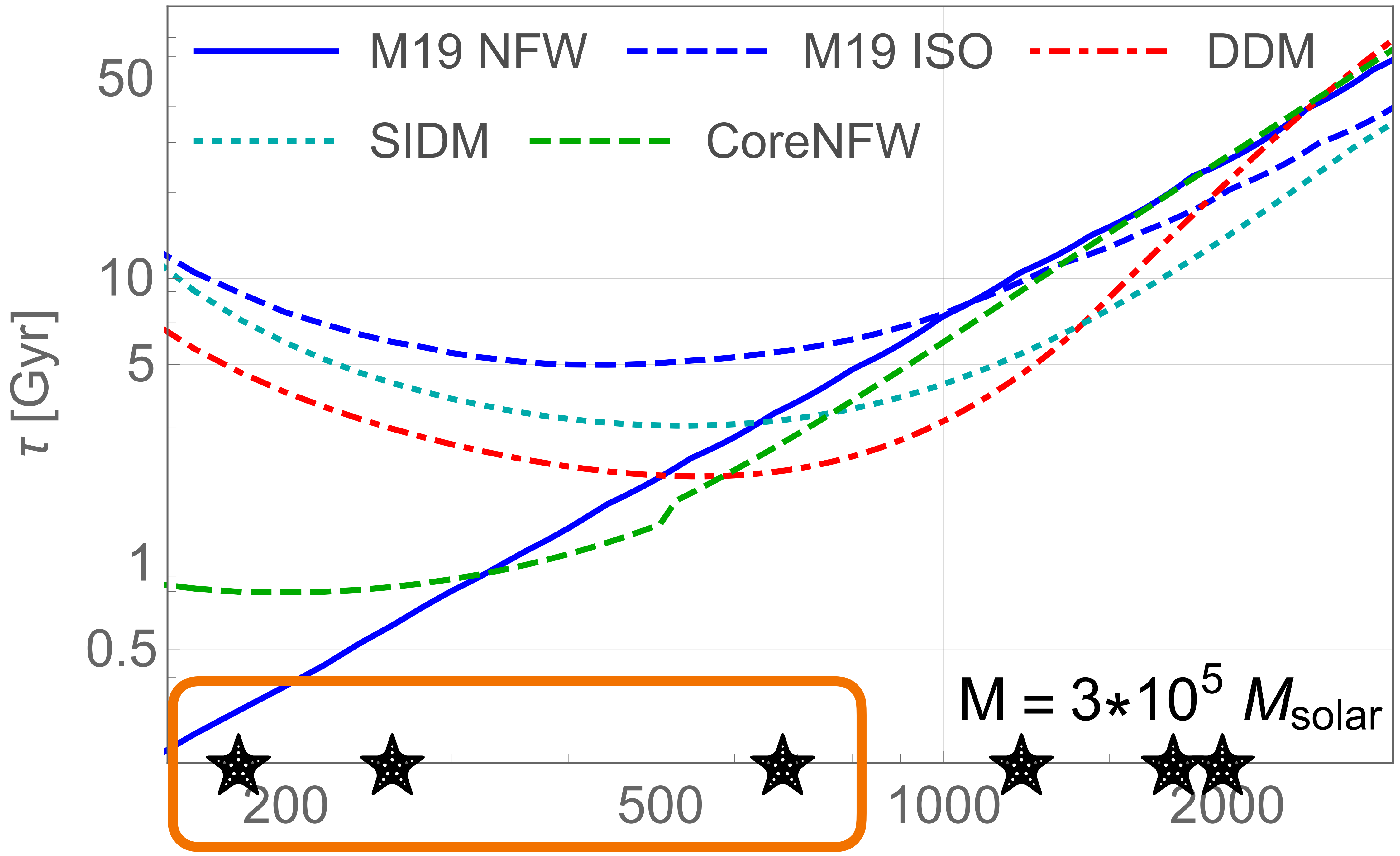
The small relaxation times in Fornax and NGC 185 seem to be inconsistent with the absence of visible nuclei in these systems. A possible explanation of this discrepancy lies in the early history of their nuclei. The first two clusters which spiral to the center will form a binary pair. When the third cluster spirals in, the resulting three-body system tends to be unstable: the binary acts as an energy source, expelling the pair and the third cluster in opposite directions (Valtonen 1974). This instability is more important in low-density systems like Fornax or NGC 185 than in M31, since the cluster motion is dominated by forces from the other clusters (rather than the galaxy) at much larger radii. Moreover, to avoid the formation of the M31

Confronted with data, a cusp is mildly tuned

For a cusp, $\tau(r) \propto r^{1.85}$

The sparse data (3 applicable GCs) favors a shallower profile, but certainly allow a cusp:

We find a Poisson probability $\sim 25\,\%$.



Inner 3 GCs fall at $r < r_{\rm cr}$

Conclusions and outlook on DM cusp in Fornax

- DM cusp predicts a power-law CDF of GCs in the center of the halo, with little dependence on initial conditions.
- Fornax requires a mild $\sim 25\,\%$ tuning to agree with this.
- Dynamical friction: $\mathcal{O}(40\%)$ of GCs to fall to the center of Fornax.

To do:

- Surface brightness modelling does not seem to predict enough light in the center of Fornax to account for fallen & disrupted GCs.
- Proper prediction of GCs in the center of Fornax would require N-body simulation.

Summary

- Revisited the Fornax timing problem. Found situation is not so bleak for a cusp.
- A **small core** due to baryonic feedback may alleviate some tension.
- A **large core** (**SIDM, DDM**) predicts little dynamical friction: GC distribution depends strongly on initial conditions. (It requires some tuning for observed radial velocity of GC4).

- Can we apply the analytical cusp prediction to more extensive data? Looking into that.

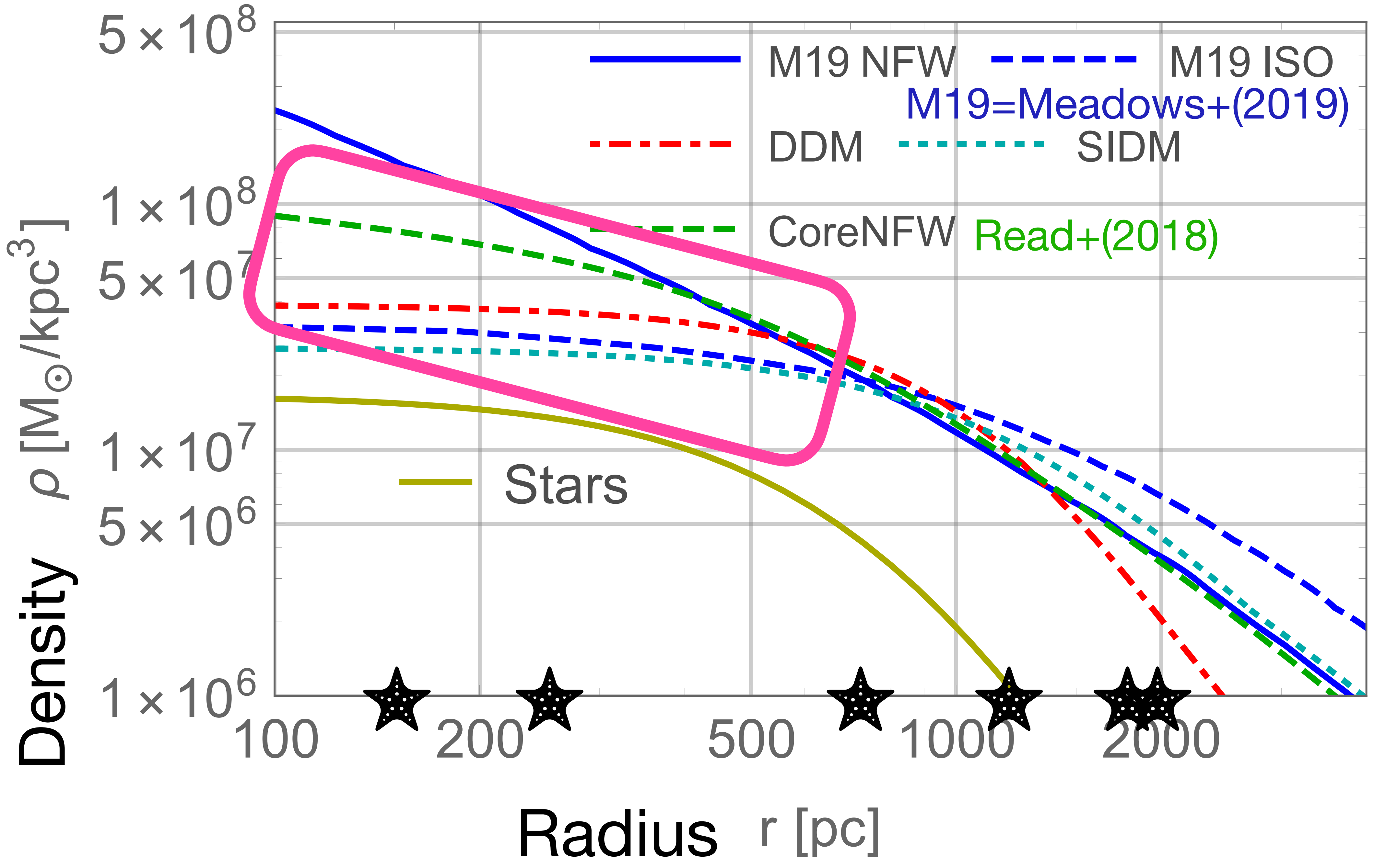
Backups

Why conclusions more mild than previous studies?

- Center of galaxy shifted a little.
- New kinematic data favors a more massive halo with less dynamical friction.
- Mass estimates of GCs were updated.
- A more accurate treatment of dynamical friction in realistic halos.
- A prediction of GC distribution for a cusp gave something concrete to test on.

Can globular cluster distribution hint to cusp vs. core?

Vanilla CDM-only simulations predict “cusp” — 1/r density.
Some indicators may hint otherwise



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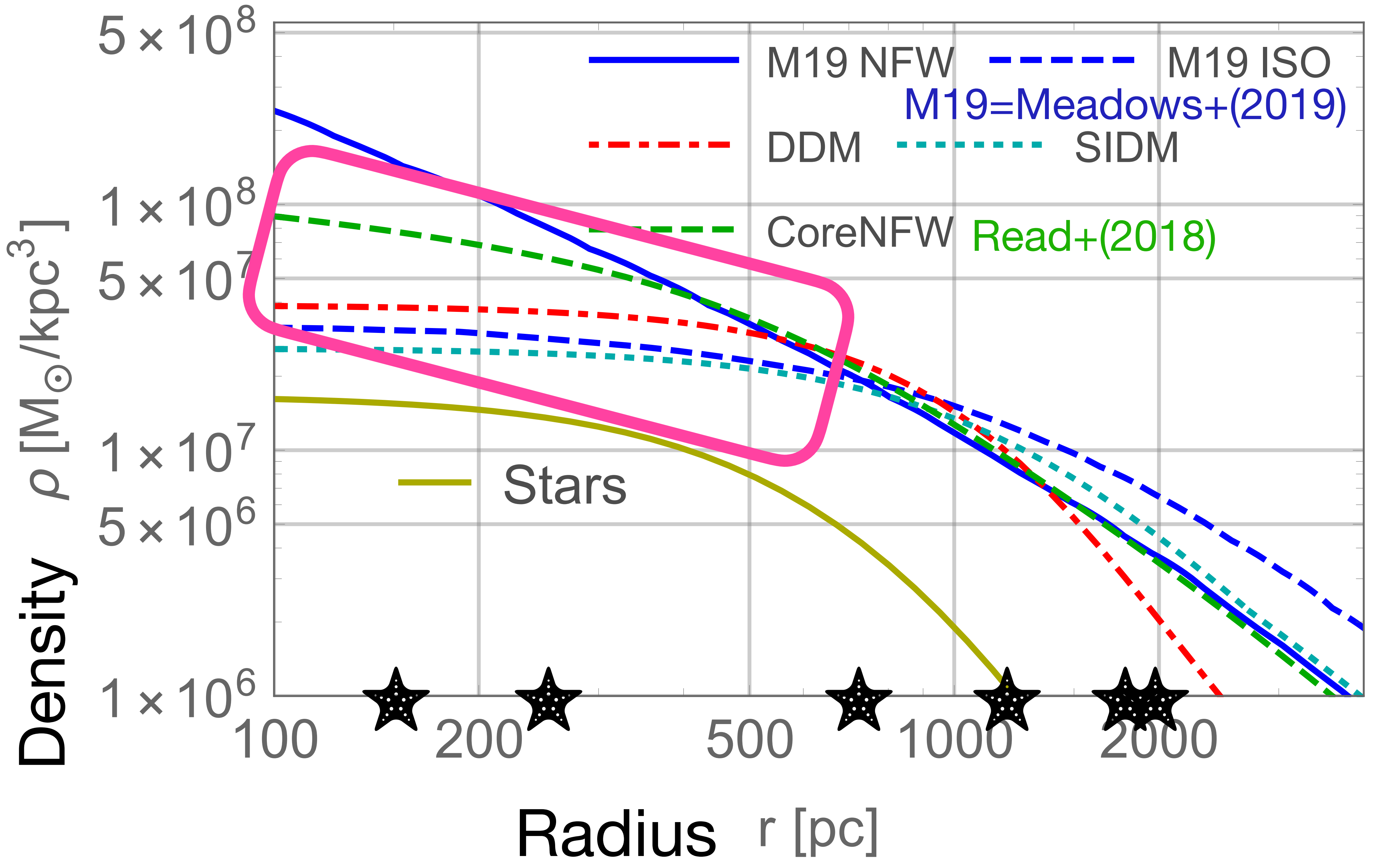
Some indicators may hint otherwise

Does the Fornax dwarf spheroidal have a central cusp or core? (MNRAS 2006)

Tobias Goerdt,[1★] Ben Moore,[1] J. I. Read,[1] Joachim Stadel[1] and Marcel Zemp[1,2]

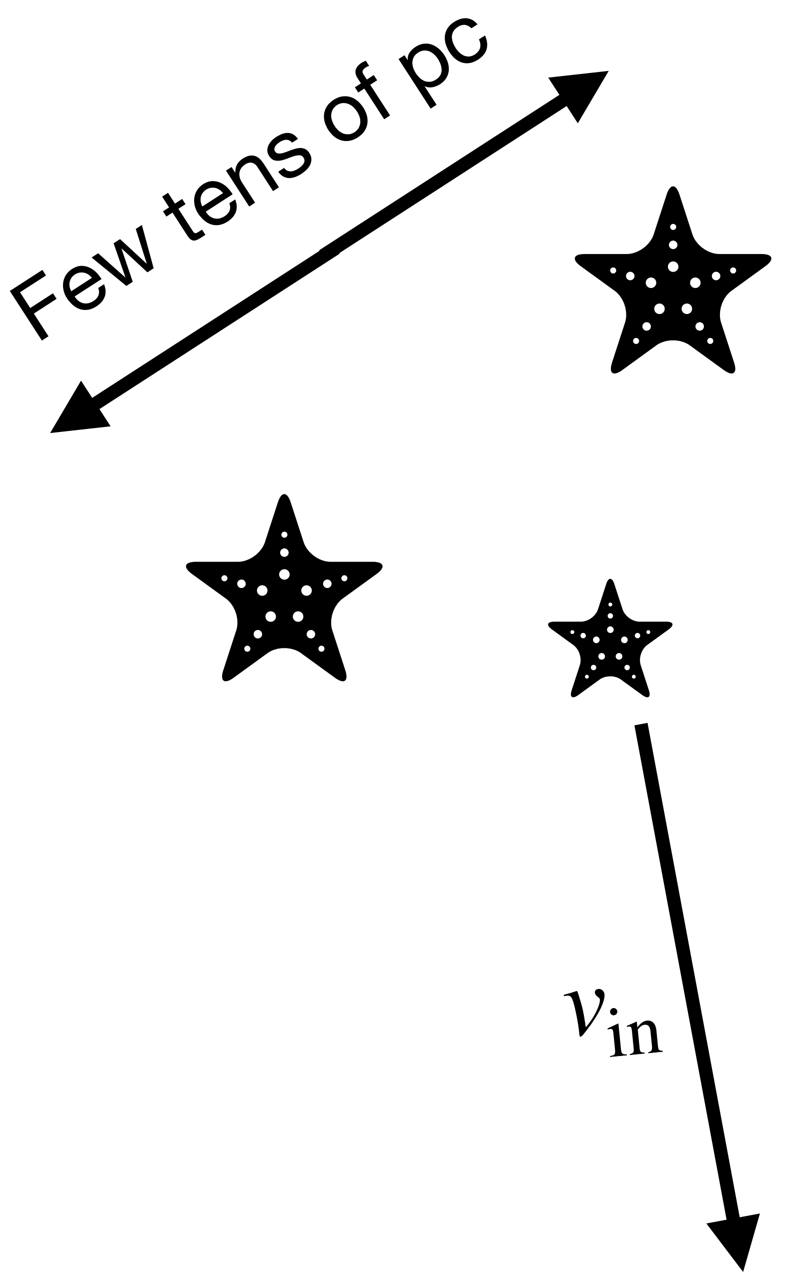
ABSTRACT
The dark matter dominated Fornax dwarf spheroidal has five globular clusters orbiting at ~1 kpc from its centre. In a cuspy cold dark matter halo the globulars would sink to the centre from their current positions within a few Gyr, presenting a puzzle as to why they survive undigested at the present epoch. We show that a solution to this timing problem is to adopt a cored dark matter halo. We use numerical simulations and analytic calculations to show that, under these conditions, the sinking time becomes many Hubble times; the globulars effectively stall at the dark matter core radius. We conclude that the Fornax dwarf spheroidal has a shallow inner density profile with a core radius constrained by the observed positions of its globular clusters. If the phase space density of the core is primordial then it implies a warm dark matter particle and gives an upper limit to its mass of ~0.5 keV, consistent with that required to significantly alleviate the substructure problem.



GCs are unlikely to be ejected from center by N-body interactions

The velocity scale is on the edge of the GC is

$$V \sim \sqrt{\frac{GM_{\rm GC}}{r_h}} = 12\sqrt{\frac{M_{\rm GC}}{10^5\ M_\odot}\frac{3\ \rm pc}{r_h}}\ \frac{\rm km}{\rm s}\ .$$



Ejecting an object from a cusp is not that easy:

$$r_{\rm eject} = \frac{v_{\rm in}^2}{4\pi G\rho_0 r_s} \sim 100\left(\frac{v_{\rm in}}{12\ \rm km/s}\right)^2\ \rm pc\ .$$

Mass of the center of Fornax consistent with stars of no more than $\sim 2-3$ GCs

Mass scale of GC $10^5\ M_\odot$.

Fornax mass inside $100\ \mathrm{pc}$ or so uncertain because of
(i) Surface brightness modeling (Plummer, Sersic, etc.)
(ii) Stellar mass uncertainty.

	$m_\star\ [10^5 M_\odot]$
GC1	0.42 ± 0.10
GC2	1.54 ± 0.28
GC3	4.98 ± 0.84
GC4	0.76 ± 0.15
GC5	1.86 ± 0.24
GC6	~ 0.29

As a very crude estimate, $0.3 - 3 \times 10^5\ M_\odot$ inside $< 100\ \mathrm{pc}$

Very dilute GCs may be tidally disrupted in the center

$$r_t \sim 40 \left(\frac{R_G}{100 \ \mathrm{pc}} \right)^{1/3} \left(\frac{M_{\mathrm{GC}}}{10^5 \ M_\odot} \right)^{1/3} \ \mathrm{pc}$$

where R_G is the distance of the GC to the halo's center.

These are very large radii, much larger than other of other GCs in Fornax.

Ly-a bounds

- Instantaneous DM free streaming is given by $k_{FS} \simeq \sqrt{\frac{3}{2}\frac{\mathcal{H}(z)}{\sigma(z)}}$
- A small core due to baryonic feedback may alleviate some tension.
- A large core predicts little dynamical friction, therefore GC distribution depends strongly on initial conditions.
  Large core also requires some tuning because of the observed radial velocity of GC4.
- Can we apply the analytical cusp prediction to more extensive data? Looking into that.

Cusp predicts 30-50% GCs fall to center — where are they?

Stellar mass in central $100\ \mathrm{pc}$ is fairly uncertain: assuming factor of two uncertainty of total mass around $4.3 \times 10^7\ M_\odot$ and different density profiles, Plummer/Sersic, we find $0.3 - 3 \times 10^5\ M_\odot$

GC masses, on the other hand, is about $10^5\ M_\odot$.

Mass budget of the center of Fornax consistent with stars of no more than $\sim 2-3$ GCs

Luminosity of a GC about $\sim 10^{4.7} = 5 \times 10^4\ L_\odot$

At* $M_V = -14.3 \pm 0.3$, $L_{\mathrm{Fornax}} \sim 4.5 \times 10^7\ L_\odot$

* Wang+(2018,DES)

Cluster	$\log L_\infty$ $(L_\odot)^{b,c}$	$\log L_m$ $(L_\odot)^b$
Fornax 1	4.07 ± 0.13	4.07 ± 0.13
Fornax 2	4.76 ± 0.12	4.75 ± 0.12
Fornax 3	5.06 ± 0.12	5.00 ± 0.11
Fornax 4	4.69 ± 0.24	$4.67^{+0.23}_{-0.24}$
Fornax 5	4.76 ± 0.20	$4.67^{+0.17}_{-0.18}$

Mackey & Gilmore (2003)

Surface brightness modeling predicts

$0.24 \pm 0.1\ \%$ of luminosity within inner $100\ \mathrm{pc} \sim 10^5\ L_\odot$

Roughly the amount of two GCs.

Mass modelling roughly agrees.

GC details in the paper

TABLE I. Some details of Fornax GCs. For the galactic center of Fornax we use an updated measurement [22], based on surface brightness modelling. This estimate is ≈ 160 pc off relative to the center defined by previous works [1, 3, 18, 20, 32, 34], leading to different projected radii of GCs. We set the distance to Fornax as 147 ± 4 kpc [19]. We estimate the error on $r_\perp$ by propagating the distance error, added in quadrature with a 13 pc [22] uncertainty on the center. For relative radial velocities Δv_r, we use the galactic radial velocity $\mathrm{RV}_{\mathrm{Fornax}} = 55.46 \pm 0.63$ km/s [51] and set $\Delta v_r = \mathrm{RV}_{\mathrm{GC}} - \mathrm{RV}_{\mathrm{Fornax}}$, adding errors in quadrature. For GC6, the values correspond to a small sample of stars, likely contaminated by background [16]. $r_{c/h}$ refers to King radius for GC1-GC5 and half-light radius for GC6. The CDM instantaneous DF time (Eq. (10)) estimates are based on the NFW profile of [18]. The instantaneous DF time of DDM and SIDM are based on Secs. IV and V.

	$m_\star$ [$10^5 M_\odot$]	$r_\perp$[kpc]	Δv_r[km/s]	$r_{c/h}$ [pc]	Refs.	τ_{CDM} [Gyr]	$\tau^{(135)}_{\mathrm{DDM}}$ [Gyr]	τ_{SIDM} [Gyr]
GC1	0.42 ± 0.10	1.73 ± 0.05	3.54 ± 1.18	10.8 ± 0.3	[19, 20, 51–53]	119	122	79.3
GC2	1.54 ± 0.28	0.98 ± 0.03	3.9 ± 0.7	6.2 ± 0.2	[19, 20, 53, 54]	14.7	7.12	8.82
GC3	4.98 ± 0.84	0.64 ± 0.02	4.94 ± 0.66	1.7 ± 0.1	[19, 20, 55, 56]	2.63	1.48	2.21
GC4	0.76 ± 0.15	0.154 ± 0.014	-8.26 ± 0.64	1.9 ± 0.2	[19, 20, 55, 56]	0.91	10.7	14.8
GC5	1.86 ± 0.24	1.68 ± 0.05	3.93 ± 0.77	1.5 ± 0.1	[19, 20, 51, 55, 56]	32.2	30.1	20
GC6	~ 0.29	0.254 ± 0.015	-1.56 ± 1.36	12.0 ± 1.4	[1, 16]	5.45	16.1	22

Analytic insight into GC distribution in a cusp

Analytic insight into GC distribution in a cusp

- For $r < r_{\rm cr}$, where $\Delta t(r_i = r_{\rm cr}; r_f = 0) = t_{\rm GC-age}$, GCs must fall to the galactic center. Then, it turns out that $N_{\rm GC}(< r)$ has an analytic approximation
  $N_{GC}(< r; \Delta t) \approx A \dfrac{\tau(r)}{\Delta t}$, where A is a normalization factor.

Analytic insight into GC distribution in a cusp

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- Projection effects correct this, but roughly keep the dependence $N_{\rm GC}(<r_\perp, \Delta t) \approx \widetilde{A}\,\dfrac{\tau(r_\perp)}{\Delta t}$ New analytic prediction.

Self-gravitating systems using Jeans equations

- In spherical symmetry, $\bar{v_\theta^2} = \bar{v_\phi^2}$ and equilibrium equation is $\frac{1}{\nu}\frac{d}{dr}(\nu \bar{v_r^2}) + 2\frac{\beta \bar{v_r^2}}{r} = -\frac{GM}{r^2}$, where $\beta = 1 - \bar{v_\theta^2}/\bar{v_r^2}$
- When $\beta \to 1$: $\bar{v_\theta^2} = \bar{v_\phi^2} = 0$, highly radial orbits
- When $\beta \to -\infty$: $\bar{v_r^2} = 0$, highly circular orbits
- $\beta = 0$: isotropic system.

New data used

- New estimate of galactic center, based on DES photometry.
- Revised GC mass estimates.
- Existence of GC6.
- New radial velocity measurements.

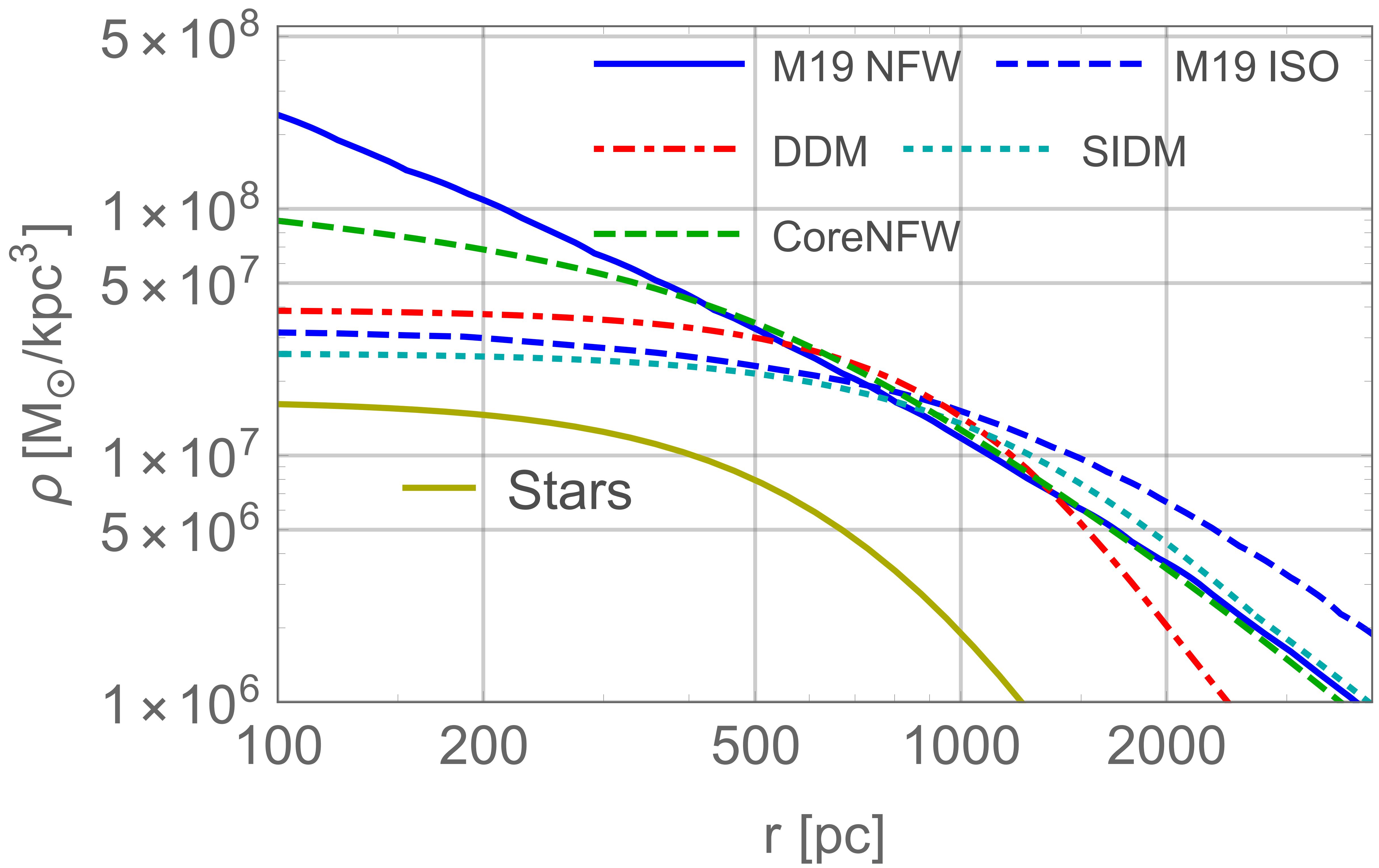
Properties of other dSph

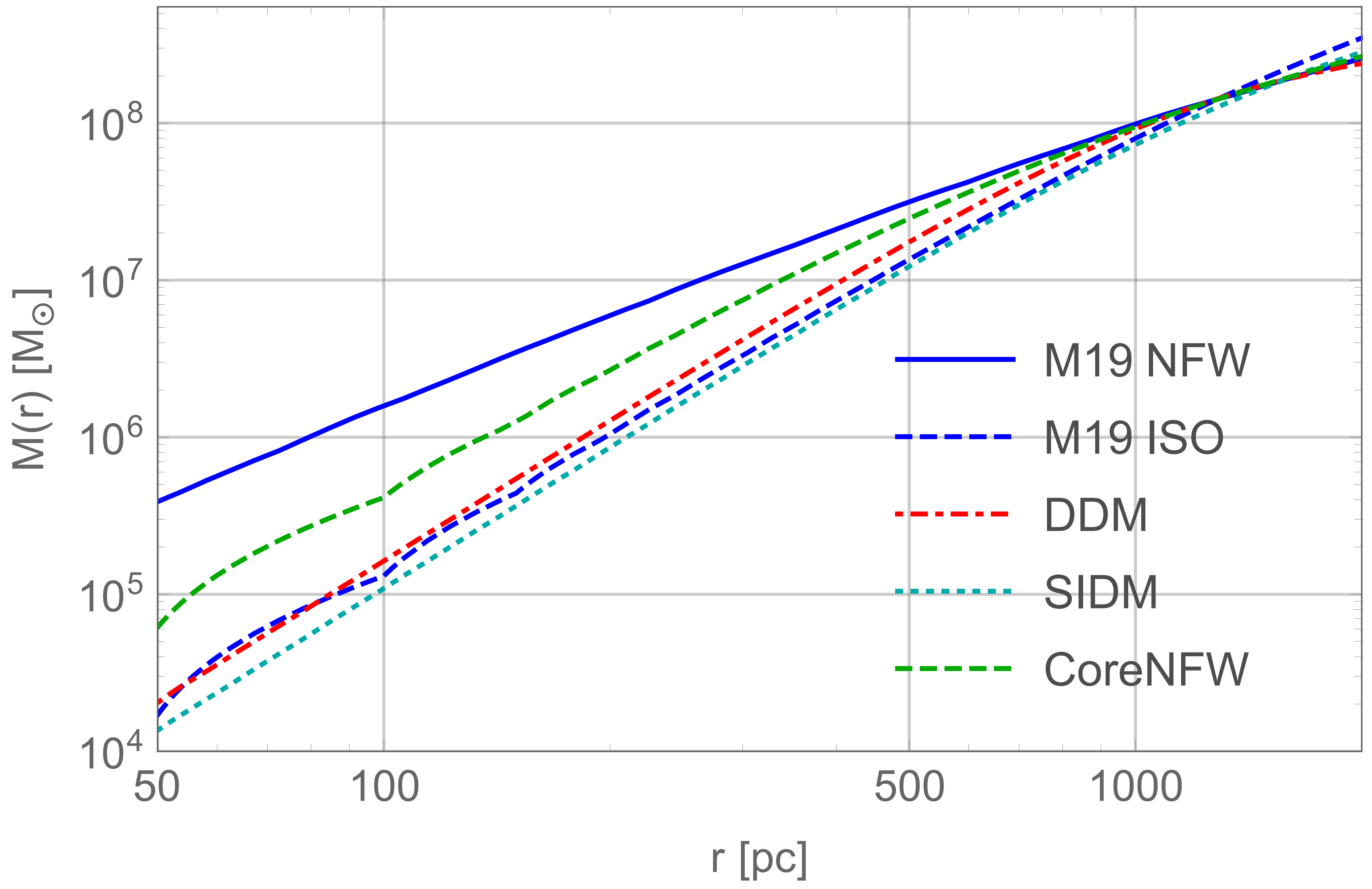
Read+2018 arXiv: 1808.06634

Galaxy	Type	D (kpc)	M_* ($10^6\,M_\odot$)	M_{gas} ($10^6\,M_\odot$)	$R_{1/2}$ (kpc)	R_{gas} (kpc)	M_{200} ($10^9 M_\odot$)	Sample size	t_{trunc} (Gyrs)	$\rho_{DM}(150\,pc)$ ($10^8 M_\odot\,kpc^{-3}$)	$\gamma_{DM}(150\,pc)$	Refs.
UMi	dSph	76 ± 3	0.29	–	0.181 ± 0.027 [0.306]	–	2.8 ± 1.1	430	12.4	$1.53^{+0.35}_{-0.32}$	$-0.71^{+0.28}_{-0.29}$	3,5
Draco	dSph	76 ± 6	0.29	–	0.221 ± 0.019 [0.198]	–	1.8 ± 0.7	504	11.7	$2.36^{+0.29}_{-0.29}$	$-0.95^{+0.25}_{-0.25}$	3,4
Sculptor	dSph	86 ± 6	2.3	–	0.283 ± 0.045 [0.248]	–	5.7 ± 2.3	1,351	11.8	$1.49^{+0.28}_{-0.23}$	$-0.83^{+0.3}_{-0.25}$	3,6
Sextans	dSph	86 ± 4	0.44	–	0.695 ± 0.044 [0.352]	–	2.0 ± 0.8	417	10.6	$1.28^{+0.34}_{-0.29}$	$-0.95^{+0.36}_{-0.41}$	3,7
Leo I	dSph	254 ± 15	5.5	–	0.251 ± 0.027 [0.298]	–	5.6 ± 2.2	328	3.1	$1.77^{+0.33}_{-0.34}$	$-1.15^{+0.33}_{-0.37}$	3,8
Leo II	dSph	233 ± 14	0.74	–	0.176 ± 0.042 [0.194]	–	1.6 ± 0.7	186	6.3	$1.84^{+0.17}_{-0.16}$	$-1.5^{+0.35}_{-0.31}$	3,8
Carina	dSph	105 ± 6	0.38	–	0.250 ± 0.039 [0.242]	–	0.8 ± 0.30	767	2.8	$1.16^{+0.20}_{-0.22}$	$-1.23^{+0.39}_{-0.35}$	3,9
Fornax	dSph	138 ± 8	43	–	0.710 ± 0.077 [0.670]	–	21.9 ± 7.4	2,573	1.75	$0.79^{+0.27}_{-0.19}$	$-0.30^{+0.21}_{-0.28}$	3,10

Much more stellar mass than the rest —
over abundance of GCs not unexpected

Mass profiles





DF in MOND - general

- Addressed in previous works: Ciotti & Binney 2004, Sanchez-Salcedo+2006, Angus & Diaferio 2009. We did not revisit it.
- Generally said to have quick orbital decay.
- Plummer star mass profile $M(r) = \frac{M_0 r^3}{(r^2 + a^2)^{3/2}}$ with $M_0 = 4.3 \times 10^7 \ M_\odot$ and $a = 0.85 \ \mathrm{kpc}$ roughly agrees with MOND ($v_{\mathrm{circ}} \approx (GMa_0)^{1/4} \sim 15 \ \mathrm{km/s}$).
- The acceleration in Fornax is $a_{\mathrm{gal}} = \frac{v_{\mathrm{circ}}^2}{r} \approx 0.1 a_0 \frac{v_{\mathrm{circ}}^2}{(15 \ \mathrm{km/s})^2} \frac{0.6 \ \mathrm{kpc}}{r}$: indeed Mondian
- Note that $a_{\mathrm{GC}} = \frac{GM_{\mathrm{GC}}}{r^2} \approx a_0 \frac{M_{\mathrm{GC}}}{10^5 \ M_\odot} \left(\frac{11 \ \mathrm{pc}}{r} \right)^2$, hinting that essentially everywhere, stars scatter on GCs in the deep Mondian regime.

Orbital characteristics of Fornax

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Astronomy & Astrophysics

Gaia DR2 proper motions of dwarf galaxies within 420 kpc

Orbits, Milky Way mass, tidal influences, planar alignments, and group infall

T. K. Fritz[1,2], G. Battaglia[1,2], M. S. Pawlowski[3,⋆], N. Kallivayalil[4], R. van der Marel[5,6], S. T. Sohn[5], C. Brook[1,2], and G. Besla[7]

Satellite	peri(1.6)(kpc)	apo(1.6)(kpc)	ecc(1.6)(kpc)	peri(0.8)(kpc)	apo(0.8)(kpc)	ecc(0.8)(kpc)
FnxI	58^{+26}_{-18}	147^{+9}_{-7}	$0.42^{+0.14}_{-0.13}$	100^{+28}_{-33}	168^{+55}_{-17}	$0.28^{+0.14}_{-0.05}$

Depends on MW potential. $(1.6) = 1.6 \times 10^{12} M_{\odot}$. Similarly (0.8).

Distance, mass to Fornax, GCs

RR Lyrae stars 1510.05642
Tip of red giant see refs in 1510.05642
Inside GCs: Mackey & Gilmore 2003 (also metallicity there)

Mass of GCs: updated in 1510.05642, using CMD and metallicity

Total mass Fornax 1510.05642

Equipartition of energy

See Binney & Tremaine (2008), Eq. 7.90 for balance of dynamical friction and dynamical heating when energy equipartition is satisfied.

The rate of change of the kinetic energy of the subject star is

$$
\begin{aligned}
D[\Delta E] &= m \sum_{i=1}^{3} \left(v_i D[\Delta v_i] + \tfrac{1}{2} D[\Delta v_i \Delta v_i] \right) \\
&= m \left(v D[\Delta v_{\parallel}] + \tfrac{1}{2} D[(\Delta v_{\parallel})^2] + \tfrac{1}{2} D[(\Delta \mathbf{v}_{\perp})^2] \right) \\
&= 16\pi^2 G^2 m m_a \ln\Lambda \left[m_a \int_v^{\infty} \mathrm{d}v_a \, v_a f_a(v_a) - m \int_0^v \mathrm{d}v_a \, \frac{v_a^2}{v} f_a(v_a) \right].
\end{aligned}
\tag{7.90}
$$

Coulomb logarithm calibration

In NFW case

$$\ln\Lambda_{\rm NFW} = \ln\frac{b_{\rm max}\sigma^2}{GM},\ b_{\rm max} = 0.5\ {\rm kpc}.$$

In Core case

$$\ln\Lambda_{\rm ISO} = \ln\frac{2rV^2}{GM}$$

Roughly agreeing with earlier definitions, but not exactly. To avoid small logarithm problems, we replace

$$\ln\Lambda \to \frac{1}{2}\ln(1+\Lambda^2)$$