Thermodynamics of (scalar-tensor) gravity: a new approach

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Work with Andrea Giusti + Jeremy Côté *Phys. Rev. D* 103, L121501 (2021); 98, 084019 (2018); 99, 064013 (2019)

16th Marcel Grossman Meeting

Outline

1. Scalar-tensor gravity as an effective fluid (dissipative, irrotational)
2. GR as an equilibrium state; temperature and shear viscosity of ST gravity
3. Approach to the GR equilibrium state, or deviations from it
4. Conclusions & open problems

Motivation

Two key ideas:

1. it is possible that gravity emerges as a sort of fluid-mechanical or thermodynamical limit ("Einstein equation as an effective equation of state")
2. on a landscape of theories of gravity, GR could be the state of equilibrium and modified gravity an excited state

(both ideas advanced in Jacobson's thermodynamics of spacetime).

Scalar-tensor gravity is the prototypical alternative to GR; $f(R)$ gravity, a subclass, is extremely popular to explain the current acceleration of the universe without an *ad hoc* dark energy.

The field equations are recast as effective Einstein equations by moving geometric terms $\neq G_{ab}$ to the r.h.s., regarding them as an effective $T_{ab}^{(eff)}$ (a fluid).

The (Jordan frame) action is

$$S_{ST} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S^{(m)}$$

where $\phi \sim G_{eff}^{-1} > 0$ is the Brans-Dicke scalar.
Field equations:

$$\begin{aligned} R_{ab} - \frac{1}{2} g_{ab} R &= \frac{8\pi}{\phi} T_{ab}^{(m)} + \frac{\omega}{\phi^2} \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi \right) \\ &+ \frac{1}{\phi} \left(\nabla_a \nabla_b \phi - g_{ab} \Box \phi \right) - \frac{V}{2\phi} g_{ab} \,, \end{aligned}$$

$$\Box \phi = \frac{1}{2\omega + 3} \left(\frac{8\pi T^{(m)}}{\phi} + \phi \frac{dV}{d\phi} - 2V - \frac{d\omega}{d\phi} \nabla^c \phi \nabla_c \phi \right)$$

Effective fluid description?

Kinematics of the scalar field fluid

The correspondence is possible if gradient $\nabla^a\phi$ is *timelike*; fluid 4-velocity is

$$u^a = \frac{\nabla^a\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}} \,,$$

$(u^c u_c = -1)$

3-D space "seen" by the comoving observers of the fluid with time direction u^a has 3-metric

$$h_{ab} \equiv g_{ab} + u_a u_b \,,$$

while $h_a{}^b$ is the usual projection operator on this 3-space,

$$\begin{aligned} h_{ab}u^a &= h_{ab}u^b = 0 \,, \\ h^a{}_b \, h^b{}_c &= h^a{}_c \,, \qquad h^a{}_a = 3 \,. \end{aligned}$$

Fluid 4-acceleration is

$$\dot{u}^a \equiv u^b \nabla_b u^a$$

(of course, orthogonal to 4-velocity, $\dot{u}^c u_c = 0$).

The (double) projection of the velocity gradient onto the 3-space orthogonal to u^c is the purely spatial tensor

$$V_{ab} \equiv h_a{}^c \, h_b{}^d \, \nabla_d u_c$$

which decomposes as

$$V_{ab} = \theta_{ab} + \omega_{ab} = \sigma_{ab} + \frac{\theta}{3} \, h_{ab} + \omega_{ab}$$

where $\theta \equiv \theta^c{}_c = \nabla^c u_c =$ expansion scalar, and these tensors are purely spatial. In general (Ellis '71)

$$\nabla_b u_a = \sigma_{ab} + \frac{\theta}{3} \, h_{ab} + \omega_{ab} - \dot{u}_a u_b = V_{ab} - \dot{u}_a u_b \, .$$

Let's specialize these general definitions to our particular case.
Kinematic quantities (not given in Pimentel '89):

$$h_{ab} = g_{ab} - \frac{\nabla_a \phi \nabla_b \phi}{\nabla^e \phi \nabla_e \phi}$$

$$\nabla_b u_a = \frac{1}{\sqrt{-\nabla^e \phi \nabla_e \phi}} \left(\nabla_a \nabla_b \phi - \frac{\nabla_a \phi \nabla^c \phi \nabla_b \nabla_c \phi}{\nabla^e \phi \nabla_e \phi} \right) .$$

4-acceleration is

$$\dot{u}_a = \left(-\nabla^e \phi \nabla_e \phi\right)^{-2} \nabla^b \phi \Big[(-\nabla^e \phi \nabla_e \phi) \nabla_a \nabla_b \phi + \nabla^c \phi \nabla_b \nabla_c \phi \nabla_a \phi \Big]$$

$$
\begin{aligned}
V_{ab} &= \frac{\nabla_a \nabla_b \phi}{\left(-\nabla^e \phi \nabla_e \phi\right)^{1/2}} \\
&\quad + \frac{\left(\nabla_a \phi \nabla_b \nabla_c \phi + \nabla_b \phi \nabla_a \nabla_c \phi\right) \nabla^c \phi}{\left(-\nabla^e \phi \nabla_e \phi\right)^{3/2}} \\
&\quad + \frac{\nabla_d \nabla_c \phi \nabla^c \phi \nabla^d \phi}{\left(-\nabla^e \phi \nabla_e \phi\right)^{5/2}} \nabla_a \phi \nabla_b \phi \, .
\end{aligned}
$$

Vorticity $\omega_{ab} \equiv V_{[ab]}$ vanishes identically, because 4-velocity originates from a gradient, so

$$
V_{ab} = \theta_{ab} \, , \qquad \nabla_b u_a = \theta_{ab} - \dot{u}_a u_b \, ,
$$

u^a is hypersurface-orthogonal.

Expansion scalar:

$$\theta = \nabla_a u^a = \frac{\Box\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{1/2}} + \frac{\nabla_a\nabla_b\phi\nabla^a\phi\nabla^b\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}} \,,$$

Shear tensor:

$$\sigma_{ab} = \left(-\nabla^e\phi\nabla_e\phi\right)^{-3/2} \Big[- \left(\nabla^e\phi\nabla_e\phi\right) \nabla_a\nabla_b\phi$$

$$-\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\,\nabla^c\phi\nabla_c\phi\right) \Box\,\phi$$

$$-\frac{1}{3}\left(g_{ab} + \frac{2\nabla_a\phi\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right) \nabla_c\nabla_d\phi\nabla^d\phi\nabla^c\phi$$

$$+\left(\nabla_a\phi\nabla_c\nabla_b\phi + \nabla_b\phi\nabla_c\nabla_a\phi\right)\nabla^c\phi\Big] \,,$$

In the vacuum field eqs written as effective Einstein eqs.

$$G_{ab} = 8\pi T^{(\phi)}_{ab} = \frac{\omega}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla^c\phi\nabla_c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}g_{ab}$$

the r.h.s. decomposes as

$$\boxed{T^{(\phi)}_{ab} = \rho^{(\phi)}u_a u_b + q_a^{(\phi)}u_b + q_b^{(\phi)}u_a + \Pi^{(\phi)}_{ab}\,,}$$

(dissipative fluid), where

$$
\begin{aligned}
\rho^{(\phi)} &= T_{ab}^{(\phi)} u^a u^b \,, \\
q_a^{(\phi)} &= -T_{cd}^{(\phi)} \, u^c h_a{}^d \,, \\
\Pi_{ab}^{(\phi)} &\equiv P^{(\phi)} h_{ab} + \pi_{ab}^{(\phi)} = T_{cd}^{(\phi)} \, h_a{}^c \, h_b{}^d \,, \\
P^{(\phi)} &= \frac{1}{3} \, g^{ab} \Pi_{ab}^{(\phi)} = \frac{1}{3} \, h^{ab} T_{ab}^{(\phi)} \,, \\
\pi_{ab}^{(\phi)} &= \Pi_{ab}^{(\phi)} - P^{(\phi)} h_{ab} \,,
\end{aligned}
$$

with

$$
q_c^{(\phi)} u^c = \Pi_{ab}^{(\phi)} u^b = \pi_{ab}^{(\phi)} u^b = \Pi_{ab}^{(\phi)} u^a = \pi_{ab}^{(\phi)} u^a = 0 \,, \qquad \pi^a{}_a = 0 \,.
$$

Calculating these quantities explicitly,

$$8\pi\rho^{(\phi)} = -\frac{\omega}{2\phi^2}\nabla^e\phi\nabla_e\phi + \frac{V}{2\phi} + \frac{1}{\phi}\left(\Box\phi - \frac{\nabla^a\phi\nabla^b\phi\nabla_a\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right),$$

$$8\pi q_a^{(\phi)} = -\frac{\nabla^c\phi\nabla_a\nabla_c\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{1/2}} - \frac{\nabla^c\phi\nabla^d\phi\nabla_c\nabla_d\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}}\nabla_a\phi,$$

$$8\pi\Pi_{ab}^{(\phi)} = \left(-\frac{\omega}{2\phi^2}\nabla^c\phi\nabla_c\phi - \frac{\Box\phi}{\phi} - \frac{V}{2\phi}\right)h_{ab} + \frac{1}{\phi}h_a{}^c h_b{}^d\nabla_c\nabla_d\phi,$$

$$8\pi P^{(\phi)} = -\frac{\omega}{2\phi^2}\nabla^e\phi\nabla_e\phi - \frac{V}{2\phi} - \frac{1}{3\phi}\left(2\Box\phi + \frac{\nabla^a\phi\nabla^b\phi\nabla_b\nabla_a\phi}{\nabla^e\phi\nabla_e\phi}\right),$$

$$
\begin{aligned}
8\pi\pi^{(\phi)}_{ab} \quad = \quad & \frac{1}{\phi\nabla^e\phi\nabla_e\phi}\left[\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\left(\Box\phi - \frac{\nabla^c\phi\nabla^d\phi\nabla_d\nabla_c\phi}{\nabla^e\phi\nabla_e\phi}\right)\right. \\
& +\nabla^d\phi\left(\nabla_d\phi\nabla_a\nabla_b\phi - \nabla_b\phi\nabla_a\nabla_d\phi - \nabla_a\phi\nabla_d\nabla_b\phi\right. \\
& \left.\left. + \frac{\nabla_a\phi\nabla_b\phi\nabla^c\phi\nabla_c\nabla_d\phi}{\nabla^e\phi\nabla_e\phi}\right)\right]
\end{aligned}
$$

The heat flux density

$$
\boxed{q^{(\phi)}_a = -\frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}\,\dot{u}_a}
$$

and the anisotropic stresses $\pi^{(\phi)}_{ab}$ do not vanish.

Thermodynamics of scalar-tensor gravity

Take the effective dissipative fluid seriously: what do we know about dissipation in GR? Dissipative fluids in GR are described by Eckart's 1st order thermodynamics (Eckart '40), notoriously plagued by non-causality and instabilities but still the most widely used approximation)

Constitutive relations in Eckart's theory:

$$\begin{aligned} P_{\text{viscous}} &= -\zeta\,\theta \\ q_a &= -K\left(h_{ab}\nabla^b\mathcal{T} + \mathcal{T}\dot{u}_a\right) \\ \pi_{ab} &= -2\eta\,\sigma_{ab} \end{aligned}$$

where

$$\begin{aligned} \zeta &= \text{bulk viscosity} \\ K &= \text{thermal conductivity} \\ \eta &= \text{shear viscosity} \end{aligned}$$

Compare with the expressions of $P^{(\phi)}, q_a^{(\phi)}, \pi_{ab}^{(\phi)} \rightarrow$

$$P_{\text{viscous}} = 0\,,$$

$$K\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi} > 0\,,$$

$$\eta = -\frac{K\mathcal{T}}{2} < 0$$

for a ST spacetime.

Negative viscosities appear in systems that exchange energy with their surroundings (atmosphere, ocean currents, liquid crystals, ...) and the non-minimally coupled ϕ-fluid is not isolated. Entropy density can decrease.

Approach to the GR equilibrium state

$$\boxed{K\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}}$$

$\phi =$ const. $\leftrightarrow \mathcal{T} = 0$ GR equilibrium state

Differentiate $\rightarrow$

$$\frac{d\left(K\mathcal{T}\right)}{d\tau} = 8\pi\left(K\mathcal{T}\right)^2 - \theta\, K\mathcal{T} + \frac{\Box\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}}$$

Physical interpretation in simplified scenarios

- Electrovacuum, $\omega =$ const., $V(\phi) = 0 \rightarrow \Box\phi = 0$.
  Then,
  $$\theta < 0 \rightarrow \frac{d(K\mathcal{T})}{d\tau} > 8\pi(K\mathcal{T})^2$$
  or, $K\mathcal{T}$ diverges away from the GR equilibrium extremely fast.
  Deviations of ST gravity from GR will be extreme near spacetime singularities.
- Electrovacuum, $\theta > 0$:
  $-\theta K\mathcal{T}$ can dominate $(K\mathcal{T})^2$ then the solution
  $K\mathcal{T}$ can approach 0: diffusion to GR equilibrium, expansion cools gravity.
  But, if $K\mathcal{T}$ is large, the positive term dominates r.h.s. and drives solution away from GR:
  approach to GR equilibrium state not always expected.
- Several analytic solutions of BD/ST gravity corroborate these ideas.

CONCLUSIONS + THE FUTURE

- Minimal assumptions: used only ST field equations and constitutive relations in Eckart's theory (not the full theory) $\rightarrow \mathcal{T}, \eta$, approach to GR equilibrium.
- Open problems:
  - Cosmology; situations with non-timelike $\nabla^c\phi\nabla_c\phi$
  - Other theories of gravity
  - Alternative approach: trade temperature with chemical potential, assign $\mathcal{T} = 0$, $S = 0$ but $\mu \neq 0$ to the effective fluid (as in Vikman *at al.* 2011-16).
- Stay tuned on arXiv!