Sixteenth Marcel Grossmann Meeting

5-10 July 2021

Effective $f(R)$ actions for modified Loop Quantum Cosmologies

Speaker: Ana Rita Ribeiro

Authors: Ana Rita Ribeiro, Daniele Vernieri, Francisco S. N. Lobo

Effective $f(R)$ actions for modified Loop Quantum Cosmologies via order reduction

Ana Rita Ribeiro*
Instituto de Astrofísica e Ciências do Espaço, Faculdade de Ciências da Universidade de Lisboa, Edifício C8, Campo Grande, P-1749-016, Lisbon, Portugal

Daniele Vernieri †
Department of Physics "E. Pancini", University of Naples "Federico II", Naples, Italy, and INFN Sez. di Napoli, Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126, Naples, Italy.

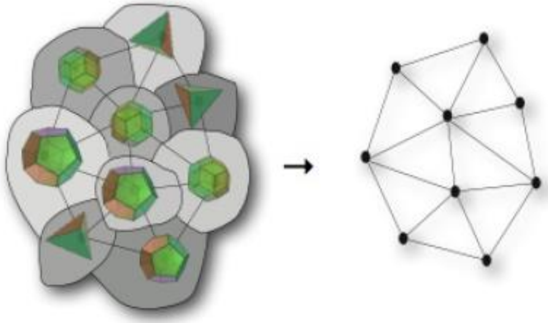
Francisco S. N. Lobo ‡
Instituto de Astrofísica e Ciências do Espaço, Faculdade de Ciências da Universidade de Lisboa, Edifício C8, Campo Grande, P-1749-016, Lisbon, Portugal and Departamento de Física, Faculdade de Ciências, Universidade de Lisboa, Edifício C8, Campo Grande, PT1749-016 Lisbon, Portugal

(Dated: LaTeX-ed May 5, 2021)

arXiv:2104.12283v2 [gr-qc] 4 May 2021

Motivations: Loop quantum cosmology and its modifications

- **Initial singularity problem**
- **Loop Quantum Gravity (LQG)**
  - Loop Quantum Cosmology (LQC)



$$H^2 = \frac{\kappa\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right) \quad (1)$$

Motivations: Loop quantum cosmology and its modifications

- **Hamiltonian constraint**

$$\mathcal{H} = \frac{\left(C_{\text{grav}} + C_M\right)}{\kappa} \quad (2)$$

- **Gravitational constraint**

$$C_{\text{grav}} = C_{\text{grav}}^{(E)} - (1+\gamma^2)C_{\text{grav}}^{(L)} \quad (3)$$

- **Euclidean and Lorentzian terms**

$$C_{\text{grav}}^{(E)} = \frac{1}{2}\int d^3x\, \epsilon_{ijk}\, F_{ab}^{i}\, \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}} \quad (4)$$

$$C_{\text{grav}}^{(L)} = \int d^3x\, K_{[a}^{j}K_{b]}^{k}\, \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}} \quad (5)$$

Natural units:
$c = \hbar = G = 1$

$\kappa = 8\pi$

Barbero Immirzi
Parameter: $\gamma \approx 0.2375$

- F_{ab}^{i} - field strength of A_a^i
- A_a^i - Ashtekar-Barbero connection
- E_a^i - conjugate triad
- K_a^i - extrinsic curvature
- $\det(q)$ - determinant of spatial metric compatible with triads

There is still no systematic derivation of LQC as a cosmological sector of LQG.

Motivations: Loop quantum cosmology and its modifications

- **LQC**

$$C_{\text{grav}}^{(E)} \propto C_{\text{grav}}^{(L)}$$

- **Modified Loop Quantum Cosmology I (mLQC-I)**

  $C_{grav}^{(E)}$ and $C_{grav}^{(L)}$ are treated separately

  Thiemann's regularization

- **Modified Loop Quantum Cosmology II (mLQC-II)**

  $C_{grav}^{(E)}$ and $C_{grav}^{(L)}$ are treated separately

  Thiemann's regularization

$$K_a^i = A_a^i/\gamma$$

$$C_{\text{grav}}^{(E)} = \frac{1}{2}\int d^3x\, \epsilon_{ijk}\, F_{ab}^i \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}} \quad (4)$$

$$C_{\text{grav}}^{(L)} = \int d^3x\, K_{[a}^j K_{b]}^k \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}} \quad (5)$$

- F_{ab}^i - field strength of A_a^i
- A_a^i - Ashtekar-Barbero connection
- E_a^i - conjugate triad
- K_a^i - extrinsic curvature
- $\det(q)$ - determinant of spatial metric compatible with triads

Motivations: Loop quantum cosmology and its modifications

- **LQC**

$$H^2 = \frac{\kappa\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right) \quad (1)$$

- **mLQC-I**

$$H_-^2 = \frac{\kappa\rho}{3}\left(1 - \frac{\rho}{\rho_c^I}\right)\left[1 + \frac{\gamma^2\rho/\rho_c^I}{(\gamma^2+1)\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right] \quad (6)$$

$$H_+^2 = \frac{\kappa\,\boldsymbol{\alpha\rho_\Lambda}}{3}\left(1 - \frac{\rho}{\rho_c^I}\right)\left[1 + \frac{\rho\left(1-2\gamma^2+\sqrt{1-\rho/\rho_c^I}\right)}{4\gamma^2\rho_c^I\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right] \quad (7)$$

- **mLQC-II**

$$H^2 = \frac{2\kappa\rho}{3}\left(1 - \frac{\rho}{\rho_c^{II}}\right)\left[\frac{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}{1+2\gamma^2\rho/\rho_c^{II}+\sqrt{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}}\right] \quad (8)$$

Natural units:
$c = \hbar = G = 1$

$\kappa = 8\pi$
$\lambda^2 \equiv 4\sqrt{3}\pi\gamma$
$\rho_\Lambda \equiv 3/[\kappa\,\alpha\lambda^2(1+\gamma^2)]$
$\alpha \equiv (1-5\gamma^2)/(\gamma^2+1)$
Barbero Immirzi Parameter: $\gamma \approx 0.2375$

Critical densities:
$\rho_c \equiv \sqrt{3}/(32\pi^2\gamma^3)$

$\rho_c^I \equiv \rho_c/[4(1+\gamma^2)]$
$\rho_c^{II} \equiv 4(1+\gamma^2)\rho_c$

The only covariant action that leads to second-order equations is the Einstein-Hilbert action.

Lagrangian formulation of General Relativity (GR)

- **Einstein-Hilbert action**

$$S_{EH} = \frac{1}{2\kappa}\int_V d^4x\,\sqrt{-g}(R + 2\Lambda) + S_M\left(g_{\mu\nu}, \Psi\right) \quad (9)$$

  - Matter action

$$S_M\left(g_{\mu\nu}, \Psi\right) = \int_V d^4x\,\sqrt{-g}\,\mathcal{L}_M\left(g_{\mu\nu}, \Psi\right) \quad (10)$$

  - Energy-momentum tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\,\mathcal{L}_M\right)}{\delta g^{\mu\nu}} \quad (11)$$

- **Variation of the action, with respect to the metric tensor, gives the Einstein field equations**

$$\delta S = 0 \Rightarrow G_{\mu\nu} - g_{\mu\nu}\Lambda = \kappa\,T_{\mu\nu} \quad (12)$$

The Friedmann equation

- **Einstein field equations**

$$G_{\mu\nu} - g_{\mu\nu}\,\Lambda = \kappa\, T_{\mu\nu} \quad (13)$$

- **Friedmann-Lemaître-Roberston-Walker metric**

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right] \quad (14)$$

- **Energy-momentum tensor for a perfect fluid**

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu + p\, g_{\mu\nu} \quad (15)$$

- **Friedmann equation ($\boldsymbol{k = \Lambda = 0}$)**

$$H^2 = \frac{\kappa\rho}{3} \quad (16)$$

$$H_-^2 = \frac{\kappa\rho}{3}\left(1-\frac{\rho}{\rho_c^I}\right)\left[1+\frac{\gamma^2\rho/\rho_c^I}{(\gamma^2+1)\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right]$$

$$H_+^2 = \frac{\kappa\,\alpha\rho_\Lambda}{3}\left(1-\frac{\rho}{\rho_c^I}\right)\left[1+\frac{\rho\left(1-2\gamma^2+\sqrt{1-\rho/\rho_c^I}\right)}{4\gamma^2\rho_c^I\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right]$$

Is it possible to derive them from an effective action, other than the Einstein-Hilbert action?

$$H^2 = \frac{2\kappa\rho}{3}\left(1-\frac{\rho}{\rho_c^{II}}\right)\left[\frac{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}{1+2\gamma^2\rho/\rho_c^{II}+\sqrt{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}}\right]$$

Metric $f(R)$ gravity

- **General relativity**

$$S_{EH} = \frac{1}{2\kappa}\int_V d^4x\,\sqrt{-g}(\boldsymbol{R + 2\Lambda}) + S_M(g_{\mu\nu},\Psi) \quad (9)$$

$$\delta S = 0 \Rightarrow G_{\mu\nu} - g_{\mu\nu}\Lambda = \kappa\, T_{\mu\nu} \quad (12)$$

Second-order field equations

- **Metric $f(R)$ gravity**

$$S = \frac{1}{2\kappa}\int_V d^4x\,\sqrt{-g}\,\boldsymbol{f(R)} + S_M(g_{\mu\nu},\Psi) \quad (17)$$

$$\boldsymbol{f(R) = R + 2\Lambda + \epsilon\varphi(R) \quad (18)}$$

$$\delta S = 0 \Rightarrow G_{\mu\nu} - g_{\mu\nu}\Lambda + \epsilon\left[-\frac{1}{2}g_{\mu\nu}\varphi(R) + \varphi'(R)R_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\varphi'(R)\right] = \kappa T_{\mu\nu} \quad (19)$$

Fourth-order field equations

We can either consider this theory as an exact theory or we can treat it as an <u>effective theory</u>.

Covariant order reduction technique

$$G_{\mu\nu} - g_{\mu\nu}\Lambda + \epsilon\left[-\frac{1}{2}g_{\mu\nu}\varphi(R) + \varphi'(R)R_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\varphi'(R)\right] = \kappa T_{\mu\nu} \quad (19)$$

Fourth-order field equations

$$\epsilon = 0 \qquad \Lambda = k = 0$$

- **Reduced Ricci scalar** $R_T = -\kappa T \quad (20)$
- **Reduced Ricci tensor** $R^T_{\mu\nu} = \kappa\, T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,\kappa\, T \quad (21)$

$$G_{\mu\nu} + \epsilon\left[-\frac{1}{2}g_{\mu\nu}\varphi(R_T) + \varphi'(R_T)\left(\kappa\, T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,\kappa\, T\right) - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\varphi'(R_T)\right] = \kappa\, T_{\mu\nu} \quad (22)$$

Second-order field equations

$$f(R) = R + \epsilon\varphi(R)$$

$$|\epsilon\, \varphi(R)| \ll |R|$$

Covariant order reduction technique

$$G_{\mu\nu} + \epsilon\left[-\frac{1}{2}g_{\mu\nu}\varphi(R_T) + \varphi'(R_T)\left(\kappa\, T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,\kappa\, T\right) - (\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box)\varphi'(R_T)\right] = \kappa\, T_{\mu\nu} \quad (22)$$

- **Friedmann-Lemaître-Roberston-Walker metric**

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right] \quad (14)$$

- **Energy-momentum tensor for a perfect fluid**

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu + p\, g_{\mu\nu} \quad (15)$$

$$\Lambda = k = 0$$

$$H^2 = \frac{\kappa\rho}{3} - \frac{\epsilon}{3}\left[\frac{1}{2}(3w+1)\varphi'(R_T)\kappa\rho + \frac{1}{2}\varphi(R_T) - 3\kappa^2\rho^2\,\varphi''(R_T)(1+w)(1-3w)\right] \quad (23)$$

Reduced Friedmann equation

$|\epsilon\,\varphi(R)| \ll |R|$

Outline of the paper

We are interested in extending the analysis outlined in T. P. Sotiriou, Phys. Rev. D **79**, 044035 (2009)

Massless scalar field ($w = 1$)

$$f(R) = R + \frac{R^2}{18\kappa\rho_c} + \cdots \quad (24) \longrightarrow H^2 = \frac{\kappa\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right) \quad (1)$$

$$R \gg -18\kappa\rho_c \Rightarrow \rho \ll 9\rho_c \quad (25)$$

Strategy to obtain effective actions

Reduced Friedmann equation from metric f(R) gravity

$$H^2 = \frac{\kappa\rho}{3} - \frac{\epsilon}{3}\left[\frac{1}{2}(3w+1)\varphi'(R_T)\kappa\rho + \frac{1}{2}\varphi(R_T) - 3\kappa^2\rho^2\,\varphi''(R_T)(1+w)(1-3w)\right] \quad (23)$$

We want them to be the same

Modified Friedmann equations from mLQCs

mLQC-I

$$H_-^2 = \frac{\kappa\rho}{3}\left(1-\frac{\rho}{\rho_c^I}\right)\left[1+\frac{\gamma^2\rho/\rho_c^I}{(\gamma^2+1)\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right] \quad (6)$$

$$H_+^2 = \frac{\kappa\,\alpha\rho_\Lambda}{3}\left(1-\frac{\rho}{\rho_c^I}\right)\left[1+\frac{\rho\left(1-2\gamma^2+\sqrt{1-\rho/\rho_c^I}\right)}{4\gamma^2\rho_c^I\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right] \quad (7)$$

mLQC-II

$$H^2 = \frac{2\kappa\rho}{3}\left(1-\frac{\rho}{\rho_c^{II}}\right)\left[\frac{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}{1+2\gamma^2\rho/\rho_c^{II}+\sqrt{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}}\right] \quad (8)$$

$$H^2 = \frac{\kappa\rho}{3} + \Psi(\rho) \quad (26)$$

Strategy to obtain effective actions

$R_T = -\kappa\rho(3w - 1)$ (27)

Solve the differential equation for $\varphi(R_T)$

$$-\frac{\epsilon}{3}\left[\frac{1}{2}\varphi(R_T) - \frac{(3w+1)}{2(3w-1)}R_T\,\varphi'(R_T) + \frac{3(1+w)}{3\mathrm{w}-1}R_T^2\,\varphi''(R_T)\right] = \Psi(R_T) \quad (28)$$

$$\varphi(R_T) \longrightarrow f(R) = R + \epsilon\varphi(R) \quad (29) \longrightarrow S = \frac{1}{2\kappa}\int_V d^4x\,\sqrt{-g}\,f(R) + S_M(g_{\mu\nu}, \Psi) \quad (17)$$

Effective $f(R)$ action that leads to a quantum bounce

Strategy to obtain effective actions

Massless scalar field ($w = 1$)

$$R_T = -2\kappa\rho \quad (30)$$

Solve the differential equation for $\varphi(R_T)$

$$-\frac{\epsilon}{3}\left[\frac{1}{2}\varphi(R_T) - R_T\,\varphi'(R_T) + 3R_T^2\,\varphi''(R_T)\right] = \Psi(R_T) \quad (31)$$

↓

$$\varphi(R_T) \longrightarrow f(R) = R + \epsilon\varphi(R) \quad (29) \longrightarrow S = \frac{1}{2\kappa}\int_V d^4x\,\sqrt{-g}\,f(R) + S_M(g_{\mu\nu},\Psi) \quad (17)$$

Effective $f(R)$ action that leads to a quantum bounce

Our results…

Effective actions for mLQC models

- **Solve the differential equation for $\varphi(R_T)$, using *Wolfram Mathematica* 12.2**

$$-\frac{\epsilon}{3}\left[\frac{1}{2}\varphi(R_T) - R_T\,\varphi'(R_T) + 3R_T^2\,\varphi''(R_T)\right] = \Psi(R_T) \quad (31) \longrightarrow \boldsymbol{\varphi(R_T)}$$

$$R_T = -2\kappa\rho \quad (30)$$

mLQC-I

$$H_-^2 = \frac{\kappa\rho}{3}\left(1 - \frac{\rho}{\rho_c^I}\right)\left[1 + \frac{\gamma^2\rho/\rho_c^I}{(\gamma^2+1)\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right] \quad (6)$$

$$H_+^2 = \frac{\kappa\,\alpha\rho_\Lambda}{3}\left(1 - \frac{\rho}{\rho_c^I}\right)\left[1 + \frac{\rho\left(1-2\gamma^2+\sqrt{1-\rho/\rho_c^I}\right)}{4\gamma^2\rho_c^I\left(1+\sqrt{1-\rho/\rho_c^I}\right)^2}\right] \quad (7)$$

mLQC-II

$$H^2 = \frac{2\kappa\rho}{3}\left(1 - \frac{\rho}{\rho_c^{II}}\right)\left[\frac{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}{1+2\gamma^2\rho/\rho_c^{II}+\sqrt{1+4\gamma^2(\gamma^2+1)\rho/\rho_c^{II}}}\right] \quad (8)$$

$$\Psi(\rho) = H^2 - \frac{\kappa\rho}{3} \quad (26)$$

Effective actions for mLQC-I (b_- branch)

$$S = \frac{1}{2\kappa}\int_V d^4x\,\sqrt{-g}\,f(R) + S_M(g_{\mu\nu},\Psi) \quad (17)$$

$$f(R) = R + \epsilon\varphi(R) \quad (29) \qquad\qquad |\epsilon\,\varphi(R)| \ll |R|$$

$$\begin{aligned}
f(R) = R + \frac{1}{90(\gamma^2+1)}\Bigg\{ & 5\left(-72\gamma^2\kappa\rho_c^I + \frac{R^2}{\kappa\rho_c^I} + 54\gamma^2 R\right) + \frac{18\gamma^2}{\sqrt{10}+10}\Bigg[3\Bigg(4\left(5-4\sqrt{10}\right)\kappa\rho_c^I \\
&+ \left(\sqrt{10}+10\right)R\Bigg)\,{}_2F_1\left[-\frac{1}{2},\frac{1}{6}\left(-\sqrt{10}-4\right);\frac{1}{6}\left(2-\sqrt{10}\right);-\frac{R}{2\kappa\rho_c^I}\right] \\
&+3\left(12\left(2\sqrt{10}+5\right)\kappa\rho_c^I + \left(\sqrt{10}+10\right)R\right)\,{}_2F_1\left[-\frac{1}{2},\frac{1}{6}\left(\sqrt{10}-4\right);\frac{1}{6}\left(\sqrt{10}+2\right);-\frac{R}{2\kappa\rho_c^I}\right] \\
&+(2\kappa\rho_c^I + R)\Bigg(9\sqrt{10}\,{}_2F_1\left[\frac{1}{2},\frac{1}{6}\left(-\sqrt{10}-4\right);\frac{1}{6}\left(2-\sqrt{10}\right);-\frac{R}{2\kappa\rho_c^I}\right] \\
&-\left(11\sqrt{10}+20\right)\,{}_2F_1\left[\frac{1}{2},\frac{1}{6}\left(\sqrt{10}-4\right);\frac{1}{6}\left(\sqrt{10}+2\right);-\frac{R}{2\kappa\rho_c^I}\right]\Bigg)\Bigg]\Bigg\} \quad (32)
\end{aligned}$$

$2F_1[a,b;c;z]$ - *Hypergeometric function*

Effective actions for mLQC-I (b_+ branch)

$$S = \frac{1}{2\kappa}\int_V d^4x \sqrt{-g}\, f(R) + S_M(g_{\mu\nu}, \Psi) \quad (17)$$

$$f(R) = R + \epsilon\varphi(R) \quad (29)$$

$$|\epsilon\, \varphi(R)| \ll |R|$$

$$\begin{aligned} f(R) = & -\alpha\kappa\rho_\Lambda + 2R + \frac{\alpha\rho_\Lambda R\left(18\left(2\gamma^2-1\right)\kappa\rho_c^I + R\right)}{72\gamma^2\kappa(\rho_c^I)^2} \\ & + \frac{\alpha\rho_\Lambda}{20\rho_c^I}\Bigg\{ -3\left[R - 2\left(\sqrt{10}-2\right)\kappa\rho_c^I\right] {}_2F_1\left[-\frac{1}{2}, \frac{1}{6}\left(-\sqrt{10}-4\right); \frac{1}{6}\left(2-\sqrt{10}\right); -\frac{R}{2\kappa\rho_c^I}\right] \\ & -3\left(2\left(\sqrt{10}+2\right)\kappa\rho_c^I + R\right) {}_2F_1\left[-\frac{1}{2}, \frac{1}{6}\left(\sqrt{10}-4\right); \frac{1}{6}\left(\sqrt{10}+2\right); -\frac{R}{2\kappa\rho_c^I}\right] \\ & +(2\kappa\rho_c^I + R)\Bigg(-\left(\sqrt{10}-1\right) {}_2F_1\left[\frac{1}{2}, \frac{1}{6}\left(-\sqrt{10}-4\right); \frac{1}{6}\left(2-\sqrt{10}\right); -\frac{R}{2\kappa\rho_c^I}\right] \\ & + \left(\sqrt{10}+1\right) {}_2F_1\left[\frac{1}{2}, \frac{1}{6}\left(\sqrt{10}-4\right); \frac{1}{6}\left(\sqrt{10}+2\right); -\frac{R}{2\kappa\rho_c^I}\right]\Bigg)\Bigg\} \quad (33) \end{aligned}$$

$2F_1[a, b; c; z]$ - *Hypergeometric function*

Effective actions for mLQC-II

$$S = \frac{1}{2\kappa}\int_V d^4x \sqrt{-g}\, f(R) + S_M(g_{\mu\nu}, \Psi) \quad (17)$$

$$f(R) = R + \epsilon\varphi(R) \quad (29)$$

$$|\epsilon\, \varphi(R)| \ll |R|$$

$$\begin{aligned} f(R) = R + \frac{1}{90\gamma^4}\Bigg\{ & 270\left(\gamma^4+\gamma^2\right)R + \frac{20\left(\gamma^6+\gamma^4\right)R^2}{\kappa\rho_c^{II}} + 90\kappa\rho_c^{II} + 9\Bigg[3\Bigg(\left(\sqrt{10}-2\right)\kappa\rho_c^{II} \\ & +2\left(\gamma^4+\gamma^2\right)R\Bigg)\, {}_2F_1\left[-\frac{1}{2},\frac{1}{6}\left(-\sqrt{10}-4\right);\frac{1}{6}\left(2-\sqrt{10}\right);\frac{2R\gamma^2\left(\gamma^2+1\right)}{\kappa\rho_c^{II}}\right] \\ & +\left(6\left(\gamma^4+\gamma^2\right)R - 3\left(\sqrt{10}+2\right)\kappa\rho_c^{II}\right)\, {}_2F_1\left[-\frac{1}{2},\frac{1}{6}\left(\sqrt{10}-4\right);\frac{1}{6}\left(\sqrt{10}+2\right);\frac{2R\gamma^2\left(\gamma^2+1\right)}{\kappa\rho_c^{II}}\right] \\ & +\left(2\left(\gamma^4+\gamma^2\right)R - \kappa\rho_c^{II}\right)\Bigg(\left(\sqrt{10}-1\right)\, {}_2F_1\left[\frac{1}{2},\frac{1}{6}\left(-\sqrt{10}-4\right);\frac{1}{6}\left(2-\sqrt{10}\right);\frac{2R\gamma^2\left(\gamma^2+1\right)}{\kappa\rho_c^{II}}\right] \\ & -\left(\sqrt{10}+1\right)\, {}_2F_1\left[\frac{1}{2},\frac{1}{6}\left(\sqrt{10}-4\right);\frac{1}{6}\left(\sqrt{10}+2\right);\frac{2R\gamma^2\left(\gamma^2+1\right)}{\kappa\rho_c^{II}}\right]\Bigg)\Bigg]\Bigg\} \end{aligned} \quad (34)$$

$2F_1[a,b;c;z]$ - *Hypergeometric function*

Effective actions for mLQCs

Taylor expansion of the solutions, using *Wolfram Mathematica* 12.2

- mLQC-I (b_- branch)

$$f(R) = R + \frac{(4+3\gamma^2)}{72(1+\gamma^2)\kappa\rho_c^I}R^2 + \frac{\gamma^2}{992(1+\gamma^2)(\kappa\rho_c^I)^2}R^3 + \cdots \quad (35)$$

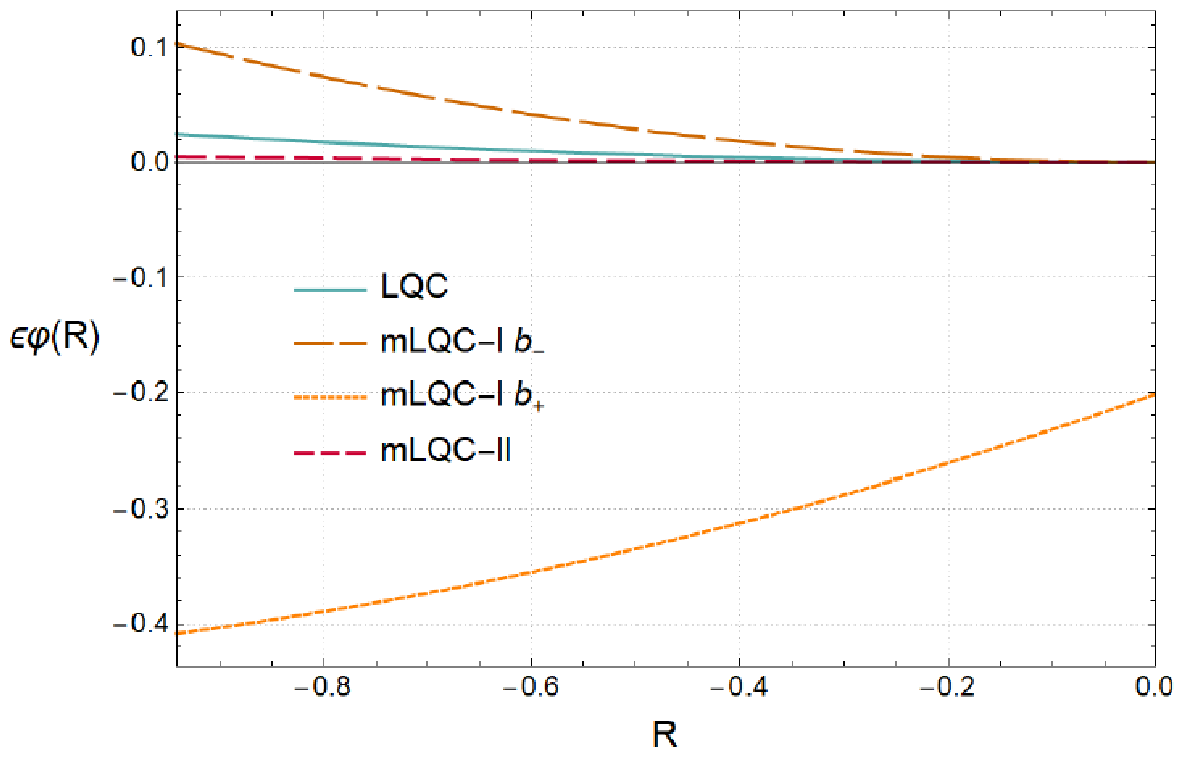
- mLQC-I (b_+ branch)

$$f(R) = -2\alpha\kappa\rho_\Lambda + \left(\frac{\alpha(5\gamma^2-1)\rho_\Lambda}{4\gamma^2\rho_c^I} + 2\right)R + \frac{\alpha(4-3\gamma^2)\rho_\Lambda}{228\gamma^2\kappa(\rho_c^I)^2}R^2 + \frac{\alpha\rho_\Lambda}{3968(1+\gamma^2)(\kappa\rho_c^I)^2}R^3 + \cdots \quad (36)$$

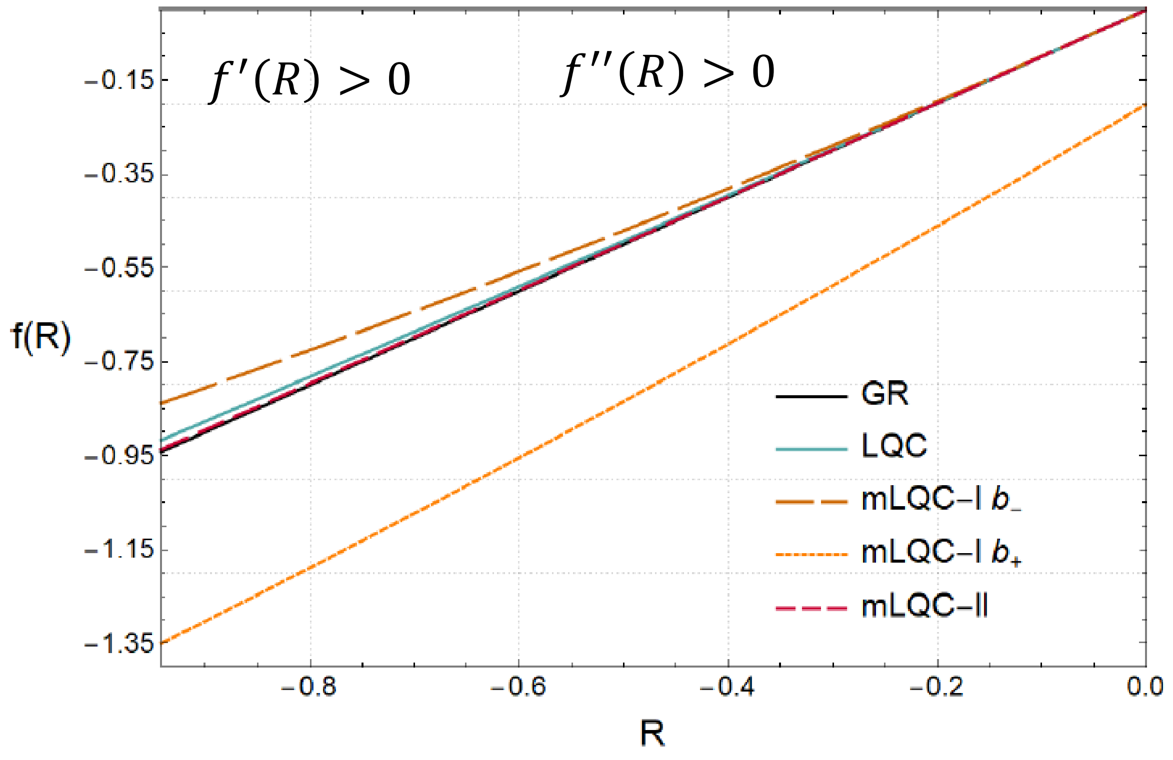
- mLQC-II

$$f(R) = R + \frac{(-3\gamma^4-2\gamma^2+1)}{18\kappa\rho_c^I}R^2 - \frac{\gamma^2(\gamma^2+1)}{62(\kappa\rho_c^{II})^2}R^3 + \cdots \quad (37)$$

Effective actions for mLQCs



(a) Deviation, from GR, of the effective Lagrangian: a comparison between all models.



(b) Effective $f(R)$ function: a comparison between all models.

Conclusions

- $f'(R) > 0$ → allows for a positive effective gravitational coupling constant:

In metric $f(R)$ *gravity, we can write* $G_{\mu\nu} = G_{eff}\left(T_{\mu\nu} + T_{\mu\nu}^{(eff)}\right)$, *where* $G_{eff} \equiv G/f'(R)$.

- $f''(R) > 0$ → avoids the Dolgov-Kawasaki instability.

Conclusions

- Gauss-Bonnet theorem

$$\begin{cases} f(R) = R + cR^2 \\ f\left(R, R^{\mu\nu}R_{\mu\nu}\right) = R + aR^2 + bR^{\mu\nu}R_{\mu\nu} \end{cases}, \qquad a + \frac{b}{3} = c$$

This family of Lagrangians leads to the same field equations:

- **mLQC-I (b_- branch)**

$$f(R) = R + \frac{(4+3\gamma^2)}{72(1+\gamma^2)\kappa\rho_c^I}R^2 + \cdots, \qquad a + \frac{b}{3} = \frac{(4+3\gamma^2)}{72(1+\gamma^2)\kappa\rho_c^I}$$

- **mLQC-II**

$$f(R) = R + \frac{(-3\gamma^4 - 2\gamma^2 + 1)}{18\kappa\rho_c^I}R^2 + \cdots, \qquad a + \frac{b}{3} = \frac{(-3\gamma^4 - 2\gamma^2 + 1)}{18\kappa\rho_c^{II}}$$

Conclusions

- Our approach is only valid when

$$|\epsilon\, \varphi(R)| \ll |R| \Rightarrow |R| \ll \rho_c \sim l_p^{-2}$$

Therefore, it breaks down in regions of very high energy densities, where $|R| \sim l_p^{-2}$.

Our claim is that at stages close enough to the bounce, the Lagrangians found in this work may effectively describe correctly the collapsing universe.

Thank you for your attention.

Any questions?

References [1/3]

1. G. Amelino-Camelia, Living Rev. Rel. 16, 5 (2013) [arXiv:0806.0339 [gr-qc]].
2. S. Carlip, Rept. Prog. Phys. 64, 885 (2001) [arXiv:gr-qc/0108040 [gr-qc]].
3. C. Rovelli, Living Rev. Rel. 1, 1 (1998) [arXiv:gr-qc/9710008 [gr-qc]].
4. T. Thiemann, [arXiv:gr-qc/0110034 [gr-qc]].
5. A. Ashtekar and J. Lewandowski, Class. Quant. Grav. 21, R53 (2004) [arXiv:gr-qc/0404018 [gr-qc]].
6. M. Bojowald, Living Rev. Rel. 8, 11 (2005) [arXiv:gr-qc/0601085 [gr-qc]].
7. M. Bojowald, Phys. Rev. Lett. 86, 5227-5230 (2001) [arXiv:gr-qc/0102069 [gr-qc]].
8. A. Ashtekar, T. Pawlowski and P. Singh, Phys. Rev. D 73, 124038 (2006) [arXiv:gr-qc/0604013 [gr-qc]].
9. A. Ashtekar, T. Pawlowski and P. Singh, Phys. Rev. D 74, 084003 (2006) [arXiv:gr-qc/0607039 [gr-qc]].
10. V. Taveras, Phys. Rev. D 78, 064072 (2008) [arXiv:0807.3325 [gr-qc]].
11. K. Banerjee, G. Calcagni and M. Martin-Benito, SIGMA 8, 016 (2012) [arXiv:1109.6801 [gr-qc]].
12. J. De Haro and J. Amoros, Phys. Rev. D 97, no.6, 064014 (2018) [arXiv:1712.08399 [gr-qc]].
13. P. Singh, Phys. Rev. D 73, 063508 (2006) [arXiv:gr-qc/0603043 [gr-qc]].
14. K. A. Meissner, Class. Quant. Grav. 21, 5245-5252 (2004) [arXiv:gr-qc/0407052 [gr-qc]].
15. J. Yang, Y. Ding and Y. Ma, Phys. Lett. B 682, 1-7 (2009) [arXiv:0904.4379 [gr-qc]].

References [2/3]

16. A. Dapor and K. Liegener, Phys. Lett. B 785, 506-510 (2018) [arXiv:1706.09833 [gr-qc]].
17. A. Dapor and K. Liegener, Class. Quant. Grav. 35, no.13, 135011 (2018) [arXiv:1710.04015 [gr-qc]].
18. E. Alesci, A. Barrau, G. Botta, K. Martineau and G. Stagno, Phys. Rev. D 98, no.10, 106022 (2018) [arXiv:1808.10225 [gr-qc]].
19. J. Bilski and A. Marciano, Phys. Rev. D 101, no.6, 066026 (2020) [arXiv:1905.00001 [gr-qc]].
20. B. F. Li, P. Singh and A. Wang, Phys. Rev. D 97, no.8, 084029 (2018) [arXiv:1801.07313 [gr-qc]].
21. B. F. Li, P. Singh and A. Wang, Phys. Rev. D 100, no.6, 063513 (2019) [arXiv:1906.01001 [gr-qc]].
22. B. F. Li, P. Singh and A. Wang, Phys. Rev. D 98, no.6, 066016 (2018) [arXiv:1807.05236 [gr-qc]].
23. G. J. Olmo and P. Singh, JCAP 01, 030 (2009) [arXiv:0806.2783 [gr-qc]].
24. G. J. Olmo, Int. J. Mod. Phys. D 20, 413-462 (2011) [arXiv:1101.3864 [gr-qc]].
25. T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. 82, 451-497 (2010) [arXiv:0805.1726 [gr-qc]].
26. F. S. N. Lobo, [arXiv:0807.1640 [gr-qc]].
27. A. De Felice and S. Tsujikawa, Living Rev. Rel. 13, 3 (2010) [arXiv:1002.4928 [gr-qc]].
28. S. Nojiri and S. D. Odintsov, Phys. Rept. 505, 59-144 (2011) [arXiv:1011.0544 [gr-qc]].
29. T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. 513, 1-189 (2012) [arXiv:1106.2476 [astro-ph.CO]].
30. S. Capozziello and M. De Laurentis, Phys. Rept. 509, 167-321 (2011) [arXiv:1108.6266 [gr-qc]].

References [3/3]

31. T. Harko and F. S. N. Lobo, Cambridge Monographs on Mathematical Physics, Cambridge, Cambridge University Press (2018).
32. L. Bel and H. S. Zia, Phys. Rev. D 32, 3128-3135 (1985)
33. J. Z. Simon, Phys. Rev. D 41, 3720 (1990)
34. J. Z. Simon, Phys. Rev. D 45, 1953-1960 (1992)
35. T. P. Sotiriou, Phys. Rev. D 79, 044035 (2009) [arXiv:0811.1799 [gr-qc]].
36. I. Terrucha, D. Vernieri and J. P. S. Lemos, Annals Phys. 404, 39-46 (2019) [arXiv:1904.00260 [gr-qc]].
37. B. J. Barros, E. M. Teixeira and D. Vernieri, Annals Phys. 419, 168231 (2020) [arXiv:1907.11732 [gr-qc]].
38. F. Bajardi, D. Vernieri and S. Capozziello, Eur. Phys. J. Plus 135, no.11, 912 (2020) [arXiv:2011.01248 [gr-qc]].
39. J. Amoros, J. de Haro and S. D. Odintsov, Phys. Rev. D 89, no.10, 104010 (2014) [arXiv:1402.3071 [gr-qc]].
40. G. J. Olmo and D. Rubiera-Garcia, Phys. Lett. B 740, 73-79 (2015) [arXiv:1405.7184 [hep-th]].