Renormalized ρ_{vac} without m^4 terms[1]

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[1] Based on *Running vacuum in quantum field theory in curved space-time: renormalizing ρ_{vac} without $\sim m^4$ terms*, Cristian Moreno-Pulido and Joan Solà Peracaula (Eur.Phys.J.C 80 (2020) 8, 692)

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The cosmological constant problems

- Observations tell us that our universe is expanding in an accelerated way due to "Dark Energy". It is usually related with the cosmological constant Λ and the vacuum energy density of quantum fields.[2]
- Observations vs predictions:

$$\rho_\Lambda^{obs} \equiv \frac{\Lambda}{8\pi G_N} \sim 10^{-47}\ \mathrm{GeV}^4, \quad \rho_{ZPE} \sim m_e^4 \approx 10^{-13}\ \mathrm{GeV}^4$$

- Coincidence problem:

$$\rho_\Lambda^{obs} \sim \rho_{CDM}^{obs},$$

But ρ_{CDM} decreases with time and ρ_Λ is assumed to be constant .

[2] We work in a flat FLRW space-time with $ds^2 = a(\tau)^2 \eta^{\mu\nu} x^\mu x^\nu$, $()' \equiv d/d\tau$ and $\mathcal{H} = a\prime/a = aH$

Dynamical vacuum energy

- The Cosmological principle still permits a function $\rho_\Lambda(t)$ or even $\rho_\Lambda(\chi(t))$, where $\chi(t)$ is a dynamical variable.[3]
- It may provide a better phenomenological account of cosmological observations and tensions. [4]
- For instance the RVM, from renormalization group arguments:

$$\rho_\Lambda(H) = \frac{3}{8\pi G_N}(C_0 + \nu H^2). \tag{1}$$

Some analyses[5] indicate that $\nu \sim 10^{-3} \ll 1$.

- More possibilities: bigger powers of H and its derivatives...

[3] some possibilities in: E. Di Valentino et al *Astropart. Phys.* **131** (2021)

[4] J. Solà et al. *Class. Quant. Grav.* **37** (2020); *EPL* **134** (2021)

[5] J. Solà, A. Gómez-Valent, and J. de Cruz Pérez, *Astrophys. J.*, **836** *(2017)*

Non-minimal coupling and Quantum Fluctuations

- In general, ρ_Λ in the effective action of QFT in curved space-time is a coupling depending on a scale M associated to the cosmological evolution[6] $(M = H, R, ...)$.
- The gravitational field equations read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + 8\pi G_N \rho_\Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{matter}, \quad (2)$$

- For simplicity, consider only one real scalar field, ϕ, contribution to $T_{\mu\nu}^{matter}$.

$$S[\phi] = -\int d^4x\sqrt{-g}\left(\frac{1}{2}g_{\mu\nu}\partial_\nu\phi\partial_\mu\phi + \frac{1}{2}(m^2 + \xi R)\phi^2\right) \quad (3)$$

where ξ is the non-minimal coupling parameter.

[6] Ilya L. Shapiro and Joan Solà, *JHEP 0202 (2002) 006*; J. Solà, *J.Phys. A41 (2008) 164066*

Non-minimal coupling and Quantum Fluctuations

- We split the field in a classical and quantum parts,

$$\phi(\tau, \mathbf{x}) = \phi_b(\tau) + \delta\phi(\tau, \mathbf{x}), \tag{4}$$

where

$$\delta\phi(\tau, \mathbf{x}) = \frac{1}{(2\pi)^{3/2} a} \int d^3k \left[A_{\mathbf{k}} e^{i\mathbf{kx}} h_k(\tau) + A_{\mathbf{k}}^{\dagger} e^{-i\mathbf{kx}} h_k^*(\tau) \right]. \tag{5}$$

- Given the former field decomposition, the EMT decomposes itself as $\langle T_{\mu\nu}^{\phi} \rangle = \langle T_{\mu\nu}^{\phi_b} \rangle + \langle T_{\mu\nu}^{\delta_\phi} \rangle$. The total vacuum contribution to the EMT can be postulated to read

$$\langle T_{\mu\nu}^{vac} \rangle \equiv -\rho_\Lambda g_{\mu\nu} + \langle T_{\mu\nu}^{\delta\phi} \rangle. \tag{6}$$

where $\rho_{vac} \sim \langle T_{00}^{vac} \rangle$.

Adiabatic Regularization

The usual commutation relations lead us to the Klein-Gordon equation for the Fourier modes

$$(\Box - m^2 - \xi R)\delta\phi(\tau, \mathbf{x}) = 0 \qquad \Longrightarrow \qquad h_k'' + \Omega_k^2 h_k = 0. \tag{7}$$

Here $\Omega_k^2 \equiv k^2 + a^2 m^2 + a^2(\xi - 1/6)R$. An ansatz is used to solve it:

$$h_k(\tau) \sim \frac{e^{i\int^\tau W_k(\tau_1)d\tau_1}}{\sqrt{W_k(\tau)}}, \tag{8}$$

with W_k being the solution of

$$W_k^2 = \Omega_k^2 - \frac{1}{2}\frac{W_k''}{W_k} + \frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2. \tag{9}$$

Adiabatic Regularization

For an adiabatic (slowly varying) Ω_k, we may use the former differential equation as a recurrence relation. In our scenario, the WKB solution is organized through adiabatic orders:

- Of order 0: k^2, m^2 and $a(\tau)$.
- Of order 1: a' and $\mathcal{H}$.
- Of order 2: a'', $(a')^2$, $\mathcal{H}'$,$\mathcal{H}^2$...
- Each aditional derivative increases the adiabatic order by one unit.
- The solution, then, is organized as follows

$$W_k = W_k^{(0)} + W_k^{(2)} + W_k^{(4)} + \cdots \tag{10}$$

the non-appearance of the odd adiabatic orders is justified by means of general covariance.

Adiabatic Regularization

- First term $W_k^{(0)} \equiv \omega_k = \sqrt{k^2 + a^2(\tau)M^2}$, M an arbitrary mass scale introduced for renormalitzation purposes[7]. Also $\Delta^2 \equiv m^2 - M^2$, of adiabatic order 2.
- Subsequent orders are obtained by iteration:

$$W_k = \underbrace{\boxed{\omega_k}}_{W_k^{(0)}} + \underbrace{\boxed{\frac{a^2\Delta^2}{2\omega_k} + \frac{a^2}{2\omega_k^2}\left(\xi - 1/6\right) R - \frac{1}{4}\frac{\omega_k''}{\omega_k^2} + \frac{3}{8}\frac{(\omega_k')^2}{\omega_k^3}}}_{W_k^{(2)}} + \ldots \tag{11}$$

- Using this procedure we can compute the expected value of $T_{00}^{\delta\phi}$ up to 4th adiabatic order:

[7] A. Ferreiro and J. Navarro-Salas, *Phys. Lett. B 792 (2019) 81*

Adiabatic Regularization

$$
\begin{aligned}
\left\langle T_{00}^{\delta\phi}\right\rangle = \frac{1}{8\pi^2 a^2}\int dk k^2 \Bigg[& 2\omega_k + \frac{a^4M^4\mathcal{H}^2}{4\omega_k^5} - \frac{a^4M^4}{16\omega_k^7}(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 + 8\mathcal{H}'\mathcal{H}^2 + 4\mathcal{H}^4) \\
& + \frac{7a^6M^6}{8\omega_k^9}(\mathcal{H}'\mathcal{H}^2 + 2\mathcal{H}^4) - \frac{105a^8M^8\mathcal{H}^4}{64\omega_k^{11}} \\
& + \left(\xi - \frac{1}{6}\right)\left(-\frac{6\mathcal{H}^2}{\omega_k} - \frac{6a^2M^2\mathcal{H}^2}{\omega_k^3} + \frac{a^2M^2}{2\omega_k^5}(6\mathcal{H}''\mathcal{H} - 3\mathcal{H}'^2 + 12\mathcal{H}'\mathcal{H}^2) \right. \\
& \left. - \frac{a^4M^4}{8\omega_k^7}(120\mathcal{H}'\mathcal{H}^2 + 210\mathcal{H}^4) + \frac{105a^6M^6\mathcal{H}^4}{4\omega_k^9} \right) \\
& + \left(\xi - \frac{1}{6}\right)^2 \left(-\frac{1}{4\omega_k^3}(72\mathcal{H}''\mathcal{H} - 36\mathcal{H}'^2 - 108\mathcal{H}^4) + \frac{54a^2M^2}{\omega_k^5}(\mathcal{H}'\mathcal{H}^2 + \mathcal{H}^4) \right) \Bigg] \\
& + \frac{1}{8\pi^2 a^2}\int dk k^2 \left[\frac{a^2\Delta^2}{\omega_k} - \frac{a^4\Delta^4}{4\omega_k^3} + \frac{a^4\mathcal{H}^2M^2\Delta^2}{2\omega_k^5} - \frac{5}{8}\frac{a^6\mathcal{H}^2M^4\Delta^2}{\omega_k^7} \right. \\
& \left. + \left(\xi - \frac{1}{6}\right)\left(-\frac{3a^2\Delta^2\mathcal{H}^2}{\omega_k^3} + \frac{9a^4M^2\Delta^2\mathcal{H}^2}{\omega_k^5} \right) \right] + \dots,
\end{aligned} \tag{12}
$$

Renormalization

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- The Cosmological constant problems
- Dynamical vacuum energy
- Non-minimal coupling and Quantum Fluctuations
- Renormalized Vacuum Energy and RVM connection
- Conclusions & remaining work

- Adiabatic regularization let us to localize the divergences explicitly of $\left\langle T_{00}^{\delta\phi} \right\rangle$. We can split it as

$$\left\langle T_{00}^{\delta\phi} \right\rangle (M) = \left\langle T_{00}^{\delta\phi} \right\rangle_{Div} (M) + \left\langle T_{00}^{\delta\phi} \right\rangle_{Non-Div} (M), \quad (13)$$

- We compute terms up to 4th order because the divergences are only present up to this adiabatic order.[8] Thus, we define the renormalized EMT in curved space-time at the scale M as follows:

$$\langle T_{00}^{\delta\phi} \rangle_{Ren}(M) \equiv \langle T_{00}^{\delta\phi} \rangle(m) - \langle T_{00}^{\delta\phi} \rangle^{(0-4)}(M), \qquad (14)$$

where superscript $(0-4)$ means up to adiabatic order 4.

[8] Bunch, T. S. Journal of Physics A: *Mathematical and General, 13(4), 12971310*; *Quantum Fields in Curved Space*, Birrell and Davies

Renormalization

- Following this prescription, the resulting renormalized value is

$$
\begin{aligned}
\langle T_{00}^{\delta\phi}\rangle_{Ren}(M) = & \frac{a^2}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4\log\frac{m^2}{M^2}\right) \\
& - \left(\xi - \frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2}\left(m^2 - M^2 - m^2\log\frac{m^2}{M^2}\right) \\
& + \left(\xi - \frac{1}{6}\right)^2 \frac{9\left(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 - 3\mathcal{H}^4\right)}{16\pi^2a^2}\log\frac{m^2}{M^2} + \dots
\end{aligned}
\tag{15}
$$

- The $\sim m^4, M^4$ terms are present as expected.
- For $M = m$ all the terms up to 4th orders vanish.

Renormalized Vacuum Energy

- The renormalization program needs to count the higher derivative terms in the classical effective action:

$$\frac{1}{8\pi G_N(M)} G_{\mu\nu} + \rho_\Lambda(M) g_{\mu\nu} + a_1(M) H^{(1)}_{\mu\nu} = T^{\phi_b}_{\mu\nu} + \left\langle T^{\delta\phi}_{\mu\nu} \right\rangle_{Ren} (M). \quad (16)$$

- As usual, we want to compare at different scales, M and M_0,

$$\begin{aligned} &\left\langle T^{\delta\phi}_{\mu\nu} \right\rangle_{Ren} (M) - \left\langle T^{\delta\phi}_{\mu\nu} \right\rangle_{Ren} (M_0) \\ &= f_{G_N^{-1}}(m, M, M_0) G_{\mu\nu} + f_{\rho_\Lambda}(m, M, M_0) g_{\mu\nu} + f_{a_1}(m, M, M_0) H^{(1)}_{\mu\nu}, \end{aligned} \quad (17)$$

where $f_X(m, M, M_0) \equiv X(M) - X(M_0)$ for each coupling.

Renormalized Vacuum Energy

Setting $\mu = \nu = 0$,

$$f_{G^{-1}}(m, M, M_0) = \left(\xi - \frac{1}{6}\right) \frac{1}{16\pi^2} \left[M^2 - M_0^2 - m^2 \log \frac{M^2}{M_0^2}\right], \tag{18}$$

$$f_{\rho_\Lambda}(m, M, M_0) = \frac{1}{128\pi^2} \left(M^4 - M_0^4 - 4m^2(M^2 - M_0^2) + 2m^4 \log \frac{M^2}{M_0^2}\right), \tag{19}$$

$$f_{a_1}(m, M, M_0) = \frac{1}{32\pi^2} \left(\xi - \frac{1}{6}\right)^2 \log \frac{M^2}{M_0^2}. \tag{20}$$

Renormalized Vacuum Energy

Our definition of vacuum energy density was

$$\rho_{vac}(M) = \rho_\Lambda(M) + \frac{\left\langle T_{00}^{\delta\phi} \right\rangle_{Ren}(M)}{a^2}, \tag{21}$$

from it, comparing two different scales we can deduce the following formula (in terms of the cosmic time)

$$\begin{aligned}\rho_{vac}(M) &= \rho_{vac}(M_0) + \frac{3}{16\pi^2}\left(\xi - \frac{1}{6}\right) H^2 \left[M^2 - M_0^2 - m^2 \ln\frac{M^2}{M_0^2}\right] \\ &- \frac{9}{16\pi^2}\left(\xi - \frac{1}{6}\right)^2 (\dot{H}^2 - 2H\ddot{H} - 6H^2\dot{H}) \ln\frac{M^2}{M_0^2}.\end{aligned} \tag{22}$$

This result relates the renormalized result at two different energy scales, M and M_0.

RVM connection

- For instance let's fix $M_0 = M_X$, where $M_X \sim 10^{16} GeV$ is a GUT scale and is also associated to the inflationary scale and $M = H_0$, at today's value of Hubble parameter:
- Neglecting $\mathcal{O}(H_0^4)$ (late universe) terms we are left with

$$\rho_{vac}(M_X) = \rho_{vac}(H_0) - \frac{3\nu_{eff}^0}{8\pi} H_0^2 M_P^2, \tag{23}$$

where $\nu_{eff}^0 \equiv \frac{1}{2\pi}\left(\frac{1}{6} - \xi\right)\frac{M_X^2}{M_P^2}\left(1 + \frac{m^2}{M_X^2}\ln\frac{H_0^2}{M_X^2}\right)$.

- The sign of ν_{eff}^0 not only depend on ξ, should be determined by data.
- For a generical value of the Hubble parameter we may write

$$\rho_{vac}(H) \approx \rho_{vac}(H_0) + \frac{3\nu_{eff}^0 M_P^2}{8\pi}(H^2 - H_0^2), \tag{24}$$

RVM connection

More geometric structures for vacuum in curved spacetime:

- We can postulate more generical expression for $T_{\mu\nu}^{vac}$,

$$T_{\mu\nu}^{vac} = -\rho_{vac} g_{\mu\nu} - \alpha_1 R g_{\mu\nu} - \alpha_2 R_{\mu\nu} + \cdots \tag{25}$$

- That lead us to

$$\rho_{vac}(H) \approx \rho_{vac}^0 + \frac{3\nu_{eff,1}}{8\pi G_N}(H^2 - H_0^2) + \frac{3\nu_{eff,2}}{8\pi G_N}(\dot{H} - \dot{H}_0), \tag{26}$$

where $\nu_{eff,1}$ and $\nu_{eff,2}$ are proportional to $\frac{M_X^2}{M_P^2}$, as before.

Conclusion & remaining work

- The renormalized value of $\rho_{vac}(M)$ has been obtained from this QFT calculation instead of a generic renormalization group argument.
- We avoid the $\sim m^4$ problem by considering the difference between two different scales.
- A richer phenomenology can be obtained from different relations between $\langle T_{00}^{vac} \rangle$ and ρ_{vac}, for instance including an additional term proportional to $\dot{H}$.
- Can we generalize these results to other fields (fermions)?

Thank you for your attention!

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