### **Explaining Time's Passage**

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#### Abstract

The way we experience time is in the accumulation of experiences and events that happen in the moment, and then are behind us. Since the time of Anaximander at least; philosophers have tried to explain both the nature of time and its origin or basis. In modern times; scientists are the ones exploring the domain of time, so now *they* attempt to explain the nature and basis of time – with varying degrees of success. This is complicated because explanations from Classical Physics or Relativity are different from, and incompatible with, answers from Quantum Mechanics, so we hope Quantum Gravity theories will help resolve this. Recent advances in Mathematics hold promise for a unified basis explaining both the thermodynamic and quantummechanical time arrows in a way that consistently informs our Philosophy. However; we may need to explore beyond the island of familiar Maths, to reconcile the divergent pictures of how and why time passes.

Keywords: origin of time, passage of time, basis of time, Philosophy of time, Physics of time

#### Introduction

While we imagine time is a single entity; the reality of time is more complex and more interesting. As far back as Anaximander; philosophers have tried to make sense of that complexity, and his analogy of stepping into a moving stream [1] actually comes very close to the reality we observe, since things keep moving while events are happening. So the simplest realities of life are governed by a rule that assures most decisions will be complex instead of simple. Coupling Plato's observation "time is the image of eternity" [2] with the insight that space is a projection of infinity can help us make sense of his deeper message. But then; we see glimpses into quantum gravity from the ancient Greek philosophers. Rovelli's book "Reality is Not What it Seems" wonderfully recounts insights from those individuals [3] that extend to topics including quantum gravity and the nature of time. While I agree with his view that we find the seeds of quantum gravity and the birth of atomism among ancient philosophers; we differ somewhat on whether time is fundamental or primordial on that basis. The resolution involves both cosmology and quantum gravity. So this paper explores how recent developments in Mathematics might have a deeper relevance than was previously appreciated to advance our understanding of time. I will discuss the examples cited above in greater detail and offer unique insights from my own research [4].

Already in 1922, concluding a debate with Bergson; Einstein said "There is no such thing as the time of the philosopher." [5] So for some it is a dead issue. But the debate goes on in the Physics community and among modern philosophers, over the consequences of how physical theory affects our understanding of time. This debate centers on the sharply differing views of time in Relativity and Quantum Mechanics. And there is strong evidence both views must be true. So if the traditional outlook is that time is a single entity for all; that view is broken. But this emphasizes the need to create new ways to conceptualize what we have learned in Physics and incorporate it into our Philosophy [6]. And yet; Anaximander's ancient view of time as a stream which continues to flow even as you are stepping in it remains enlightening in modern times. So there is hope for reconciling Physics with Philosophy, and knowledge that by developing quantum gravity theories; we gain a better philosophical understanding of time. I think the reverse is also true; it is easier to choose or develop better approaches to quantum gravity, by understanding the Philosophy of time better. Prevailing QG theories share many features and predictions with considerable common ground to explore [7]. So I examine ideas of Connes [8], Longo [9], Gisin [10], as well as Drossel and Ellis [11], on the emergence of time from quantum gravity, with some unique insights on wavefunction collapse from Dieter Zeh [12], and a discussion of the possibility of global asymmetry as it applies to whether time is primordial or fundamental [13].

#### What is there to Explain?

If we had a better understanding of the extreme micro- and macro-scale in Physics; time would not be an enigma at all. But from my view; people are unaware they live in the 'Goldilocks zone' on an island of comfortable norms. The realm of familiar Maths is perfectly sufficient for exploring anything anywhere on the island; so who could want more? Theoretical Physics seeks the origins of events of our current surface-layer reality in deeper layers or causal elements further back in time. But if we cut ourselves off from the possibility those origins are higher-dimensional; we omit some probable realities that solve standing problems in Physics. This is because evolutive properties arise unavoidably when the underlying or governing algebra is non-commutative or non-associative. Time's passage may thus be caught up in how higher-order spaces are reduced in dimension through cosmological transitions to obtain a cosmos with familiar properties, such as the one we inhabit. Evolution arises automatically if the dimensions are high enough, as any system's degrees of freedom approach infinity; so it becomes almost certain that a space or its underlying algebra will evolve, or is self-evolving, once certain initial conditions are met. This involves a unit of minimum uncertainty or action, and at least a modicum of energy; but physicists observe that these conditions are handily met at the Planck scale. One might question creating something from nothing, but we are uncertain absolute nothing ever existed. So there is little argument, about reasons for time to move inexorably forward, if we assume the universe started from variations or variability and moved toward conditions.

And yet; people ask "what can cause numbers to change on their own or physical parameters change of their own accord?" It seems more reasonable to accept that virtual particles can pop in and out of existence in pairs, rather than that objects and spaces are evolving processes. But to explain time; we may need to accept that everything evolves on its own, unless it is otherwise constrained. To understand why time passes; I suggest we start with the assumption that objects and spaces are not necessarily static or fixed, and could be inherently time-evolving. This is precisely what Alain Connes proposed, when in 2000 he emphatically wrote "Noncommutative measure spaces evolve with time!" [14] This result is based largely on the findings of Tomita [15] and Takesaki [16] studying modular Hilbert algebras, but Connes has generalized that work to create a new kind of differential geometry. This methodology is more nearly universal than people are aware of, however. Even if some of his ideas are not verified by current evidence; his line of reasoning is not falsified thereby, because intrinsic time evolution undeniably arises in higher-d Maths. This idea or the broader context it suggests provides a framework for a more realistic yet more flexible view of time. In this view time is, at its root, inherently or automatically progressing; originating from the dynamism of intrinsic evolution of forms and spaces in higher dimensions because the non-commutativity and/or non-associativity of their underlying algebra induces algebraic and geometric directionality and/or sequentiality.

This approach offers a unified basis for the quantum-mechanical and thermodynamic arrows of time that resolves some paradoxes or quandaries. If we see time in the tendency for objects and spaces to evolve automatically, once the degrees of freedom are high enough; we have a way to unite the Math and the Physics of time that resolves the paradox of differing time arrows from various models. This is perhaps a more major advancement than has been appreciated to this point. It restores seeing time as an accumulation of experiences or events that occur and are then behind us in a view of Quantum Mechanics where Heisenberg's uncertainty or 'unsharp measure' is due to spacetime being non-commuting. All interactions or observations accumulate because quantum-mechanical reality is like Anaximander's stream. But if we assume the evolution of the early cosmos was higher-dimensional too; we can extend intrinsic time to canonical time evolution in the initial origin at the Planck time through the inflationary period at least until baryogenesis, and likely continuing to play a part until decoupling or recombination occurred, or at a 5-d  $\rightarrow$  4-d boundary [17] that is seen as a black hole  $\rightarrow$  white hole transition [18] in some theories. Time evolves from higher-d spaces through cosmological transitions, in these models. The Aikyon theory of Singh [19] employs Connes' intrinsic time and the octonions to explain both cosmic origins and particle physics without such transitions, but affirms other aspects of my present work. So we now examine a proposed basis for these varied ideas.

#### **Explaining What We Observe**

Great successes in Physics assume what we observe is built and can be explained from the bottom up, from the smallest components or the simplest applicable formulas, but there is another way to explain what we see, where nature employs top-down methods taking advantage of higher-order and higher-dimensional mathematical forms, as well. If we allow a broader spectrum of possibilities at the cosmos' origin, by assuming that both bottom-up and top-down strategies are at work at the same time; a picture emerges where dimensionality has an upper and lower limit at the outset of geometrogenesis, which later converges toward a single value [20]. This lets emergent forms be shaped by higher-d figures like the Monster Group and E<sub>8</sub>, during the earliest phases of cosmological evolution, even while they are being built from lower-d components. From this radically top-down outlook, even when coupled with causal structure theories of quantum gravity; the conventional view using bottom-up reasoning is inside-out, and we need a view where we see the universe from the outside-in, to fully understand it. The disparity is partly due to the way most people have learned Maths, where the simplest elements are taught first and advanced ideas are derived from the simplicities. However; we know there are patterns that emerge in complex structures from which simple rules can be derived.

This dichotomy is called additive vs. formant synthesis. In the one case; you add bits of clay to make a sculpture or individual harmonic tones to create the desired waveform, while in the other you chip bits away from the outside or remove some overtones from a more complex waveform, to obtain the desired shape. Here I propose that nature has utilized and always uses both methods to shape the cosmos. It builds larger and more complex forms using smaller pieces, but it also pares physical reality down from a palette of possible forms, and borrows structure from more complex forms to create simpler ones. In this way; nature can employ the full range of Mathematics and exploit the true organizing power of what resides in higher dimensions. But the outside-in view is difficult to obtain on the island of common Maths where mathematical structures are built from the bottom-up, rather than inherited from orderly patterns in higher-level structure. To gain *that* view we need to look beyond the familiar island where features like commutativity and associativity can be taken for granted, and explore the unfamiliar expanse of non-commutative and non-associative algebras and geometry. Luckily mathematicians have already charted a lot of the content that exists in higher-d spaces, and we are learning how some of it works to shape the laws of Physics. A clear example is found in the normed division algebras, and in the reduction of the octonions to the quaternions, the complex numbers, and the reals.

$$\mathbb{O}\supset\mathbb{H}\supset\mathbb{C}\supset\mathbb{R}$$

The octonions are the most general number type, 8-dimensional with one real and seven imaginary parts but are non-associative, the quaternions are 4-d with three imaginary parts but are non-commutative, the complex numbers have one real and one imaginary part,

but are well-behaved, and only the reals encode a constant value. Imaginary numbers encode variation or the freedom to vary by a specified amount in a certain direction or orientation. Though they are the granddaddy of familiar types, the octonions are thought to be weird and difficult [21]. Viewing the imaginary dimensions as rotations; the octonions reduce to the quaternions if 4 of their 7 axes are fixed, that reduce to the complex numbers if 2 of the remaining 3 are fixed, and if the last rotation is halted, only the real-numbered value remains. The octonion algebra itself is a reduction because 480 possible multiplication tables before choosing a starting place and direction reduce to 16, 8 left- and 8 right-handed algebras [22]. And once any table is chosen, we must employ only that one table thereafter. Each step in any octonion calculation thus proceeds from optiony toward specificity.

# $Smooth \supseteq Top \supseteq Meas$ $Gas \supseteq Liquid \supseteq Solid$

In the relations above; we see this pattern of moving from variability or variations to definite conditions as a more general bridge between the fields of Math and Physics that needs to be further explored. Moving from left to right; we see that acquiring a surface or boundary makes a space or object topological, or indicates a phase change from a gas to a liquid state. Then having a fixed metric makes objects or spaces measurable, or indicates the transition or phase change to the solid state. This reflects the sensibility of the earlier expression where applying successive constraints to the octonions eventually gets us to the real numbers. We observe that evolutive properties arising from directionality in higher-d Maths project onto or influence what comes after, such that things tend to evolve from highly variable states toward specific conditions and discrete possibilities. One might question the applicability of this line of reasoning to real-world Physics or everyday life. It certainly has a place for answering questions in early universe Cosmology, in Quantum Gravity, and the underpinnings of Quantum Mechanics. But the actual footprint of non-commutative and nonassociative algebra and geometry is scarcely known. That is why it is important for today's Physics researchers to look into evolutive properties that arise in these Maths, as a possible source or root cause for the evolution of time.

The idea that spaces with certain properties have a built-in time evolution is neither well-known nor well-understood, except among a handful of ardent researchers in this arena. But when Tomita opened that door, by discovering intrinsic evolution in modular Hilbert algebras [23]; Connes and others were quick to apply that notion to a broader class of objects and spaces. To walk through that door, we must go beyond the island of familiar Maths and embrace the idea that non-commutativity and non-associativity are more of a blessing than a curse to Physics [24], because they assure directed evolution. But we must first accept that if reality is quantum-mechanical, non-commutative ordering factors automatically arise due to Heisenberg uncertainty making the space we live in non-commutative. Notably; this issue also arises in Relativity as factor-ordering problems sometimes [25], when applying the

equivalence principle. However, it happens often. Cycles of action must be undertaken in a specific sequence both when doing octonion algebra and with common tasks like painting or baking [26], where a process of ratcheting accumulation happens in both cases. So the demands of additional rules of order and sequence one must learn to use non-commutative and non-associative algebras are not unnatural or onerous, and in fact represent the same laws of directional evolution all natural processes must follow.

## $\delta \colon \mathbb{R} \to \operatorname{Out}(M)$

Connes uses the above expression [27], where Out(M) = Aut(M) / Int(M) the quotient group of automorphisms of M by its normal subgroup of inner automorphisms, to show how modular or Tomita flow gives rise to intrinsic time evolution. He emphatically asserts that non-commutative spaces evolve with time and this is greatly expanded in recent work by Longo [28], showing why non-commutative spaces must evolve, and explaining the basis for time's emergence in detail. Non-commutativity is enough to assure time will evolve, and we underlably live in a space that is non-commutative. The background state of physical reality is not what Classical Physics would imagine a vacuum to be, a static space absent of contents. We can point to quantum uncertainty or virtual particles to explain this, but time's directionality can also be seen to result from residing in a higher-d spacetime instead of 3-d space plus time. Longo extends Connes' arguments to a more robust connection between quantum mechanics and thermodynamics using the language of operator algebras, and shows how time emerges as a result. But he sees this as resulting from wavefunction collapse, and I do not see emergent time as dependent upon that feature. Here I remind the reader of an outside-in view offered by Dieter Zeh [29] suggesting that the global wavefunction is more real or fundamental than particles or quantum transitions, and persists while we measure. But recent work by Peter Morgan [30], using the Koopman-von Neumann formalism, suggests we can choose between collapse and no-collapse models depending on the context, so long as we talk about subsequent measurements rather than states.

Things go from possibility toward actuality. However; this dynamism does not depend on properties arising in higher-d spaces and hyper-complex algebras or from infinities, as in the above examples, so higher-d attributes are not essential for top-down mechanisms to work. Gisin [31] points out that even real numbers, which are commonly viewed as fixed quantities, are only fixed by a process of determination. From an intuitionist view; the exact or precise value of a quantity is not known before it is determined in a process. So if there is a specific real number with a large number of digits; we may not find out what extended values are for a long time, and information on what is beyond a given point is no better than random data. This is like a stochastic background in the realm of the indeterminate, which influences our view of time. Gisin speaks to this issue [32] and also has novel ideas about wavefunction collapse [33]. This work informs the Contextual Collapse model of Drossel and Ellis [34], a top-down approach which treats the quantum measurement problem in detail,

where measurement is by nature a multi-stage process. Their work addresses shortcomings in decoherence theory and other models, which they handle individually, concluding that a robust description includes both Quantum and Classical elements because any physical measurement unavoidably includes a mix of both, depending on the context. They further extend the conversation to how quantum measurements are a special case, projections of the wavefunction; of a larger class they call events which are ubiquitous in nature.

#### What Kind of Philosophy is best for Physics?

Is there a single best Philosophy of Physics? While some would argue that learning Philosophy is a side trip for those learning Physics; that in itself is a Philosophy about how Science is done. So it is unavoidable that the two are interwoven. This is aptly illustrated by Rovelli [35] in his paper arguing that Philosophy and Physics need each other, contrasting the schools of Isocrates and Plato. While Isocrates taught that practical skills and their application should be cultivated; Plato advocated the quest for knowledge of how or why things work, and how they came to be as they are. While Plato was among the first to be called a philosopher; we put both in that category in modern times, and their teachings are seen as two competing schools of Philosophy. Aristotle argued that general theory supports practice, and one can argue that both approaches are needed to make progress in some areas. But the debate about which philosophy is best goes on today, writ large on the face of politics. This influences funding for all areas of Science, depending on the philosophy of the prevailing political party. This split is seen to be connected to our perception of time, in that liberals have a more forward-facing view, being focused on creating a better future, while conservatives tend to focus on preserving the legacy and traditions of our past, which is more rearward-facing. Here again; people fiercely debate which philosophy is correct, or benefits us most, rather than seeing that both viewpoints are needed.

If asked what kind of Philosophy is best for Physics; I would advise people to seek an inclusive or encompassing view of reality. We need to be explorers, unafraid to look beyond the boundaries, if we want to make important discoveries or developments. And we must be wary of information silos that present a self-consistent but wrong-headed view of the world. The incorrect usage of the word recombination persists in astrophysics, for example, to describe what happens at the horizon of last scattering during decoupling; even though early-universe cosmology tells us nuclei and electrons never were combined into atoms before then. This illustrates how skilled scholars work on an island of familiar and comfortable norms, with fierce pressure to remain within the bounds of their area of specialization. The self-consistent view within the silo is often reinforced by walls that are polished mirror smooth. An outsider and generalist like myself gets a different view from others when attending quantum gravity lectures, for example, because the common ground stands out as starkly as the differences between approaches, and I have no need for a clear winner. I think all of the current lines of research inform us in useful ways, and are not mutually-exclusive. But we

should not stop exploring or seeking other options to explore, just because we have some viable explanations for gravity on the table.

So the ideal Philosophy for Physics always leaves the door open to learning more about what we think we know, discovering something new or unexpected, and so on. We are unavoidably inside a construction that is the cosmos, and further limited because we can see only what is within the Hubble radius. We are on the inside, looking out at the larger cosmos. This is why I advocate seeking the outside-in view of Math and Physics, as a radical extension of the top-down view, and as a possible cure for the view that making things from the bottom-up can not only create all form but aptly explains all we see. A basic fact about explorers is needing somewhere to explore. Perhaps we need to get off-planet to obtain the perspective we need for the outside-in view beyond our limitations. Or maybe we only need to seek that perspective from higher ground philosophically. To see a circle in its entirety; one must be off the page, in the 3<sup>rd</sup> dimension. Likewise; to draw that circle, one must be in a dimension one higher than the surface on which you are drawing. If this as a general pattern; there is always a need to discern what the structure we are living in would look like from one dimension higher. But the block universe of General Relativity, of spacetime as seen from the outside, is a severe limitation. So we need to frame other constructs that give us better scaffolding for our ideas of the view from out there. And that is the main thrust of this paper; to inspire others who might help to create that structure.

When entertaining questions about Philosophy and Physics; one cannot forget the way Mathematics always makes its way onto the scene. Plato brought to light the notion of a mathematical ideal or archetype which is projected onto physical reality. We now have a much more sophisticated notion of how that works, but the idea persists to this day. I think we need to be hyper-dimensional Platonists, with a guide to the Atlas of Lie Groups [36] in every College library where they teach higher-level Math and Physics subjects. In this way; we will speed up the process of discovery fueled by what we know. But we need to introduce some ideas at an earlier age, so tomorrow's young people will have the benefit of what we as adults already know. This is what David Blair seeks to do with the "Einstein First" program, which teaches advanced concepts based on our latest knowledge [37], and then works that back into the structure of what students have already learned. This allows young people to benefit from the understanding of their elders, in a context that is not intimidating for teachers with a limited knowledge of Relativity. We should attempt to do the same with other areas of Mathematics and Physics so the next generation will have the benefit of what we have learned. This might not be possible if we teach things solely from the bottom-up view. What if we had young people building Zome models [38] of  $E_8$  instead? At least we need to give kids a chance to see what these figures look like, at Science Centers and Math Museums.

The nexus of Philosophy and Physics must be strong, for either topic to provide useful information. This means it is incumbent on the organizers of events and publications to provide a context for insightful contributions that forge a stronger connection between what appear to be separated views, or to push the envelope of developments on the outskirts. The 16<sup>th</sup> Marcel Grossmann conference was exemplary on both of these fronts, in its spirit of cooperation, openness, and inclusivity. The organizers deliberately included minority views and off-limits topics in both the choice of plenary speakers and in the various breakout sessions. So people were encouraged to look outside of their silos somewhat, and to explore what is beyond the island of comfortable mainstream assumptions. During MG16; a Philosophy of Physics approaching the ideal was observed. But in the world at large; there is a lot of progress to be made forging connections to support a higher ideal. What we see now is a world that largely ignores the advice of both philosophers and physicists, in favor of following popular icons. There is much to do, if we want the ideas that enlighten physicists to be meaningful for everyone, but things being discovered in Physics have a meaning that unavoidably affects all equally. There is no point to arguing Physics is devoid of meaning. Therefore our Philosophy must include the notion that Physics is relevant.

We owe much of our current understanding to the philosophers of ancient Greece, not least the notion of atomism with indivisibly small bits or increments in the cosmos or the way it is constructed, as was well-explained in Rovelli's book [39]. Leucippus likely brought the seed of this idea to Democritus from Anaximander's school. But we can see that atomism applied to Plato's comment that time is the projected image of eternity, shows up as every atom in the stream of Anaximander having a duration or persistence in time. This is observed as a half-life, when examining the same phenomenon in nuclear or sub-atomic Physics, due in part to the relativistic effects of time dilation. But here I note that parcels of space needed to have persistence in time already for those particles to exist. That is why it is important to continue our exploration of quantum gravity theories and try to understand the cosmological context for these theories in a broader way. So physicists should see it as a responsibility to build bridges between the philosophically disconnected islands of thought, by applying a Philosophy of Physics that supports our common endeavor to learn the secrets of the cosmos. Encouraging students as well as researchers to explore beyond what is known is important, and willingness to question our own boundaries is an important first step. We must be ready to leave the world of comfortable norms behind, when examining cosmic origins, to fully understand how things got to be the way they are now.

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