BD-ΛCDM and Running Vacuum Models: Theoretical background and current observational status

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Brans and Dicke model

The most important features of the Brans and Dicke (BD) theory

- It is a particular type of scalar-tensor theory.
- Gravity is mediated by the metric tensor field and also by a scalar field.
- G_N is replaced by that scalar field $G(t) = 1/\psi(t) = G_N/\varphi(t)$.
- The theory contains a dimensionless parameter $\omega_{\rm BD}$ and General Relativity can be retrieved in the limit $\omega_{\rm BD} \to \infty$ and $\varphi \to 1$.

BD action and the field equations

The action in the Jordan frame reads

$$S_{\rm BD} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}\left(R\psi - \frac{\omega_{\rm BD}}{\psi}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi\right) - \rho_\Lambda\right] + S_m.$$

The variation with respect to the metric yields

$$\psi G_{\mu\nu} + \left(\Box\psi + \frac{\omega_{\rm BD}}{2\psi}g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi\right)g_{\mu\nu} - \nabla_\mu\nabla_\nu\psi - \frac{\omega_{\rm BD}}{\psi}\nabla_\mu\psi\nabla_\nu\psi$$
$$= 8\pi\left(T_{\mu\nu} - g_{\mu\nu}\rho_\Lambda\right)$$

whereas the variation with respect to the scalar field

$$\Box\psi = \frac{8\pi}{2\omega_{\rm BD}+3}(T - 4\rho_\Lambda)$$

Cosmological equations for BD gravity

For convenience we will use the following definitions $\varphi(t) \equiv G_N\psi(t)$ and $\epsilon_{\rm BD} \equiv \frac{1}{\omega_{\rm BD}}$. In the spatially flat FLRW metric, $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, the cosmological equations can be written as: (The dots represent derivative *w.r.t.* the cosmic time)

$$
\begin{aligned}
3H^2 + 3H\frac{\dot\varphi}{\varphi} - \frac{1}{2\epsilon_{\rm BD}}\left(\frac{\dot\varphi}{\varphi}\right)^2 &= \frac{8\pi G_N}{\varphi}\rho \\
2\dot H + 3H^2 + \frac{\ddot\varphi}{\varphi} + 2H\frac{\dot\varphi}{\varphi} + \frac{1}{2\epsilon_{\rm BD}}\left(\frac{\dot\varphi}{\varphi}\right)^2 &= -\frac{8\pi G_N}{\varphi}p \\
\ddot\varphi + 3H\dot\varphi &= \frac{8\pi G_N\epsilon_{\rm BD}}{2+3\epsilon_{\rm BD}}(\rho - 3p)
\end{aligned}
$$

with $\rho \equiv \rho_m + \rho_\gamma + \rho_\nu + \rho_\Lambda$ and $p \equiv p_m + p_\gamma + p_\nu + p_\Lambda$.

$$
\dot\rho + 3H(\rho + p) = \sum_N [\dot\rho_N + 3H(\rho_N + p_N)] = 0
$$

Cosmological perturbations

To study the evolution of the perturbations we have to consider $\varphi = \bar{\varphi} + \delta\varphi$. At deep subhorizon scales we can obtain the differential equation for the matter density contrast $\delta_m \equiv \delta\rho_m/\bar{\rho}_m$

$$\delta_m'' + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)\delta_m' - \frac{4\pi}{H^2(a)}\frac{G_N}{\bar{\varphi}}\left(\frac{2+4\epsilon_{\rm BD}}{2+3\epsilon_{\rm BD}}\right)\frac{\bar{\rho}_m}{a^2}\delta_m = 0.$$

It is possible to define

$$G_{\rm eff} = \frac{G_N}{\bar{\varphi}}\left(\frac{2+4\epsilon_{\rm BD}}{2+3\epsilon_{\rm BD}}\right)$$

Can the CC be seen as a dynamical quantity in the BD framework ?

We can approximate the evolution of φ from the MDE as:

$$\varphi(a) = \varphi_0 a^{-\epsilon} \quad (|\epsilon| \ll 1).$$

Plugging this ansatz into the cosmological equations and neglecting terms of order $\mathcal{O}(\epsilon^2)$[1]

$$H^2(a) = \frac{8\pi G_N}{3\varphi_0}\left(\rho_m^0 a^{-3+\epsilon} + \rho_{\rm DE}(H)\right)$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3\varphi_0}\left(\rho_m^0 a^{-3+\epsilon} + \rho_{\rm DE}(H) + 3p_\Lambda\right)$$

[1]For more details see J. Solà IJMP D27 (2018) 14, JdCP and J. Solà 1847029 and MPLA 33 (2018) 38, 1850228.

being the effective DE density

$$\rho_{\rm DE}(H) = \rho_\Lambda + \frac{3\nu_{\rm eff}\varphi_0}{8\pi G_N} H^2$$

with

$$\nu_{\rm eff} \equiv \epsilon\left(1 + \frac{1}{6}\omega_{\rm BD}\epsilon\right).$$

We can compute the effective EoS parameter defined as:

$$w_{\rm eff}(z) = \frac{p_\Lambda}{\rho_{\rm DE}(H)} \simeq -1 + \frac{3\nu_{\rm eff}\varphi_0}{8\pi G_N \rho_\Lambda} H^2(z) = -1 + \frac{\nu_{\rm eff}}{\Omega_\Lambda}\frac{H^2(z)}{H_0^2}$$

Running Vacuum Models

In analogy with the Renormalization Group equation we can consider the following expression which contains the Quantum effects that make the vacuum energy density a dynamical quantity [2]

$$\frac{d\rho_{\rm vac}}{d\ln\mu^2} = \frac{1}{4\pi^2}\sum_i \left[a_i M_i^2 \mu^2 + b_i \mu^4 + c_i \frac{\mu^6}{M_i^2} + \ldots\right],$$

Identifying the running scale with $\mu \sim H^2 + \dot{H}$ and integrating the above expression we end up with:

$$\rho_{\rm vac}(H) = \frac{3}{8\pi G_N}\left(c_0 + \nu H^2 + \tilde{\nu}\dot{H}\right) + \mathcal{O}(H^4)$$

[2] See I.L. Shapiro and J. Solà, JHEP 0202 (2002) 006, J. Solà, J. Phys. A41 (2008) 164066 and I.L. Shapiro and J. Solà, Phys. Lett. B682 (2009) 105, C. Moreno-Pulido and J. Solà, Eur.Phys.J.C. 80 (2020) 8, 692.

By imposing the relation $\tilde{\nu} = \nu/2$ and employing the definition of the Ricci scalar in the FLRW metric, $\mathcal{R} = 12H^2 + 6\dot{H}$ we have

$$\rho_{\rm vac} = \frac{3}{8\pi G_N}\left(c_0 + \frac{\nu}{12}\mathcal{R}\right).$$

To allow the evolution of the vacuum component, not present in the standard model, we need to consider an extra ingredient.

Type I RRVM

In this scenario the CDM is in interaction with vacuum

$$\dot{\rho}_{dm} + 3H\rho_{dm} = -\dot{\rho}_{\rm vac}.$$

Neglecting the contribution of massive neutrinos in front of the one of CDM+baryons:

$$\rho_{dm}(a) = \rho_m^0 a^{-3\xi} - \rho_b^0 a^{-3}$$
$$\rho_{\rm vac}(a) = \rho_{\rm vac}^0 + \left(\frac{1}{\xi} - 1\right)\rho_m^0\left(a^{-3\xi} - 1\right)$$

where

$$\xi \equiv \frac{1-\nu}{1-\frac{3}{4}\nu} \simeq 1 - \frac{\nu}{4} + \mathcal{O}(\nu^2) \equiv 1 - \nu_{\rm eff} + \mathcal{O}(\nu_{\rm eff}^2).$$

Threshold redshift scenario for type I model

We keep deactivated the interaction between vacuum and CDM for most of the cosmic history until the late time. We model this situation as:

$\underline{a < a_*}$

$$\rho_{dm}(a) = \rho_{dm}(a_*)\left(\frac{a}{a_*}\right)^{-3}$$
$$\rho_{\rm vac} = \rho^0_{\rm vac} + \left(\frac{1}{\xi} - 1\right)\rho^0_m\left(a_*^{-3\xi} - 1\right)$$

$\underline{a > a_*}$

$$\rho_{dm}(a) = \rho^0_m a^{-3\xi} - \rho^0_b a^{-3}$$
$$\rho_{\rm vac}(a) = \rho^0_{\rm vac} + \left(\frac{1}{\xi} - 1\right)\rho^0_m\left(a^{-3\xi} - 1\right)$$

being the constant

$$\rho_{dm}(a_*) = \rho^0_m a_*^{-3\xi} - \rho^0_b a_*^{-3}.$$

Type II RRVM

In this scenario matter evolve in the standard way, however vacuum can still evolve due to the dynamics of the gravitational coupling, $\varphi(H) \equiv G_N/G(H)$

$$\frac{\dot{\varphi}}{\varphi} = \frac{\dot{\rho}_{\rm vac}}{\rho_t + \rho_{\rm vac}}.$$

The approximate behaviour of the vacuum term close to the present time is

$$\rho_{\rm vac}(a) \simeq \bar{C}(1 + 4\nu_{\rm eff}) + \nu_{\rm eff}\,\rho_m^0 a^{-3}$$

regarding the gravitational coupling

$$\varphi \sim a^{-\epsilon} \simeq 1 - \epsilon \ln(a) \quad 0 < \epsilon \ll 1.$$

Cosmological perturbations

The dynamics of the vacuum energy density affects nontrivially the cosmological perturbations. If we consider the perturbed metric $ds^2 = -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j$, the modified equation for CDM takes the form

$$\dot{\delta}_{dm} + \frac{\dot{h}}{2} - \frac{\dot{\rho}_{\rm vac}}{\rho_{dm}}\delta_{dm} = 0,$$

where h is the trace of h_{ij}.

Cosmological data

The data employed in the analysis:

- **SNIa**: We use the full Pantheon likelihood Pantheon, which contains 1048 SNIa in addition to 207 SNIa more from the DES survey.
- **BAO**: We use data on both isotropic and anisotropic BAO, covering the redshift range $z \sim$0.122-2.334.
- **Cosmic Chronometers**: 31 data points on $H(z_i)$.
- **CMB**: We consider the full likelihood Planck 2018 TT+lowE.
- **LSS**: A set of $f(z_i)\sigma_8(z_i)$ data points in a redshift range $z \sim$0.035-1.48.
- **Prior on H_0**: The Hubble parameter measured by the SH0ES team collaboration $H_0 = (73.5 \pm 1.4)$km/s/Mpc obtained with the cosmic distance ladder method.

Numerical results (Baseline)

Baseline

Parameter	GR-ΛCDM	type I RRVM	type I RRVM$_{\rm thr.}$	type II RRVM	BD-ΛCDM
H_0(km/s/Mpc)	$68.37^{+0.38}_{-0.41}$	$68.17^{+0.50}_{-0.48}$	$67.63^{+0.42}_{-0.43}$	$69.02^{+1.16}_{-1.21}$	$69.30^{+1.38}_{-1.33}$
ω_b	$0.02230^{+0.00019}_{-0.00018}$	$0.02239^{+0.00023}_{-0.00024}$	$0.02231^{+0.00020}_{-0.00019}$	$0.02245^{+0.00025}_{-0.00027}$	0.02248 ± 0.00025
ω_{dm}	$0.11725^{+0.00094}_{-0.00084}$	$0.11731^{+0.00092}_{-0.00087}$	$0.12461^{+0.00201}_{-0.00210}$	$0.11653^{+0.00158}_{-0.00160}$	$0.11629^{+0.00148}_{-0.00151}$
$\nu_{\rm eff}$	-	$0.00024^{+0.00039}_{-0.00040}$	$0.02369^{+0.00625}_{-0.00563}$	0.00029 ± 0.00047	-
$\epsilon_{\rm BD}$	-	-	-	-	$-0.00109 \pm^{+0.00135}_{-0.00141}$
$\varphi_{\rm ini}$	-	-	-	$0.980^{+0.031}_{-0.027}$	$0.972^{+0.030}_{-0.037}$
φ_0	-	-	-	$0.973^{+0.036}_{-0.033}$	$0.963^{+0.036}_{-0.041}$
$\tau_{\rm reio}$	$0.049^{+0.008}_{-0.007}$	$0.051^{+0.008}_{-0.009}$	$0.058^{+0.007}_{-0.009}$	0.051 ± 0.008	0.051 ± 0.008
n_s	$0.9698^{+0.0039}_{-0.0036}$	$0.9716^{+0.0044}_{-0.0047}$	0.9703 ± 0.038	$0.9762^{+0.0081}_{-0.0091}$	
σ_8	0.796 ± 0.007	$0.789^{+0.013}_{-0.014}$	$0.768^{+0.010}_{-0.009}$	$0.791^{+0.013}_{-0.012}$	$0.790^{+0.013}_{-0.012}$
S_8	0.796 ± 0.011	$0.791^{+0.014}_{-0.013}$	$0.797^{+0.012}_{-0.011}$	$0.781^{+0.021}_{-0.020}$	$0.777^{+0.021}_{-0.022}$
r_s (Mpc)	$147.90^{+0.30}_{-0.31}$	$147.99^{+0.35}_{-0.36}$	147.81 ± 0.30	$146.30^{+2.39}_{-2.30}$	$145.72^{+2.44}_{-2.90}$
$\chi^2_{\rm min}$	2290.20	2289.72	2272.44	2288.74	2289.40
ΔDIC	-	-2.70	+13.82	-4.59	-3.53

The mean values and 68.3% confidence limits for the models under study using our Baseline dataset. We display the fitting values for the usual parameters and a couple of useful derived parameters. For all the RRVM's we show $\nu_{\rm eff}$, and for the type II and BD-ΛCDM we also report the initial and current values of φ, $\varphi_{\rm ini}$ and φ_0, respectively. We provide the corresponding values of $\chi^2_{\rm min}$ and ΔDIC.

Numerical results (Baseline+H_0)

Baseline + H_0

Parameter	GR-ΛCDM	type I RRVM	type I RRVM$_{\rm thr.}$	type II RRVM	BD-ΛCDM
H_0 (km/s/Mpc)	$68.75^{+0.41}_{-0.36}$	$68.77^{+0.49}_{-0.48}$	$68.14^{+0.43}_{-0.41}$	$70.93^{+0.93}_{-0.87}$	$71.23^{+1.01}_{-1.02}$
ω_b	$0.02240^{+0.00019}_{-0.00021}$	$0.02238^{+0.00021}_{-0.00023}$	$0.02243^{+0.00019}_{-0.00018}$	$0.02269^{+0.00025}_{-0.00024}$	$0.02267^{+0.00026}_{-0.00023}$
ω_{dm}	$0.11658^{+0.00080}_{-0.00083}$	$0.11661^{+0.00084}_{-0.00085}$	$0.12299^{+0.00197}_{-0.00203}$	$0.11602^{+0.00162}_{-0.00163}$	$0.11601^{+0.00161}_{-0.00157}$
$\nu_{\rm eff}$	-	$-0.00005^{+0.00040}_{-0.00038}$	$0.02089^{+0.00553}_{-0.00593}$	$0.00038^{+0.00041}_{-0.00044}$	-
$\epsilon_{\rm BD}$	-	-	-	-	$-0.00130\pm^{+0.00136}_{-0.00140}$
$\varphi_{\rm ini}$	-	-	-	$0.938^{+0.018}_{-0.024}$	$0.928^{+0.024}_{-0.026}$
φ_0	-	-	-	$0.930^{+0.022}_{-0.029}$	$0.919^{+0.028}_{-0.033}$
$\tau_{\rm reio}$	$0.050^{+0.008}_{-0.007}$	$0.049^{+0.009}_{-0.008}$	$0.058^{+0.008}_{-0.009}$	0.052 ± 0.008	0.052 ± 0.008
n_s	$0.9718^{+0.0035}_{-0.0038}$	0.9714 ± 0.0046	$0.9723^{+0.0040}_{-0.0039}$	$0.9868^{+0.0072}_{-0.0074}$	$0.9859^{+0.0073}_{-0.0072}$
σ_8	0.794 ± 0.007	0.795 ± 0.013	0.770 ± 0.010	$0.794^{+0.013}_{-0.012}$	$0.792^{+0.013}_{-0.012}$
S_8	$0.788^{+0.010}_{-0.011}$	0.789 ± 0.013	0.789 ± 0.011	$0.761^{+0.018}_{-0.017}$	$0.758^{+0.019}_{-0.018}$
r_s (Mpc)	$147.97^{+0.29}_{-0.31}$	$147.94^{+0.35}_{-0.36}$	$147.88^{+0.33}_{-0.29}$	$143.00^{+1.54}_{-1.96}$	$142.24^{+1.99}_{-2.12}$
$\chi^2_{\rm min}$	2302.14	2301.90	2288.82	2296.38	2295.36
ΔDIC	-	-2.36	+10.88	+5.52	+6.25

Same as in Table 1, but also considering the prior on $H_0 = (73.5 \pm 1.4)$ km/s/Mpc from SH0ES [3]

[3] M.J. Reid, D.W. Pesce and A.G. Riess, ApJL 886 (2019) L27.

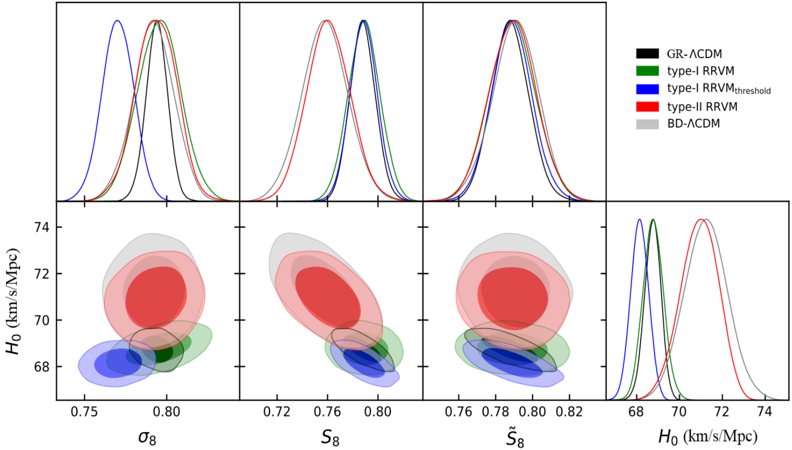

Figure: 1σ and 2σ contours in the H_0-$\sigma_8, S_8, \tilde{S}_8 \equiv S_8/\sqrt{\varphi_0}$ planes and the corresponding one-dimensional posteriors for the GR- and BD- ΛCDM and the RRVM's obtained from the fitting analyses with our Baseline+H_0 data set.

- The BD-ΛCDM is able to fit the different cosmological data even with a $G_{\rm eff} \sim G_N/\varphi_0$ being higher than in the GR-ΛCDM model.
- We find significant evidence that a dynamical $\rho_{\rm vac}$ can improve the description of the cosmological data in comparison to the ΛCDM.
- With such dynamics it is possible to loosen the σ_8-tension.
- The type II model and the BD model can deal with the σ_8-tension and the H_0-tension at a time.

Thank you for your attention!