BD-ΛCDM and Running Vacuum Models: Theoretical background and current observational status

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The most important features of the Brans and Dicke (BD) theory

- It is a particular type of scalar-tensor theory.
- Gravity is mediated by the metric tensor field and also by a scalar field.
- $G_N$ is replaced by that scalar field $G(t) = 1/\psi(t) = G_N/\varphi(t)$.
- The theory contains a dimensionless parameter $\omega_{BD}$ and General Relativity can be retrieved in the limit $\omega_{BD} \to \infty$ and $\varphi \to 1$. 
BD action and the field equations

The action in the Jordan frame reads

$$S_{BD} = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi} \left( R\psi - \frac{\omega_{BD}}{\psi} \right) g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right] \right] + S_m.$$

The variation with respect to the metric yields

$$\psi G_{\mu\nu} + \left( \Box \psi + \frac{\omega_{BD}}{2\psi} g^{\mu\nu} \nabla_\mu \psi \nabla_\nu \psi \right) g_{\mu\nu} - \nabla_\mu \nabla_\nu \psi - \frac{\omega_{BD}}{\psi} \nabla_\mu \psi \nabla_\nu \psi = 8\pi \left( T_{\mu\nu} - g_{\mu\nu} \rho_\Lambda \right)$$

whereas the variation with respect to the scalar field

$$\Box \psi = \frac{8\pi}{2\omega_{BD} + 3} (T - 4\rho_\Lambda)$$
Cosmological equations for BD gravity

For convenience we will use the following definitions $\varphi(t) \equiv G_N \psi(t)$ and $\epsilon_{BD} \equiv \frac{1}{\omega_{BD}}$. In the spatially flat FLRW metric, $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, the cosmological equations can be written as: (The dots represent derivative w.r.t. the cosmic time)

$$3H^2 + 3H \frac{\dot{\varphi}}{\varphi} - \frac{1}{2\epsilon_{BD}} \left( \frac{\dot{\varphi}}{\varphi} \right)^2 = \frac{8\pi G_N}{\varphi} \rho$$

$$2\dot{H} + 3H^2 + \frac{\ddot{\varphi}}{\varphi} + 2H \frac{\dot{\varphi}}{\varphi} + \frac{1}{2\epsilon_{BD}} \left( \frac{\dot{\varphi}}{\varphi} \right)^2 = - \frac{8\pi G_N}{\varphi} p$$

$$\ddot{\varphi} + 3H \dot{\varphi} = \frac{8\pi G_N \epsilon_{BD}}{2 + 3\epsilon_{BD}} (\rho - 3p)$$

with $\rho \equiv \rho_m + \rho_\gamma + \rho_\nu + \rho_\Lambda$ and $p \equiv p_m + p_\gamma + p_\nu + p_\Lambda$.

$$\dot{\rho} + 3H(\rho + p) = \sum_N [\dot{\rho}_N + 3H(\rho_N + p_N)] = 0$$
Cosmological perturbations

To study the evolution of the perturbations we have to consider \( \varphi = \bar{\varphi} + \delta \varphi \). At deep subhorizon scales we can obtain the differential equation for the matter density contrast \( \delta_m \equiv \delta \rho_m / \bar{\rho}_m \)

\[
\delta_m'' + \left( \frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta_m' - \frac{4\pi}{H^2(a)} \frac{G_N}{\bar{\varphi}} \left( \frac{2 + 4\epsilon_{BD}}{2 + 3\epsilon_{BD}} \right) \frac{\bar{\rho}_m}{a^2} \delta_m = 0.
\]

It is possible to define

\[
G_{\text{eff}} = \frac{G_N}{\bar{\varphi}} \left( \frac{2 + 4\epsilon_{BD}}{2 + 3\epsilon_{BD}} \right)
\]
Can the CC be seen as a dynamical quantity in the BD framework?

We can approximate the evolution of $\varphi$ from the MDE as:

$$\varphi(a) = \varphi_0 a^{-\epsilon} \quad (|\epsilon| \ll 1).$$

Plugging this ansatz into the cosmological equations and neglecting terms of order $O(\epsilon^2)^1$

$$H^2(a) = \frac{8\pi G_N}{3\varphi_0} \left( \rho_m^0 a^{-3+\epsilon} + \rho_{DE}(H) \right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3\varphi_0} \left( \rho_m^0 a^{-3+\epsilon} + \rho_{DE}(H) + 3p_\Lambda \right)$$

---

being the effective DE density

\[ \rho_{DE}(H) = \rho_{\Lambda} + \frac{3\nu_{\text{eff}}\varphi_0}{8\pi G_N} H^2 \]

with

\[ \nu_{\text{eff}} \equiv \epsilon \left( 1 + \frac{1}{6\omega_{\text{BD}}\epsilon} \right). \]

We can compute the effective EoS parameter defined as:

\[ w_{\text{eff}}(z) = \frac{p_{\Lambda}}{\rho_{DE}(H)} \simeq -1 + \frac{3\nu_{\text{eff}}\varphi_0}{8\pi G_N \rho_{\Lambda}} H^2(z) = -1 + \frac{\nu_{\text{eff}}}{\Omega_{\Lambda}} \frac{H^2(z)}{H_0^2} \]
Running Vacuum Models

In analogy with the Renormalization Group equation we can consider the following expression which contains the Quantum effects that make the vacuum energy density a dynamical quantity:

\[ \frac{d\rho_{\text{vac}}}{d \ln \mu^2} = \frac{1}{4\pi^2} \sum_i \left[ a_i M_i^2 \mu^2 + b_i \mu^4 + c_i \frac{\mu^6}{M_i^2} + \ldots \right], \]

Identifying the running scale with \( \mu \sim H^2 + \dot{H} \) and integrating the above expression we end up with:

\[ \rho_{\text{vac}}(H) = \frac{3}{8\pi G_N} \left( c_0 + \nu H^2 + \tilde{\nu} \dot{H} \right) + \mathcal{O}(H^4) \]

By imposing the relation \( \tilde{\nu} = \nu/2 \) and employing the definition of the Ricci scalar in the FLRW metric, \( R = 12H^2 + 6\dot{H} \) we have

\[
\rho_{\text{vac}} = \frac{3}{8\pi G_N} \left( c_0 + \frac{\nu}{12} R \right).
\]

To allow the evolution of the vacuum component, not present in the standard model, we need to consider an extra ingredient.
Type I RRVM

In this scenario the CDM is in interaction with vacuum

\[ \dot{\rho}_{dm} + 3H\rho_{dm} = -\dot{\rho}_{\text{vac}}. \]

Neglecting the contribution of massive neutrinos in front of the one of CDM+baryons:

\[ \rho_{dm}(a) = \rho_{m}^0 a^{-3\xi} - \rho_{b}^0 a^{-3} \]

\[ \rho_{\text{vac}}(a) = \rho_{\text{vac}}^0 + \left( \frac{1}{\xi} - 1 \right) \rho_{m}^0 \left( a^{-3\xi} - 1 \right) \]

where

\[ \xi \equiv \frac{1 - \nu}{1 - \frac{3}{4}\nu} \simeq 1 - \frac{\nu}{4} + \mathcal{O}(\nu^2) \equiv 1 - \nu_{\text{eff}} + \mathcal{O}(\nu_{\text{eff}}^2). \]
Threshold redshift scenario for type I model

We keep deactivated the interaction between vacuum and CDM for most of the cosmic history until the late time. We model this situation as:

\( a < a_\ast \)

\[
\rho_{dm}(a) = \rho_{dm}(a_\ast) \left( \frac{a}{a_\ast} \right)^{-3}
\]

\[
\rho_{vac} = \rho_{vac}^0 + \left( \frac{1}{\xi} - 1 \right) \rho_m^0 \left( a_\ast^{-3 \xi} - 1 \right)
\]

\( a > a_\ast \)

\[
\rho_{dm}(a) = \rho_m^0 a^{-3 \xi} - \rho_b^0 a^{-3}
\]

\[
\rho_{vac}(a) = \rho_{vac}^0 + \left( \frac{1}{\xi} - 1 \right) \rho_m^0 \left( a^{-3 \xi} - 1 \right)
\]

being the constant

\[
\rho_{dm}(a_\ast) = \rho_m^0 a_\ast^{-3 \xi} - \rho_b^0 a_\ast^{-3}.
\]
Type II RRVM

In this scenario matter evolve in the standard way, however vacuum can still evolve due to the dynamics of the gravitational coupling, \( \varphi(H) \equiv G_N/G(H) \)

\[
\frac{\dot{\varphi}}{\varphi} = \frac{\dot{\rho}_{\text{vac}}}{\rho_t + \rho_{\text{vac}}}.
\]

The approximate behaviour of the vacuum term close to the present time is

\[
\rho_{\text{vac}}(a) \simeq \bar{C}(1 + 4\nu_{\text{eff}}) + \nu_{\text{eff}}\rho_m^0 a^{-3}
\]

regarding the gravitational coupling

\[
\varphi \sim a^{-\epsilon} \simeq 1 - \epsilon \ln(a) \quad 0 < \epsilon \ll 1.
\]
The dynamics of the vacuum energy density affects nontrivially the cosmological perturbations. If we consider the perturbed metric 
\[ ds^2 = -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j, \]
the modified equation for CDM takes the form
\[ \dot{\delta}_{dm} + \frac{\dot{h}}{2} - \frac{\dot{\rho}_{\text{vac}}}{\rho_{dm}} \delta_{dm} = 0, \]
where \( h \) is the trace of \( h_{ij} \).
Cosmological data

The data employed in the analysis:

- **SNIa**: We use the full Pantheon likelihood Pantheon, which contains 1048 SNIa in addition to 207 SNIa more from the DES survey.
- **BAO**: We use data on both isotropic and anisotropic BAO, covering the redshift range $z \sim 0.122-2.334$.
- **Cosmic Chronometers**: 31 data points on $H(z_i)$.
- **CMB**: We consider the full likelihood Planck 2018 TT+lowE.
- **LSS**: A set of $f(z_i)\sigma_8(z_i)$ data points in a redshift range $z \sim 0.035-1.48$.
- **Prior on $H_0$**: The Hubble parameter measured by the SH0ES team collaboration $H_0 = (73.5 \pm 1.4)\text{km/s/Mpc}$ obtained with the cosmic distance ladder method.
Numerical results (Baseline)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GR-LCDM</th>
<th>type I RRVM</th>
<th>type I RRVM thr.</th>
<th>type II RRVM</th>
<th>BD-LCDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$ (km/s/Mpc)</td>
<td>68.37$^{+0.38}_{-0.41}$</td>
<td>68.17$^{+0.50}_{-0.48}$</td>
<td>67.63$^{+0.42}_{-0.43}$</td>
<td>69.02$^{+1.16}_{-1.21}$</td>
<td>69.30$^{+1.38}_{-1.33}$</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>0.02230$^{+0.00019}_{-0.00018}$</td>
<td>0.02239$^{+0.00023}_{-0.00024}$</td>
<td>0.02231$^{+0.00020}_{-0.00019}$</td>
<td>0.02245$^{+0.00025}_{-0.00027}$</td>
<td>0.02248$^{±0.00025}$</td>
</tr>
<tr>
<td>$\omega_{dm}$</td>
<td>0.11725$^{+0.00099}_{-0.00084}$</td>
<td>0.11731$^{+0.00092}_{-0.00087}$</td>
<td>0.12461$^{+0.00201}_{-0.00210}$</td>
<td>0.11653$^{+0.00158}_{-0.00160}$</td>
<td>0.11629$^{+0.00148}_{-0.00151}$</td>
</tr>
<tr>
<td>$\nu_{eff}$</td>
<td>-</td>
<td>0.00024$^{+0.00009}_{-0.00040}$</td>
<td>0.02369$^{+0.00029}_{-0.00063}$</td>
<td>0.00029$^{±0.00047}$</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon_{BD}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.00109$^{±0.000135}$</td>
</tr>
<tr>
<td>$\varphi_{ini}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.980$^{+0.031}_{-0.027}$</td>
<td>0.972$^{+0.030}_{-0.037}$</td>
</tr>
<tr>
<td>$\varphi_0$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.973$^{+0.036}_{-0.033}$</td>
<td>0.963$^{+0.036}_{-0.041}$</td>
</tr>
<tr>
<td>$\tau_{reio}$</td>
<td>0.049$^{+0.008}_{-0.007}$</td>
<td>0.051$^{+0.008}_{-0.009}$</td>
<td>0.058$^{+0.007}_{-0.009}$</td>
<td>0.051$^{±0.008}$</td>
<td>0.051$^{±0.008}$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.9698$^{+0.0039}_{-0.0036}$</td>
<td>0.9716$^{+0.0044}_{-0.0047}$</td>
<td>0.9703$^{±0.0038}$</td>
<td>0.9762$^{+0.0081}_{-0.0091}$</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.796$^{±0.007}$</td>
<td>0.789$^{+0.013}_{-0.014}$</td>
<td>0.768$^{+0.010}_{-0.009}$</td>
<td>0.791$^{+0.013}_{-0.012}$</td>
<td>0.790$^{+0.013}_{-0.012}$</td>
</tr>
<tr>
<td>$S_8$</td>
<td>0.796$^{±0.011}$</td>
<td>0.791$^{+0.014}_{-0.013}$</td>
<td>0.797$^{+0.012}_{-0.011}$</td>
<td>0.781$^{+0.021}_{-0.020}$</td>
<td>0.777$^{+0.021}_{-0.022}$</td>
</tr>
<tr>
<td>$r_s$ (Mpc)</td>
<td>147.90$^{+0.30}_{-0.31}$</td>
<td>147.99$^{+0.35}_{-0.36}$</td>
<td>147.81$^{±0.30}$</td>
<td>146.30$^{+2.39}_{-2.30}$</td>
<td>145.72$^{+2.44}_{-2.90}$</td>
</tr>
<tr>
<td>$\chi^2_{min}$</td>
<td>2290.20</td>
<td>2289.72</td>
<td>2272.44</td>
<td>2288.74</td>
<td>2289.40</td>
</tr>
<tr>
<td>$\Delta$DIC</td>
<td>-</td>
<td>-2.70</td>
<td>+13.82</td>
<td>-4.59</td>
<td>-3.53</td>
</tr>
</tbody>
</table>

The mean values and 68.3% confidence limits for the models under study using our Baseline dataset. We display the fitting values for the usual parameters and a couple of useful derived parameters. For all the RRVM's we show $\nu_{eff}$, and for the type II and BD-LCDM we also report the initial and current values of $\varphi$, $\varphi_{ini}$ and $\varphi_0$, respectively. We provide the corresponding values of $\chi^2_{min}$ and $\Delta$DIC.
### Numerical results (Baseline + $H_0$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GR-$\Lambda$CDM</th>
<th>type I RRVM</th>
<th>type I RRVM$_{thr}$</th>
<th>type II RRVM</th>
<th>BD-$\Lambda$CDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$ (km/s/Mpc)</td>
<td>68.75$^{+0.41}_{-0.36}$</td>
<td>68.77$^{+0.49}_{-0.48}$</td>
<td>68.14$^{+0.43}_{-0.41}$</td>
<td>70.93$^{+0.93}_{-0.87}$</td>
<td>71.23$^{+1.01}_{-1.02}$</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>0.02240$^{+0.00019}_{-0.00021}$</td>
<td>0.02238$^{+0.00021}_{-0.00023}$</td>
<td>0.02243$^{+0.00019}_{-0.00018}$</td>
<td>0.02269$^{+0.00025}_{-0.00024}$</td>
<td>0.02267$^{+0.00026}_{-0.00023}$</td>
</tr>
<tr>
<td>$\omega_{dm}$</td>
<td>0.11658$^{+0.00080}_{-0.00083}$</td>
<td>0.11661$^{+0.00084}_{-0.00085}$</td>
<td>0.12299$^{+0.00197}_{-0.00203}$</td>
<td>0.11602$^{+0.00102}_{-0.00163}$</td>
<td>0.11601$^{+0.00161}_{-0.00157}$</td>
</tr>
<tr>
<td>$\nu_{\text{eff}}$</td>
<td>-</td>
<td>$-0.00005^{+0.000040}_{-0.000038}$</td>
<td>0.02089$^{+0.00553}_{-0.00593}$</td>
<td>0.00038$^{+0.00041}_{-0.00044}$</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon_{\text{BD}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$-0.00130^{+0.00136}_{-0.00140}$</td>
</tr>
<tr>
<td>$\phi_{\text{ini}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.938$^{+0.018}_{-0.024}$</td>
<td>0.928$^{+0.024}_{-0.026}$</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.930$^{+0.022}_{-0.029}$</td>
<td>0.919$^{+0.028}_{-0.033}$</td>
</tr>
<tr>
<td>$\tau_{\text{reio}}$</td>
<td>0.050$^{+0.008}_{-0.007}$</td>
<td>0.049$^{+0.009}_{-0.008}$</td>
<td>0.058$^{+0.008}_{-0.009}$</td>
<td>0.052$^{+0.007}_{-0.008}$</td>
<td>0.052$^{+0.008}_{-0.008}$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.9718$^{+0.0035}_{-0.0038}$</td>
<td>0.9714$^{+0.0039}_{-0.0038}$</td>
<td>0.9723$^{+0.0040}_{-0.0040}$</td>
<td>0.9868$^{+0.0072}_{-0.0074}$</td>
<td>0.9859$^{+0.0073}_{-0.0072}$</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.794$^{+0.007}_{-0.007}$</td>
<td>0.795$^{+0.013}_{-0.013}$</td>
<td>0.770$^{+0.011}_{-0.010}$</td>
<td>0.794$^{+0.013}_{-0.012}$</td>
<td>0.792$^{+0.013}_{-0.012}$</td>
</tr>
<tr>
<td>$S_8$</td>
<td>0.788$^{+0.010}_{-0.011}$</td>
<td>0.789$^{+0.013}_{-0.013}$</td>
<td>0.789$^{+0.011}_{-0.011}$</td>
<td>0.761$^{+0.018}_{-0.017}$</td>
<td>0.758$^{+0.019}_{-0.018}$</td>
</tr>
<tr>
<td>$r_s$ (Mpc)</td>
<td>147.97$^{+0.29}_{-0.31}$</td>
<td>147.94$^{+0.35}_{-0.36}$</td>
<td>147.88$^{+0.33}_{-0.29}$</td>
<td>143.00$^{+1.34}_{-1.96}$</td>
<td>142.24$^{+1.99}_{-2.12}$</td>
</tr>
<tr>
<td>$\chi^2_{\text{min}}$</td>
<td>2302.14</td>
<td>2301.90</td>
<td>2288.82</td>
<td>2296.38</td>
<td>2295.36</td>
</tr>
<tr>
<td>$\Delta \text{DIC}$</td>
<td>-</td>
<td>-2.36</td>
<td>+10.88</td>
<td>+5.52</td>
<td>+6.25</td>
</tr>
</tbody>
</table>

Same as in Table 1, but also considering the prior on $H_0 = (73.5 \pm 1.4)$ km/s/Mpc from SH0ES.

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Figure: 1σ and 2σ contours in the $H_0$-$\sigma_8$, $S_8$, $\tilde{S}_8 \equiv S_8/\sqrt{\varphi_0}$ planes and the corresponding one-dimensional posteriors for the GR- and BD- $\Lambda$CDM and the RRVM’s obtained from the fitting analyses with our Baseline+ $H_0$ data set.
The BD-ΛCDM is able to fit the different cosmological data even with a $G_{\text{eff}} \sim G_N/\varphi_0$ being higher than in the GR-ΛCDM model.

We find significant evidence that a dynamical $\rho_{\text{vac}}$ can improve the description of the cosmological data in comparison to the ΛCDM.

With such dynamics, it is possible to loosen the $\sigma_8$-tension.

The type II model and the BD model can deal with the $\sigma_8$-tension and the $H_0$-tension at a time.
Thank you for your attention!