ENTANGLEMENT ENTROPY AT CRITICAL POINTS IN THE MULTIVERSE

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MG16

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MOTIVATION

- SCENARIO: A PAIR OF UNIVERSES WHICH BORN TOGETHER $\Rightarrow$ ENTANGLED

1. HOW DO WE FIND THE ENTANGLEMENT ENTROPY OF THE PAIR?
2. IS HIGH AT THEIR BIRTH?
3. IS HIGH AT THEIR CRITICAL POINTS? (KIEFER 1988)

OUTLINE

1. CANONICAL QUANTUM GRAVITY
2. THIRD QUANTIZATION
3. INVARIANT REPRESENTATION
4. ENTANGLEMENT ENTROPY
5. CONCLUSIONS AND PROSPECTIVES

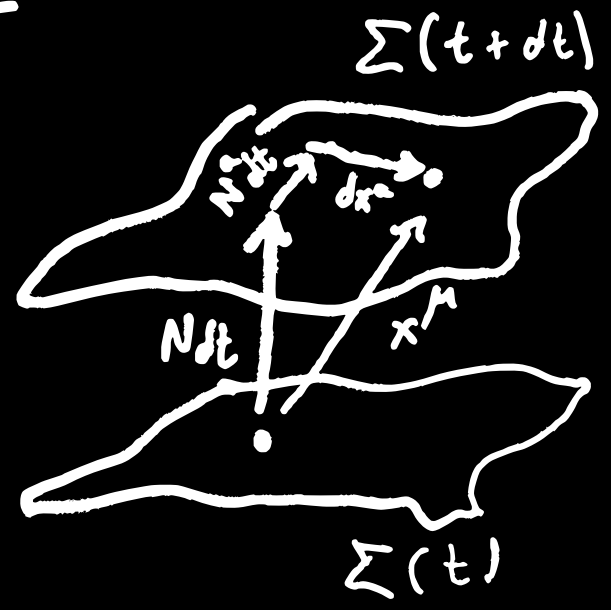
1 CANONICAL QUANTUM GRAVITY

- HAMILTONIAN FORMULATION OF G.R.
- ADM FORMALISM

} 50's

$\Rightarrow$ (DEWITT) CANONICAL QUANTUM GRAVITY !!

$$\boxed{\{g_{\mu\nu}\} \to \{N_a, h_{ab}\}, \quad \tilde{p}^{ab} = -i \frac{\partial}{\partial h_{ab}}}$$



$\Rightarrow$ WHEELER-DEWITT EQUATION

$$\boxed{\mathcal{H}\Psi = 0}$$

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→ THE UNIVERSE IS DESCRIBED BY :

$\Psi \equiv$ WAVE FUNCTION OF THE UNIVERSE

→ EACH MODEL : $\boxed{\mathcal{H}}$ HAMILTONIAN

- PROBLEM : INFINITE DEGREES OF FREEDOM

  ||

  REALITY

⇒ $\boxed{\text{MINISUPERSPACE}}$ FLRW UNIVERSE

- MASSLESS SCALAR FIELD:

<u>W-DW EQ:</u> $$\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} + f(\alpha, \Lambda, k, \dots)\right]\Psi = 0$$

WHERE: $[\alpha = \ln(a)]$.

$\Rightarrow$ $\boxed{\text{SEPARABLE}}$: $\mathcal{H}_\alpha \otimes \mathcal{H}_\phi$

$$\Rightarrow \mathcal{H}_\alpha \Psi = E_\alpha \Psi \quad , \quad \mathcal{H}_\phi \Psi = E_\phi \Psi$$

$$\Rightarrow E_\alpha = -E_\phi$$

$$\Rightarrow \boxed{\left[\frac{\partial^2}{\partial\alpha^2} + f(\alpha, \dots, E_\phi)\right]\Psi = 0}$$

H.O.-LIKE DIFFERENTIAL EQUATION

2 THIRD QUANTIZATION

- REMEMBER:

QM	$\longrightarrow$	QFT
$\psi(\vec{x},t)$		$\hat{\psi}(\lvert\varphi_i\rangle)$
- PARTICLES -		- FIELDS -

$\Rightarrow$ WE USE: $\begin{cases} a,\ a^{+} & \text{OPERATORS} \\ \lvert\varphi_i\rangle & \text{FOCK SPACE} \end{cases}$

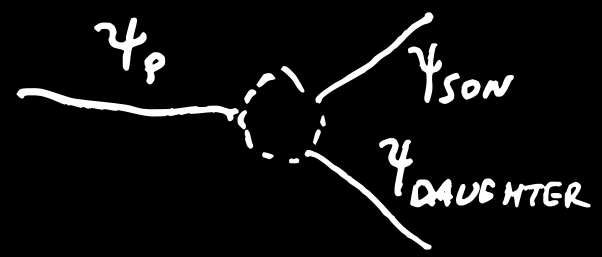
$\rightarrow$ INFINITE HILBERT SPACE (COUNTABLE) !!

- FOR CQG:

$\psi(a, \phi) \longrightarrow \hat{\psi}(|\varphi_i\rangle)$

(OUR ONLY AND BELOVED UNIVERSE)

FOCK SPACE!!



- QFTCS: $\eta_{\mu\nu} \rightarrow g_{\mu\nu}$

$\Rightarrow \langle \hat{N} \rangle \neq$ CONSTANT !!

$\Rightarrow$ PARTICLES ARE NOT WELL DEFINED

$\Rightarrow$ SOLUTIONS
- ADIABATIC EXPANSIONS $\Rightarrow$ RENORMALIZATION
- INVARIANT REPRESENTATION

$$\{|\varphi_i\rangle\}_{inv} \Rightarrow \langle\hat{N}\rangle = \text{CONSTANT.}$$

- FOR THIRD QUANTIZED UNIVERSES:

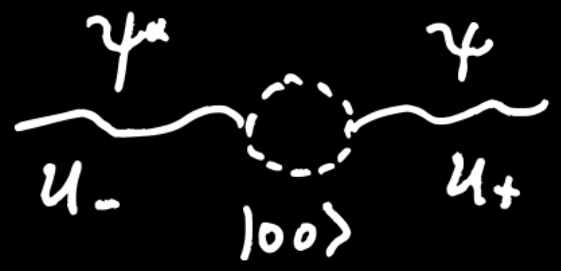
$$\langle\hat{N}\rangle \neq \text{CONSTANT} \Rightarrow \text{UNNATURAL}$$

(FROM THE ANTHROPOLOGICAL POINT OF VIEW:

N REMAINS CONSTANT !!)

3 INVARIANT REPRESENTATION

- NOTATION: $|u_- u_+\rangle$
- $|00\rangle$: "VACUUM" = NO SPACE/TIME.



- LEWIS & REISENFELD (1969):

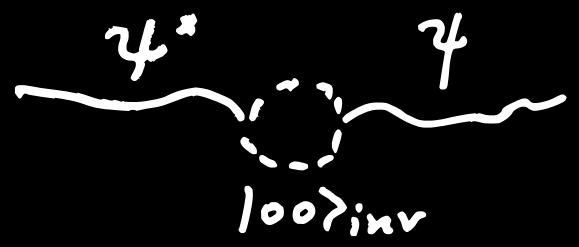
$$\left[\begin{array}{c}\text{INVARIANT REPRESENTATION FOR}\\ \text{TIME-DEPENDENT H.O.}\end{array}\right] \Rightarrow |00\rangle_{inv} = \sum_i c_i |n_- n_+\rangle_i$$

$\Downarrow$

INFINITE SUM.

$\Rightarrow$ FOR OUR W-DW EQS. !!

4 ENTANGLEMENT ENTROPY



- $\hat{\rho} = |00\rangle_{inv}\langle 00|$

$$\Rightarrow \quad \hat{\rho}_{\mathcal{H}_+} = \mathrm{Tr}_{\mathcal{H}_-}[\hat{\rho}]$$

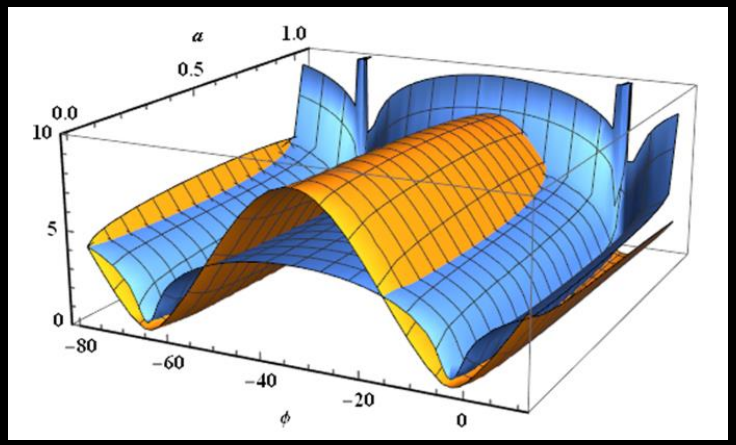
BIPARTITE SYSTEM $\Rightarrow$ VON NEUMANN ENTROPY:

$$\boxed{S_{ENT} = -\mathrm{Tr}\left[\rho_{\mathcal{H}_+} \ln \rho_{\mathcal{H}_+}\right]}$$

WE FOUND:

- TWO-DIMENSIONAL SYSTEM: — QUANTUM ϕ —

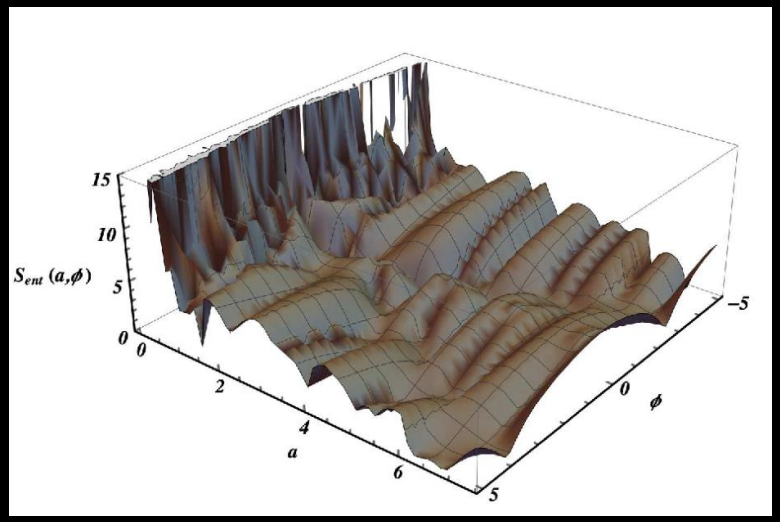
$k=1 \Rightarrow$ CRITICAL POINT: $a=1$



MASSLESS ϕ $[V(\phi)=0]$

→ FINITE AT $a \to 0$

→ INFINITE AT CRITICAL POINTS !!



MASSIVE ϕ $[V(\phi)=\frac{1}{2}m\phi^2]$

→ INFINITE AT $a \to 0$

$k=0 \Rightarrow$ NO CRITICAL POINTS

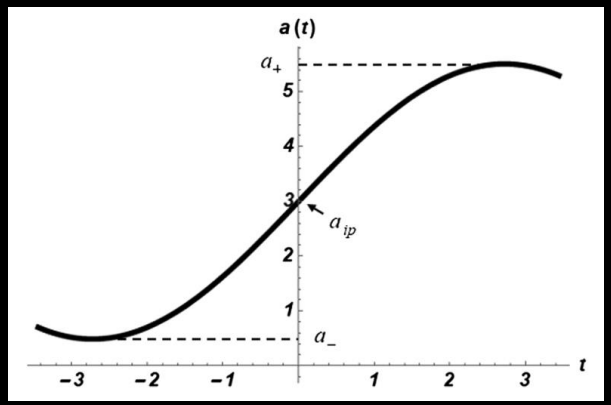
- SINUSOIDAL EVOLUTION

DĄBROWSKI (1996)

– CLASSICAL ϕ –

$$H_\phi(\dot\phi, \phi) \to H_\phi(\rho_\phi) \to H_\phi(\omega)$$

$$P_\phi = \omega \rho_\phi$$



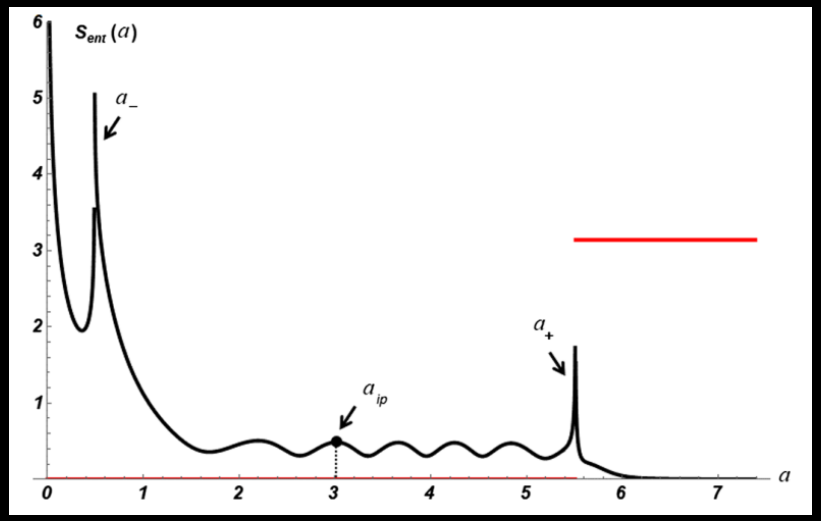
$$a(t) \sim \sin(\omega t)$$

PARAMETERS:

$\Lambda < 0$, $K = 1$

WALL-LIKE MATTER $(\rho = \rho_0/a)$

STRING-LIKE MATTER $(\rho = \rho_0/a^2)$



$$\Rightarrow \begin{cases} S_{ENT} \not\to \infty & \text{if } H \not\to 0 \\ S_{ENT} \to \infty & \text{iff } H \to 0. \end{cases}$$

$$\Rightarrow \boxed{S_{ENT}(H)\Big|_{H=0} \; ?}$$

$S_{ENT}(H)$ AROUND $H=0$?

- $\phi \Rightarrow \Psi(\phi,\alpha) = \int dK \cdot A(K) \cdot e^{iK\phi} \cdot \varphi_K(\alpha)$ } SIMPLEST MODEL

  WHERE: $A(K) = \delta(K-1)$

- FOUND: $\boxed{S_{ENT}(H) \sim c_0 - \ln(H)}$ ALMOST PERFECTLY !!

$$\Rightarrow \boxed{S_{ENT}(H) \sim -I(H)}$$

$H \equiv$ PROBABILITY !!

- **FURTHERMORE:**

→ ANOTHER MEASUREMENTS OF THE ENTANGLEMENT ENTROPY: (TSALLIS, RENYI...) EQUIVALENT ?

→ TEMPERATURE OF ENTANGLEMENT:

$$T(\alpha) = \frac{\omega(\alpha)}{2\ln[\coth(r)]}, \quad r = \operatorname{arctanh}\left(\frac{|\beta_B|}{|\alpha_B|}\right)$$

SEEMS NOT BE A GOOD MEASURE OF THE QUANTUMNESS: !!

⇒ SOMETIMES: $S_{ENT} \to \infty$ ⇒ $T \not\to \infty$ AND DECREASES

5 CONCLUSIONS & PROSPECTIVES

- EXACT ENTROPY OF ENTANGLEMENT OF A PAIR
- IT IS HIGH AT THE INITIAL SINGULARITY
  - → INFINITE WHEN ϕ IS CLASSICAL
  - → FINITE BUT HIGH WHEN ϕ IS QUANTUM
- AT CRITICAL POINTS OF THE CLASSICAL EVOLUTION:

$$S_{ENT} \longrightarrow \infty .$$

(CONFIRMING KIEFER'S HYPOTHESIS !!)

- WE FOUND THE RELATION:

$$\boxed{S_{ENT}(H) = C - \ln(H)}$$

AROUND MAXIMA AND MINIMA.

- SOME OTHER MEASUREMENTS OF THE ENTANGLEMENT SEEM NOT TO BEHAVE AS EXPECTED.

PROSPECTIVES

- MORE REALISTIC SCENARIOS

  $\Rightarrow$ FULL QFT OF UNIVERSES

- IMPRINTS ON THE CMB (OR ANY BACKGROUND)

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