
Presenting Different time steps, at the start of inflation, Using Kiefer Density Matrix, for the use of an Inflaton, in determining different conceivable time intervals for time flow Analysis and GWs

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We are using the book “Towards Quantum Gravity with an article by Claus Kiefer as to a quantum gravity interpretation of the density matrix in the early universe. The density matrix we are using is a one loop approximation, with inflaton value and potential terms, like $V(\phi)$ using the Padmanabhan values one can expect if the scale factor is $a \sim a(\text{Initial}) \times t^\gamma$, from early times. In doing so, we isolate out presuming an exceedingly small initial time step candidates initial time values which are from a polynomial for time values due to the Kiefer Density value. A gravity wave analysis concludes our article with inflaton decay.

Keywords: Minimum scale factor, cosmological constant, space-time bubble, Arrow of time

1. Introduction

Our initial goal is to obtain, via a Kieffer Density function candidate minimum time steps which will be for the purpose of giving input into an uncertainty principle of the form [1][2][3]

$$\Delta E \Delta t \approx 4\hbar \quad (1)$$

Whereas our candidate [4] for a density matrix uses

$$H^2 = V_0 \exp\left(-\sqrt{\frac{16\pi G}{\nu}}\phi\right) = V_0 \cdot \left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t\right)^{\frac{1}{2} \cdot \sqrt{\frac{\nu}{\pi G}} - 4 \sqrt{\frac{\pi G}{\nu}}} \quad (2)$$

Where [5][6] in turn is referenced directly to having use of the following

$$\begin{aligned} a(t) &= a_{initial} t^\nu \\ \Rightarrow \phi &= \ln\left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t\right)^{\sqrt{\frac{\nu}{16\pi G}}} \\ \Rightarrow \dot{\phi} &= \sqrt{\frac{\nu}{4\pi G}} \cdot t^{-1} \\ \Rightarrow \frac{H^2}{\dot{\phi}} &\approx \sqrt{\frac{4\pi G}{\nu}} \cdot t \cdot T^4 \cdot \frac{(1.66)^2 \cdot g_*}{m_p^2} \approx 10^{-5} \end{aligned} \quad (3)$$

In doing all of this we are making full use of the following from [4] due to a one loop approximation

$$\rho(\phi, \phi) \approx \left\{ \exp \left(\frac{\pm 3M_P^4}{8 \cdot V(\phi)} \right) \right\} \cdot \phi^{-\tilde{Z}-2} \quad (4)$$

Which after we isolate out Δt makes use of Eq. (1), which is derived as given in [1][2][3]

$$\begin{aligned} \Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_P^2 \frac{\Delta E}{\hbar} &\Rightarrow (\Delta E)^2 - \frac{\hbar \Delta t}{\gamma t_P^2} (\Delta E) + \frac{\hbar^2}{\gamma t_P^2} = 0 \\ \Rightarrow \Delta E &= \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 + \sqrt{1 - \frac{4\hbar^2}{\gamma t_P^2 \cdot \left(\frac{\hbar \Delta t}{2\gamma t_P^2}\right)^2}} \right) = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \sqrt{1 - \frac{16\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2}} \right) \end{aligned} \quad (5)$$

$$\begin{aligned} \Delta E &\approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \left(1 - \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \right) \right) \\ \Rightarrow \Delta E &\approx \text{either } \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2}, \text{ or } \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(2 - \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \right) \end{aligned} \quad (6)$$

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \equiv \frac{4\hbar}{\Delta t} \quad (7)$$

We will be applying Eq. (4) to obtain Δt , and then from this step applying Eq.(1) to say foundational import issues of time flow in the beginning, as

it affects initial energy values and from there obtain some matters of observational import in GW astronomy

2. Understanding the import of Eq. (2), Eq. (3) and Eq. (4) for Δt

Our assumption is that time, t , which becomes Δt is extremely small. Hence without loss of generality we write, if as an example, $\tilde{z} \approx 2$

And we simplify time dependence by setting $v = \frac{3\pi}{\sqrt{2}}$ in Eq.(2), Eq. (3) and Eq.(4)

Then, without loss of generality, if we observe this, and set Θ as a probability density value of Eq. (4), we then have

$$\left(\frac{1}{4} \cdot \exp\left(\pm \frac{3}{8} \cdot \frac{M_P^4}{V_0} \right) \right) \cdot \left(1 - 4 \cdot \left(\frac{\sqrt{\frac{8\pi V_0}{\left(\frac{3\pi}{\sqrt{2}}\right) \cdot \left(\frac{9\pi}{\sqrt{2}} - 1\right)}}}{\left(\frac{3\pi}{\sqrt{2}}\right) \cdot \left(\frac{9\pi}{\sqrt{2}} - 1\right)}\right) \cdot \Delta t \right) \approx \Theta \quad (8)$$

If so, then we have a minimum time step of the form

$$\Delta t \approx 4 \cdot \left(\frac{\sqrt{\left(\frac{3\pi}{\sqrt{2}}\right) \cdot \left(\frac{9\pi}{\sqrt{2}} - 1\right)}}{8\pi V_0} \right) \cdot \left(1 - 4\Theta \exp\left(\pm \frac{3}{8} \cdot \frac{M_P^4}{V_0} \right) \right) \quad (9)$$

3. Interpreting Eq. (9) in terms of the affects it has on Eq. (1)

We have to consider what Θ may or may not be. The core of the derivation of Eq. (4) in [4] due to [7] is dependent on having the following, namely

Quote

The quantum gravitational scale of inflation is calculated by finding a sharp probability peak in the distribution function of chaotic inflationary cosmologies driven by a scalar field with large negative constant Ξ of nonminimal interaction. In the case of the no-boundary state of the universe this peak corresponds to the eternal inflation, while for the tunnelling quantum state it generates a standard inflationary scenario. The sub-Planckian parameters of this peak (the mean value of the corresponding Hubble constant $H \cong 10^{-5}mp$, its quantum width $\Delta H/H \cong 10^{-5}$ and the number of inflationary e-foldings $N \cong 60$) are found to be in good correspondence with the observational status of inflation theory, provided the coupling constants of the theory are constrained by a condition which is likely to be enforced by the (quasi) supersymmetric nature of the sub-Planckian particle physics model

End of quote

Notice here that this is akin to making use of Eq.(3.4) of [7] so that we have restrictions on particle manufacturing for the theory. This is in line with

$$\Delta E \approx \hbar \cdot \left(\frac{8\pi V_0}{\sqrt{\left(\frac{3\pi}{\sqrt{2}}\right) \cdot \left(\frac{9\pi}{\sqrt{2}} - 1\right)}} \right) \cdot \left(1 + 4\Theta \exp\left(\pm \frac{3}{8} \cdot \frac{M_P^4}{V_0}\right) \right) \quad (10)$$

The consequences for frequency of signals is as follows. First, we make an estimation as to the width of the wavefront of a DeBroglie wave, which

may be a consequence of a signal, as well as the position of the phenomenon of generation of say Gravitons. In doing so we wish to refer to the following as motivation in order to link this to graviton mass and other such concerning heavy gravity, In [8] we have the following, as to how to obtain the mass of an inflaton, namely use, if

$$m^2 = \left. \frac{d^2V}{d\phi^2} \right|_{\phi=0, t=3.9776 \cdot t_P} = \frac{16\pi}{\left(3\pi / \sqrt{2}\right)} \quad (11)$$

$$\Rightarrow m = 2.74635619187$$

IMO the inflaton mass is 2.746356 times Planck Mass, and this is a starting value of inflaton mass at $t = 3.9776$ Planck time

Having this value of inflaton mass should be compared with the value of energy density as given by

$$\rho \approx \frac{\dot{\phi}^2}{2} + V(\phi) \equiv \frac{\gamma}{8\pi G} \cdot t^2 + V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \quad (12)$$

Using the coefficient of the scale factor chosen, this above becomes simplified to become, if.

$$V_0 \approx M_P^4 \xrightarrow{\hbar=\ell_P=t_P=k_B=G=1} 1 \quad (12a)$$

This is a way to obtain the following value for density as can be seen below

$$\rho = \frac{3\sqrt{2}}{8} \cdot \left(\frac{(3.997 + \varpi)}{t_P \xrightarrow{\hbar=\ell_P=M_P=k_B=G=1} 1} \right)^{\frac{3\pi}{\sqrt{2}}} + \left(V_0 \approx M_P^4 \xrightarrow{\hbar=\ell_P=t_P=k_B=G=1} 1 \right) \quad (12b)$$

We then can look at how this will be larger than Planck energy where our starting point will be if we set ϖ close to zero, and then have, due to how close the time is to Planck time, a situation for which we are looking at nearly Planck length, cubed as a starting volume, then we have initially, having a near Planck sized initial volume, we have then

$$1 + \left(\frac{3\sqrt{2}}{8}\right) \cdot (3.997 + \varpi)^{\frac{3\pi}{\sqrt{2}}} \approx \sqrt{\frac{8\pi}{\left(\frac{3\pi}{\sqrt{2}}\right) \cdot \left(\frac{9\pi}{\sqrt{2}} - 1\right)}} \cdot \left(1 + 4\Theta \cdot \exp\left(\pm \frac{3}{8}\right)\right) \quad (13)$$

Then up to an initial round off error, in the beginning, we can have

$$\Theta \approx \frac{1}{4} \cdot \left[\exp\left(\pm \frac{3}{8}\right) \right] \cdot \{ \}$$

$$\{ \} = \left\{ -1 + \sqrt{\frac{\left(\frac{3\pi}{\sqrt{2}}\right) \cdot \left(\frac{9\pi}{\sqrt{2}} - 1\right)}{8\pi}} \cdot \left[1 + \left(\frac{3\sqrt{2}}{8}\right) \cdot (3.997 + \varpi)^{\frac{3\pi}{\sqrt{2}}} \right] \right\} \quad (14)$$

Θ is in the initial setting a way to bring up what Θ is as a probability density value, and this is the point to remember.

In **quantum mechanics**, a probability amplitude is a complex number used in describing the behaviour of systems. The modulus squared of this quantity represents a **probability density**. Note that the Interpretation of **values** of a wave function as the probability amplitude is a pillar of the Copenhagen interpretation of **quantum mechanics**

We are then reproducing in this method the idea of [9] namely that the following is true.

Probability Density of Particles. The **probability density** (or probably distribution) is given by taking the square of the absolute value of

the **wave function**. It gives us the likelihood of finding an electron (or some other system) at some given point in space.

4. Making use of the idea of a nonzero probability density of “masses” of some ‘particle within the Copenhagen interpretation of Quantum Mechanics, in the vicinity of the big bang

Here what we are going to say, is that due to a fluctuation in time, given by Eq. (9), that the probability density of finding, say traces of the inflaton, as given in Eq.(11) will be nonzero, and varying in ways which could be experimentally tested. Given a mass, m , as in Eq. (11) could be interpreted as being of value of effective space-time mass > 1 Planck mass, and if these are broken apart, in the matter of [10] for black holes, we could have, with the evolution of time a template for investigating the applicability of graviton masses being generated by black holes being broken up in the vicinity of a quantum bounce, with each quantum generated

$$m_g = \frac{\hbar\sqrt{\Lambda}}{c} \quad (15)$$

Whereas we have a thermality relationship which may be useful for analysis of the form given by

$$\frac{H^2}{\dot{\phi}} \approx 10^{-5} \quad (16)$$

And also the use of the following at the start of inflation, as given in [11] and [12] as well as the ideas given by Uptal Sarkar in [13] and [14]. We will also be examining if our construction will allow for the development of spin off of ideas given by Eq. (13) and (14)

$$H = 1.66\sqrt{g_*} \cdot \frac{T_{temp}^2}{m_P} \quad (17)$$

The term g_* can refer to the initial degrees of freedom and can go as high as 110, whereas Eq. (13) is a bound in the amount of inhomogeneity. Whereas our future research objective is to find a way to allow the idea of a nonzero probability density, to ascertain different values of say graviton production if we make use of having from Freeze [10] of a mechanism of breaking up initially speaking black holes, due to the following criteria as given by Freeze

$$\rho_{BH-breakup-density} = \frac{M_P^4}{32\pi} \cdot \left(\frac{M_P^4}{m^4} \right) \cdot \frac{1}{|1 + 3\omega_Q|} \quad (18)$$

This breakup of black holes by the physics so outlined may give us a way to ascertain if the following entropy, initially is verifiable experimentally, whereas we wish to examine in full ideas given in the series of multiple references [15],[16],[17],[18],[19],[20],[21],[22],[23],[24], and [25], starting with verification of

$$S \sim 3 \cdot [1.66\sqrt{g_*}]^2 T^3 \quad (19)$$

5. Conclusion: Examining the origins of density fluctuations, as given in this formalism

In doing this final part of the inquiry we are making use of [26] and in doing so we are looking at the following on page 299 of [26], namely

$$\hat{a} \cdot \ddot{\phi} + 3 \cdot \hat{a} \cdot H \cdot \dot{\phi} + \frac{dV}{d\phi}(\phi) + \left(\mathfrak{S}/\dot{\phi} \right) = 0 \quad (20)$$

Then, if one has a time ~ 10 times Planck time, one has

$$\begin{aligned} \mathfrak{J} &\approx \frac{2}{(10 \cdot t_P)} - \left(3 \cdot \left(\frac{3\pi}{\sqrt{2}} - 1 \right) \right) \cdot \left(\frac{\hat{a} = 15/2}{(10 \cdot t_P)^3} \right) \\ &\equiv \xrightarrow{t_P \rightarrow 1} \frac{2}{(10)} - \left(3 \cdot \left(\frac{3\pi}{\sqrt{2}} - 1 \right) \right) \cdot \left(\frac{15/2}{(10)^3} \right) > 0 \end{aligned} \quad (21)$$

\mathfrak{J} in this situation is a way of representing the energy drain created by the radiation of “particles” which couple to the field given by ϕ , i.e. this is the big news, namely that as early as a time value past the onset of the big bang, measured as about 10 times Planck time, i.e. roughly 10^{-42} seconds, we have an energy drain caused by something which is generated by the inflaton “mass” likely breaking up into radiated “particles” which are generated as the inflaton mass is created by Eq.(11) above

Furthermore, making use of the Equation which has \textcircled{H} we have a way of representing the likelihood of particles being in existence in a particular regime of space-time. Taking into account having time t in between 4 to 10 times Planck time, we can state that some version of the inflaton itself would exist in some order of magnitude of say 10 to 100 times Planck length, from the view point of [27], namely

Quote

We propose that there may be a substantial stochastic gravitational wave (GW) background from particle origin, mainly from the gravitational three-body decay of the inflaton. The emitted gravitons could constitute a sizable contribution to dark radiation if the mass of inflaton is close to the Planck scale, which may be probed by future CMB experiments that have a sensitivity on the deviation of the effective number of neutrinos in the standard cosmology, $\delta N_{\text{eff}} \sim 0.02 - 0.03$. We have also illustrated the spectrum of the radiated gravitational waves, in comparison to the current and future experiments, and found that GWs from

particle origin could be the dominant contribution to the energy density at high-frequency domain, but beyond the sensitivity regions of various GW experiments in the near future

End of quote

The frequency range could be as high, initially as 10^{27} GHz based upon the extremely small regime of space-time given in the initial creation of an inflaton which would be by 60 or more e folds [28] down graded in the present era down to about 1 to 10 GHz, from its initial configuration

Keep in mind that the inflaton, as given in Eq. (11) is over 2 times Planck mass, which is by Reference [27] more than enough to insure that there is a GW signal from inflaton decay as initiated by the procedures given in [27]. The e fold values could be as low as 50 e folds, if we literally interpret [28], whereas what we are considering is a signal which may be so created. Keep in mind that the differential time flow, as specified in this problem for reasons we will investigate in future publications may make the difference between 10 GHz and 1 GHz relic inflaton decay generated GW. i.e. the fact is, that this possibility cannot be dismissed out of hand until we obtain data sets. Also keep in mind what was brought up in [28]

Quote

If future cosmological observations point to a particular inflaton potential for which our Universe is not generic under the canonical measure, that would shed light on even higher-scale physics. Such a circumstance would tell us that our Universe is tuned — on a non-generic trajectory — from the point of view of the classical measure. This would indicate the importance of intrinsically quantum gravitational processes or some ultimate theory of initial conditions.

End of quote

We hope that our discussion of a density functional as given in Eq. (14) may be a step in this direction

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