Growth of Linear Perturbations in a Universe with Superfluid Dark Matter

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Talk presented at 16th Marcel Grossmann Meeting, 2021

Motivation

- Standard CDM (Cold Dark Matter) explains cosmic history and rotation curves in galaxies, ***but has other problems at galactic scales like Baryonic Tully-Fisher relation, cusp-core problem, too big to fail problem,....***
- MOND (MOdified Newtonian Dynamics) can explain galactic dynamics, ***but fails to explain CMB observations-Proper relativistic extension missing*****.**
- We want to work with a theory that can address the issues of both CDM and MOND, while being successful both at cosmological scales and small scales**-** ***CDM-MOND hybrids***.
- This class of models take into consideration the ***interacting dark matter-baryon picture*** where a MOND-like force is mediated through this new interaction term.

Dark Matter (DM) as superfluids: Overview

[L. Berezhiani and J. Khoury,Phys. Rev.D, 2015; L. Berezhiani, et al., JCAP, 2018]

- ☐ The dark matter is microscopically made up of ***very light axion-like particles***.
- ☐ ***Undergoes a phase transition*** at some specific scale (defined by the temperature) and forms superfluid.
- ☐ At galactic scales, owing to the superfluid behaviour, ***DM can be described as collective excitations or phonons***, rather than individual particles.
- ☐ ***The phonon modes mediate a MONDian force*** (Modified Newtonian force) between visible matter by coupling to baryons.
- ☐ ***In clusters, DM consists of mostly normal fluid*** compared to superfluid. Hence the DM is best described as particles.

Phonon Effective Theory of DM superfluid (SDM)

In the non-relativistic regime, the effective theory of superfluid phonons is described by a scalar field θ.

- The Lagrangian is of the form:

$$\mathscr{L} = P(X) \tag{1}$$

where $X = \dot{\theta} - m\Phi - \frac{(\nabla\theta)^2}{2m}$

m : mass of the particle, Φ : gravitational potential

In this model,

$$P(X) = \frac{2\Lambda(2m)^{3/2}}{3} X\sqrt{|X|} \tag{2}$$

The interaction Lagrangian is given by,

$$\mathscr{L}_{int} = -\alpha \frac{\Lambda}{M_{pl}} \theta \rho_b \tag{3}$$

Motivation behind the choice of the Lagrangian

- θ ***is the phase of the wavefunction*** describing the superfluid phonon modes; Φ ***is the gravitational potential*** in which the DM particle sits.
- ***The power*** of the Lagrangian is defined by ***the choice of the equation of state*** (EoS), and a fractional power of 5/2 is indeed obtained in superfluids formed by ultra cold atoms.
- In the case of CDM superfluid, the choice of ***the power*** 3/2 in the Lagrangian is somewhat arbitrary, but ***motivated by the fact that the superfluid DM should give rise to MOND-like dynamics*** at galactic scales when baryons are also included.
- Interaction term ensures a MOND force.
- There are ***three free parameters*** of the model: m, Λ (range) and α (coupling strength).

Background Cosmology

Define an equation of state of the condensate:

$$w = \frac{P}{\rho} = \frac{\rho^2}{12\Lambda_0^2 m^6} \tag{4}$$

For $\rho \ll \Lambda_0 m^3$, we have $w \ll 1$ effectively showing that DM superfluid behaves as dust.

Assuming this condition to be satisfied at matter-radiation equality when $\rho_{eq} \approx 0.4eV^4$, we get a lower bound on Λ_0

$$\Lambda_0 \gg 0.1 \left(\frac{m}{eV}\right)^{-3} \quad eV \tag{5}$$

For $m = 0.6$ eV, we get $\Lambda_0 \gg 0.5$ eV

Evolution of density

Considering $\theta = \theta(t)$, the phonon action can be written as,

$$\mathscr{L} = \frac{2\Lambda_0(2m)^{3/2}}{3} a^3 \dot{\theta}^{3/2} - \alpha_0 \frac{\Lambda_0}{M_{pl}} a^3 \theta \rho_b \tag{6}$$

The resulting equation of motion for θ has a solution,

$$(2m)^{3/2} \dot{\theta}^{1/2} = -(\alpha_0/M_{Pl})\rho_b t + C/a^3 \tag{7}$$

In the non-relativistic approx., the density is given by
$\rho = mn = m\Lambda_0(2m)^{3/2}\dot{\theta}^{1/2}$
This gives,

$$\rho = -(\alpha_0\Lambda_0/M_{Pl})mt\rho_b + \rho_{dust} \tag{8}$$

Cosmological bound on α_0

If the superfluid has to behave as ordinary dust, at present time, we need

$$(\alpha_0 \Lambda_0 / M_{Pl}) m t_0 (\rho_b / \rho_{dust}) \leq 1 \tag{9}$$

Considering $\rho_b/\rho_{dust} = 1/6$, $m = 0.6$ eV and $\Lambda_0 \gg 0.5$ eV, we get an upper bound on α_0

$$\alpha \ll 2.4 \times 10^{-4} \left(\frac{m}{\mathrm{eV}} \right)^2 \tag{10}$$

Cosmological Perturbations

Lagrangian in an FLRW matter dominated universe-

$$\mathscr{L} = \frac{2\Lambda(2m)^{3/2}}{3}\left(\dot{\theta} - \frac{(\nabla\theta)^2}{2m} - m\Phi\right)^{3/2} - \alpha\frac{\Lambda}{M_{Pl}}\rho_b\theta \tag{11}$$

Hamiltonian for interacting superfluid and baryonic matter-

$$H = \frac{\Pi_\theta^3}{3\Lambda^2(2m)^3} + \left(m\Phi + \frac{(\nabla\theta)^2}{2m}\right)\Pi_\theta + \frac{\alpha\Lambda}{M_{pl}}\rho_b\theta \tag{12}$$

Hamiltonian Eqs. Motion:

$$\dot{\theta} = \frac{\Pi_\theta^2}{\Lambda^2(2m)^3} + m\Phi + \frac{(\nabla\theta)^2}{2m}; \quad \dot{\Pi}_\theta = \frac{1}{m}\nabla\cdot(\Pi_\theta\nabla\theta) - \frac{\alpha\Lambda}{M_{pl}}\rho_b \tag{13}$$

Fluid Equations

We identify the terms as corresponding hydro-dynamical variables

$$\rho_m = m\Pi_\theta, \quad \vec{u} = -\frac{\nabla\theta}{m}. \tag{14}$$

The potential and four velocity can be split in two parts-

$$\Phi = -\frac{1}{2}(\dot{H} + H^2)l^2 + \phi; \quad \vec{u} = H\vec{l} + \vec{v} \tag{15}$$

Two fluid Eqs:

$$\dot{\rho}_m + 3H\rho_m + \frac{1}{a}\nabla\cdot(\rho_m\vec{v}) = -\frac{\alpha\Lambda m}{M_{pl}}\rho_b \tag{16}$$

$$\dot{\vec{v}} + H\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\vec{v} = -\frac{\rho_m\nabla\rho_m}{4a\Lambda^2m^6} - \frac{\nabla\phi}{a} \tag{17}$$

Evolution of Perturbations

[S. Banerjee, S. Bera, D. F. Mota, JCAP, 2020]

Consider perturbations around homogeneous solution-

$$\theta(x,t) = \bar{\theta}(t) + \delta\theta(x,t); \quad \rho_m = \bar{\rho}_m + \delta\rho_m; \quad \rho_b = \bar{\rho}_b + \delta\rho_b$$

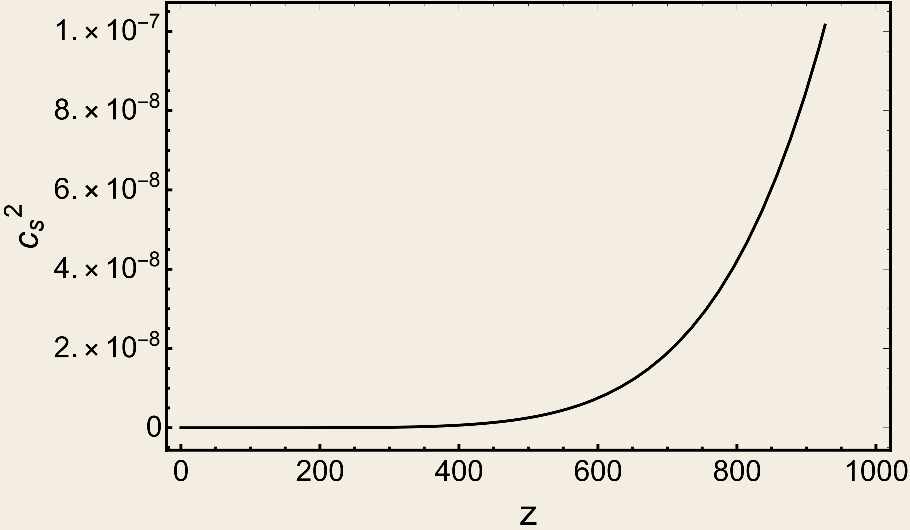
The evolution Eq:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{\bar{\rho}_m\delta_m}{2M_{pl}^2} - \frac{c_s^2\nabla^2\delta_m}{a^2} = -\frac{\alpha\Lambda m\dot{\delta}_b}{M_{pl}} - 2H\frac{\alpha\Lambda m\delta_b}{M_{pl}} + \frac{\bar{\rho}_b\delta_b}{2M_{pl}^2} \quad (18)$$

where $\delta_i = \rho_i/\bar{\rho}$ for $i = m,\ b$ and the sound speed is given by

$$c_s^2 = \frac{\bar{\rho}_m^2}{4\Lambda^2 m^6} \quad (19)$$

Sound Speed c_s^2 for $m = 1$ eV and $\Lambda = 500$ eV



Analytical Solutions

□ **Without Baryons**: setting $\bar{\rho}_b = 0$ and $\delta\rho_b = 0$ with $c_s^2 \ll 1$:-

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{\bar{\rho}_m \delta_m}{2M_{pl}^2} = 0 \tag{20}$$

• evolution equation of non-relativistic CDM in the ΛCDM model with **growing solution** $\delta \propto a$ for a matter dominated universe.

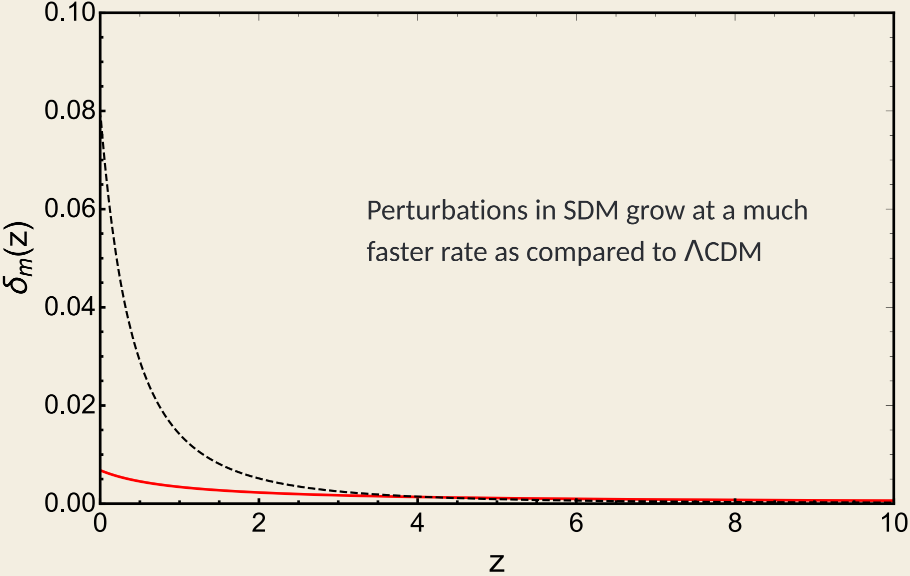
• **SDM thus behaves exactly like CDM in the absence of baryons**.

□ **With Baryons**: The time evolution of the DM perturbation growth is captured by the following forms:

$$\delta_m \propto a \quad \text{for no interaction} \tag{21}$$

$$\delta_m \propto a^{5/2} \quad \text{for strong interaction} \tag{22}$$

$\delta_m(z)$ for $m = 1\,\text{eV}$, $\Lambda = 500\,\text{eV}, \alpha = 10^{-6}$



Numerical Solutions

Considering the following Equations:

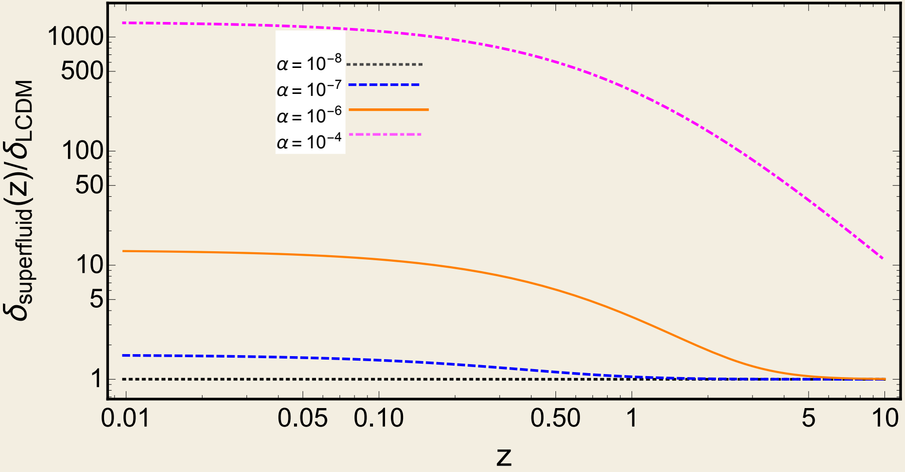
$$-H(1+z)\frac{d\delta_m}{dz}+\frac{\bar{\rho}_m}{\bar{\rho}}(ikv)=-\frac{\alpha\Lambda m}{M_{pl}}\delta_b \tag{23}$$

$$-ikH(1+z)\frac{dv}{dz}+ikHv=\frac{k^2\bar{\rho}_m\bar{\rho}\delta_m}{4\Lambda^2m^6}+\frac{\bar{\rho}}{2M_{pl}^2(1+z)^2}\left(\delta_m+\delta_b+\frac{3iHv}{k(1+z)}\right) \tag{24}$$

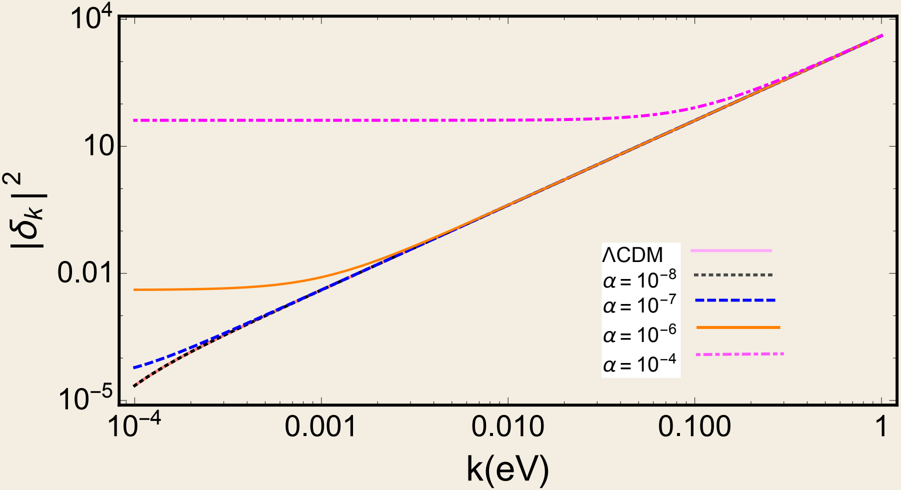
- **Parameters and Initial Conditions**:

$m=1$ eV and $\Lambda=500$ eV, epoch of equality $z=1000$.
We set $\delta_b(z=1000)=\delta_m(z=1000)=10^{-5}$ and
$H(z=1000)=m=1$ eV.
Since $\bar{\rho_m}\gg\bar{\rho}_b$, we assume $\bar{\rho}=\bar{\rho}_m+\bar{\rho}_b\approx\bar{\rho}_m$, $\bar{\rho}_m\propto 1/a^3$ and
$H\propto 1/a^{3/2}$. Furthermore, we take $\delta_b\propto a$. We keep k fixed at 0.0001 eV.

Evolution of $\delta_{SDM}(z)/\delta_{LCDM}$



Matter Power Spectrum



Results:

$\triangle$ The plot for $\alpha = 10^{-8}$ coincides with ΛCDM (deviation from ΛCDM is 0.13% at $z = 0.01$). When α is large enough, the growth is very steep-ensuring that structure formation takes place at an earlier epoch as compared to ΛCDM.

$\triangle$ This is different from the value quoted in Khoury et al. The bound obtained in Khoury et al, for $m = 1$ eV, is $\alpha \leq 10^{-4}$, which, strongly deviates from ΛCDM in the context of perturbation growth.

$\triangle$ As α is increased to 10^{-7}, the deviation from ΛCDM becomes much larger ($\sim$ 62%).

$\triangle$ The power spectrum plot reaffirms the previous results. In general the power increase approximately linearly with k. However, for large enough α, it becomes relatively flatter.

Ongoing Works and Future Directions

- Constraining with CMB observations
- A thorough study of the gravitational lensing
- Effect on the frequency/velocity of the gravitational waves passing through a superfluid medium-more stringent bounds on SDM parameters

Thank you for your attention!!!