Interacting dark sector in the late Universe: Mapping fields and fluids, and observational signatures

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in collaboration with

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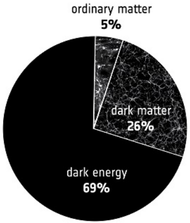
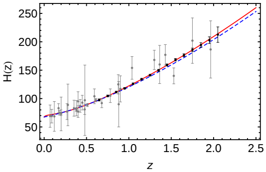
Dark sector in the late universe



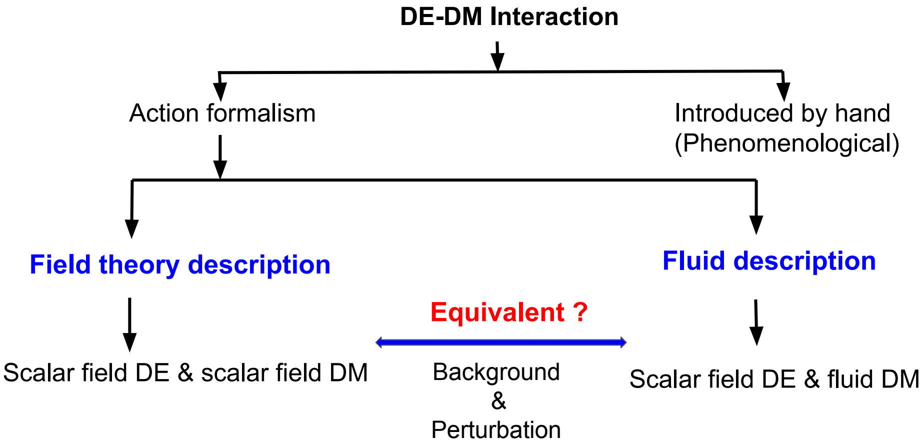


- Energy budget of the late universe is dominated by the dark energy and dark matter ($\sim 95\%$).
- Interaction between the dark sector and ordinary matter is purely gravitational.
- Current observations does not rule out the interaction between dark matter and dark energy.

- Equivalent fluid and field theory descriptions of interacting dark sector.
- Can interacting dark sector alleviate H_0 tension ?
- Potential observational signatures of interacting dark sector.

Dark energy - dark matter interaction

$$\nabla^{\mu} T_{\mu\nu}^{(\mathrm{DE,DM})} = Q_{\nu}^{(\mathrm{DE,DM})}, \nabla^{\mu} T_{\mu_{\nu}}^{(tot)} = \quad Q_{\nu}^{(\mathrm{DE})} + Q_{\nu}^{(\mathrm{DM})} = 0$$



$f(\tilde{R}, \tilde{\chi})$ model $\rightarrow$ interacting scalar fields

[JPJ & SS-2020]

- We consider the following action

$$S_J = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}f(\tilde{R},\tilde{\chi}) - \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\chi}\tilde{\nabla}_\nu\tilde{\chi} - V(\tilde{\chi})\right]$$

- Do a conformal transformation

$$g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}, \quad \text{where} \quad \Omega^2 = F(\tilde{R},\tilde{\chi}) \equiv \frac{\partial f(\tilde{R},\tilde{\chi})}{\partial\tilde{R}}$$

- Redefine the scalar fields

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - U(\phi)\right.$$
$$\left. - \frac{1}{2}e^{2\alpha(\phi)}g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - e^{4\alpha(\phi)}V(\chi)\right).$$

Evolution of the scalar fields

- Dark energy represented by ϕ; dark matter represented by χ.

- Equations of motion for the scalar fields

$$-\nabla^{\mu}\nabla_{\mu}\chi - 2\alpha_{,\phi}(\phi)\nabla_{\mu}\phi\nabla^{\mu}\chi + e^{2\alpha(\phi)}V_{,\chi}(\chi) = 0$$

$$-\nabla^{\mu}\nabla_{\mu}\phi + 4e^{4\alpha}\alpha_{,\phi}(\phi)V(\chi) + e^{2\alpha}\alpha_{,\phi}(\phi)\nabla^{\mu}\chi\nabla_{\mu}\chi + U_{,\phi}(\phi) = 0$$

- Compare these equations with the non-interacting scenario to obtain the interaction function.

$$Q_{\nu}^{(\mathrm{F})} = \nabla_{\mu}T_{\nu}^{(m)\mu} = -e^{2\alpha(\phi)}\alpha_{,\phi}(\phi)\nabla_{\nu}\phi\left[\nabla^{\sigma}\chi\nabla_{\sigma}\chi + 4e^{2\alpha(\phi)}V(\chi)\right]$$

Fluid description of the interacting dark sector

[JPJ & SS-2020]

- Define the energy density ρ_m, pressure p_m and four velocity u_μ of the dark matter fluid as

$$\rho_m = -\frac{1}{2}e^{2\alpha}\left[g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - e^{2\alpha}V(\chi)\right], \quad p_m = -\frac{1}{2}e^{2\alpha}\left[g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi + e^{2\alpha}V(\chi)\right]$$

$$u_\mu = -\left[-g^{\alpha\beta}\nabla_\alpha\chi\nabla_\beta\chi\right]^{-\frac{1}{2}}\nabla_\mu\chi$$

- Energy momentus tensor $T^{(m)}_{\mu\nu} = p_m g_{\mu\nu} + (\rho_m + p_m)u_\mu u_\nu$
- Interaction term

$$\begin{aligned} Q^{(\mathrm{F})}_\nu = \nabla_\mu T^{(m)\mu}_\nu &= -e^{2\alpha(\phi)}\alpha_{,\phi}(\phi)\nabla_\nu\phi\left[\nabla^\sigma\chi\nabla_\sigma\chi + 4e^{2\alpha(\phi)}V(\chi)\right] \\ &= -\alpha_{,\phi}(\phi)\nabla_\nu\phi(\rho_m - 3p_m) = T^{(m)}\nabla_\nu\alpha(\phi) \end{aligned}$$

Interacting dark sector in literature

[JPJ & SS-2020]

Interacting DE-DM model	DE-DM Interaction $\nabla^{\mu} T_{\mu\nu}^{(\mathrm{DE,DM})} = Q_{\nu}^{(\mathrm{DE,DM})}$	Is $Q_\nu \propto Q_\nu^{(\mathbf{F})}$?
Amendola - 1999 [32]	$\dot{\rho}_m + 3H\rho_m = -C\rho_m\dot{\phi}$	Yes
Amendola - 1999 [33]	$\dot{\rho}_m + 3H\rho_m = -C\rho_m\dot{\phi}$	Yes
Billyard & Coley -1999 [34]	$\dot{\phi}(\ddot{\phi} + 3H\dot{\phi} + kV) = \frac{(4-3\gamma)}{2\sqrt{\omega+\frac{3}{2}}}\dot{\phi}\mu$	Yes
Olivares.etal - 2005 [35]	$\frac{d\rho_c}{dt} + 3H\rho_c = 3Hc^2\left(\rho_c + \rho_x\right)$	No
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Boehmer.etal - 2008 [38]	$\dot{\rho}_c + 3H\rho_c = -\sqrt{2/3}\kappa\beta\rho_c\dot{\varphi}$	Yes
	$\dot{\rho}_c + 3H\rho_c = -\alpha H\rho_c$	No
Caldera-Cabral.etal - 2008 [39]	$\dot{\rho}_c = -3H\rho_c + 3H\left(\alpha_x\rho_x + \alpha_c\rho_c\right)$	No
	$\dot{\rho}_c = -3H\rho_c + 3\left(\Gamma_x\rho_x + \Gamma_c\rho_c\right)$	No

Cosmological evolution with interacting dark sector

Background evolution

- Evolution in the field theory description

$$\begin{aligned} \ddot{\overline{\chi}} + 3H\dot{\overline{\chi}} + e^{2\alpha}V_{,\chi}(\overline{\chi}) + 2\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}\dot{\overline{\chi}} &= 0 \\ \ddot{\overline{\phi}} + 3H\dot{\overline{\phi}} + U_{,\phi}(\overline{\phi}) + 4e^{4\alpha}\alpha_{,\phi}(\overline{\phi})V(\overline{\chi}) - e^{2\alpha}\alpha_{,\phi}(\overline{\phi})\dot{\overline{\chi}}^2 &= 0\,. \end{aligned}$$

- Evolution in the fluid description

$$\ddot{\overline{\phi}}\dot{\overline{\phi}} + 3H\dot{\overline{\phi}}^2 + U_{,\phi}(\overline{\phi})\dot{\overline{\phi}} = \overline{Q}^{(\mathrm{F})}, \quad \dot{\overline{\rho}}_m + 3H(\overline{\rho}_m + \overline{p}_m) = -\overline{Q}^{(\mathrm{F})}\,.$$

- Interaction term

$$\overline{Q}^{(\mathrm{F})} = -\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}(\overline{\rho}_m - 3\overline{p}_m) = \alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}e^{2\alpha(\overline{\phi})}\left[\dot{\overline{\chi}}^2 - 4e^{2\alpha}V(\overline{\chi})\right].$$

- Matter energy density $\overline{\rho}_m = \overline{\rho}_{m_0}a^{-3(1+\omega_m)}e^{[\alpha(\overline{\phi})-\alpha_0](1-3\omega_m)}$

Evolution of first order perturbations : Fluid description

- Evolution of dark energy scalar field and dark matter fluid

$$\delta\dot{\rho}_m + 3H(\delta p_m + \delta\rho_m) + (\overline{p}_m + \overline{\rho}_m)\left[\frac{\nabla^2 \delta u^s}{a^2} - 3\dot{\Psi}\right] = -\delta Q$$

$$\dot{\overline{\phi}}\left(\delta\ddot{\phi} - \frac{\nabla^2\delta\phi}{a^2} - 2\Phi\ddot{\overline{\phi}} + U_{,\phi\phi}(\overline{\phi})\delta\phi\right) + \delta\dot{\phi}\left(\ddot{\overline{\phi}} + 6H\dot{\overline{\phi}} + U_{,\phi}(\overline{\phi})\right) - \frac{\dot{\overline{\phi}}^2}{2}\left(3\dot{\Psi} + \dot{\Phi} + 6H\Phi\right) = \delta Q,$$

- where

$$\delta Q^{(\mathrm{F})} = -(\delta\rho_m - 3\delta p_m)\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}} - (\overline{\rho}_m - 3\overline{p}_m)\left[\alpha_{,\phi\phi}(\overline{\phi})\dot{\overline{\phi}}\delta\phi + \alpha_{,\phi}(\overline{\phi})\delta\dot{\phi}\right]$$

Evolution of observables

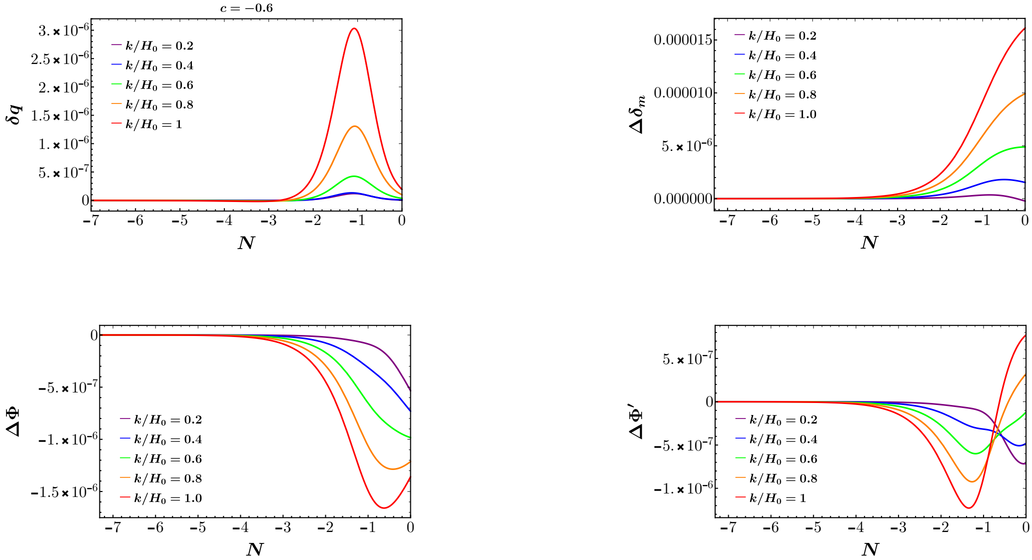
[JPJ.etal-2021]

- Three perturbed quantities related to:
  1. Structure formation: δ_m
  2. Weak gravitational lensing: $\Phi + \Psi$
  3. Integrated Sachs Wolfe effect: $\Phi' + \Psi'$

- Evolve the first order perturbations in the range $0 < z < 1500$

- Study the effect of DE-DM interaction on the evolution of perturbed quantities at different length scales

$$\Delta\delta_m = \delta_{m_i} - \delta_{m_{ni}}, \quad \Delta\Phi = \Phi_i - \Phi_{ni}, \quad \Delta\Phi' = \Phi'_i - \Phi'_{ni}$$



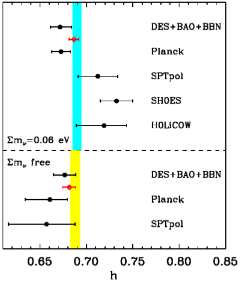
c = −0.6
δq
3.×10^-6
2.5×10^-6
2.×10^-6
1.5×10^-6
1.×10^-6
5.×10^-7
0
k/H0 = 0.2
k/H0 = 0.4
k/H0 = 0.6
k/H0 = 0.8
k/H0 = 1
−7 −6 −5 −4 −3 −2 −1 0
N
Δδm
0.000015
0.000010
5.×10^-6
0.000000
k/H0 = 0.2
k/H0 = 0.4
k/H0 = 0.6
k/H0 = 0.8
k/H0 = 1.0
−7 −6 −5 −4 −3 −2 −1 0
N
ΔΦ
0
−5.×10^-7
−1.×10^-6
−1.5×10^-6
k/H0 = 0.2
k/H0 = 0.4
k/H0 = 0.6
k/H0 = 0.8
k/H0 = 1.0
−7 −6 −5 −4 −3 −2 −1 0
N
ΔΦ′
5.×10^-7
0
−5.×10^-7
−1.×10^-6
k/H0 = 0.2
k/H0 = 0.4
k/H0 = 0.6
k/H0 = 0.8
k/H0 = 1
−7 −6 −5 −4 −3 −2 −1 0
N

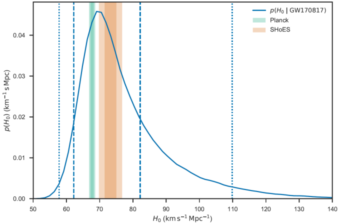
Key points

- Interacting dark sector can be obtained from a classical field theory action.
- Field theory action can be obtained from $f(\tilde{R}, \tilde{\chi})$ via conformal transformation.
- Energy-momentum of individual components of dark sector is not conserved.
- There is a one to one mapping between field theory description and fluid description of interacting dark sector.
- This mapping exists for a unique interaction term $Q^{(F)}$.
- Cosmological observations related to the LSS, Lensing and ISW effect can potentially detect the signatures of dark matter - dark energy interaction.

Thank You

H_0 tension





- Current value of Hubble parameter(H_0) can be estimated from various cosmological measurements.
- $H_0^{SNe} > H_0^{GW} > H_0^{CMB}$
- Resolving H_0 tension
  - Rectify the observational errors.
  - Self-interacting neutrinos.
  - Extension to ΛCDM model.
  - Dark energy - dark matter interaction

Fig Source : DES 2017, LIGO 2017

Autonomous system for the background evolution

[JPJ & SS-2020, JPJ.etal-2020]

- Define the dimensionless variables

$$x = \sqrt{\frac{C_1}{6}} \frac{\dot{\phi}}{H M_{Pl}}, \quad y = \sqrt{\frac{C_1}{3}} \frac{\sqrt{U}}{H M_{Pl}}$$

$$\lambda = -\frac{M_{Pl}}{\sqrt{C_1}} \frac{U_{,\phi}}{U}, \quad \Gamma = \frac{U U_{,\phi\phi}}{U_{,\phi}^2}$$

$$\alpha = \alpha(\phi), \quad \beta = -\frac{M_{Pl}}{\sqrt{C_1}} \frac{\alpha_{,\phi}}{\alpha}, \quad \gamma = \frac{\alpha \alpha_{,\phi\phi}}{\alpha_{,\phi}^2}$$

- These parameters are evolved as functions of number of e-foldings

$$N \equiv \ln a = \ln \left(\frac{1}{1+z} \right)$$

more details

Stability of background solution

Fixed point	x^*	y^*	Ω_r^*	Ω_ϕ^*	Ω_m^*	λ^*	α^*	β^*	ϵ^*
2a	0	0	1	0	0	-	-	-	2
2b	0	0	0	0	1	-	0	-	1.5
2c	0	0	0	0	1	-	-	0	1.5
2d	0	1	0	1	0	0	-	-	0
2e	-1	0	0	1	0	0	-	0	3
2f	0	0	0	0	1	0	-	0	1.5
2g	0	0	1	0	0	0	-	0	2
2h	0	1	0	1	0	0	-	0	0
2i	1	0	0	1	0	0	-	0	3

Radiation dominated saddle point$\rightarrow$ Matter dominated saddle point$\rightarrow$ Accelerated attractor

Interacting dark sector : mapping fields to fluids

Dark sector in the late universe

- Energy budget of the late universe is dominated by dark energy and dark matter.
- Interaction between the dark sector and ordinary matter is purely gravitational.
- Current observations do not rule out the interaction between dark matter and dark energy.
- Can interacting dark sector alleviate H_0 tension ?
- Constraining the interaction between dark energy and dark matter.

Dark energy-dark matter interaction

[JPJ & SS - 2019]

- Total energy of the universe is conserved. $\nabla^\mu T^{(tot)}_{\mu\nu} = 0$
- Energy corresponding to dark matter and dark matter are not conserved separately $\nabla^\mu T^{(\mathrm{DE,DM})}_{\mu\nu} = Q^{(\mathrm{DE,DM})}_\nu, \quad Q^{(\mathrm{DE})}_\nu + Q^{(\mathrm{DM})}_\nu = 0$
- Possible options
  - Phenomenological interaction term introduced by hand.
  - $f(R, \mathcal{X})$ gravity
  - Interacting scalar fields
    - Derived from an action
    - Can describe a variety of models
    - No ambiguity in the perturbative analysis

$f(\tilde{R}, \tilde{\chi})$ model $\to$ interacting scalar fields

[JPJ & SS-2020]

- We consider the following action

$$S_J = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2} f(\tilde{R}, \tilde{\chi}) - \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\chi} \tilde{\nabla}_\nu \tilde{\chi} - V(\tilde{\chi}) \right]$$

- Do a conformal transformation

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}, \quad \text{where} \quad \Omega^2 = F(\tilde{R}, \tilde{\chi}) \equiv \frac{\partial f(\tilde{R}, \tilde{\chi})}{\partial \tilde{R}}$$

- Redefine the scalar fields

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - U(\phi) \right.$$
$$\left. - \frac{1}{2} e^{2\alpha(\phi)} g^{\mu\nu} \nabla_\mu \chi \nabla_\nu \chi - e^{4\alpha(\phi)} V(\chi) \right).$$

Cosmological evolution

- Perturbed FLRW metric in synchronous gauge

$$g_{00} = -1, \quad g_{0i} = 0, \quad g_{ij} = a^2 \left[(1 + A)\delta_{ij} + \frac{\partial^2 B}{\partial x^i \partial x^j} \right].$$

$$a \equiv a(t), \quad H = \frac{\dot{a}}{a}, \quad A \equiv A(t, x, y, z), \quad B \equiv B(t, x, y, z)$$

- Perturbations of the scalar fields, dark matter fluid and the interaction term.

$$\phi = \overline{\phi} + \delta\phi, \quad \chi = \overline{\chi} + \delta\chi, \quad \rho_m = \overline{\rho}_m + \delta\rho_m, \quad p_m = \overline{p}_m + \delta p_m$$

$$Q_\nu = \overline{Q}_\nu + \delta Q_\nu$$

- Four-velocity of dark matter fluid

$$u_\mu = \overline{u}_\mu + \delta u_\mu, \quad \overline{u}_0 = -1, \quad \delta u_0 = 0, \quad \overline{u}_i = 0$$

$$\delta u_i = \frac{\partial \delta u^s}{\partial x^i}, \quad \delta u^s = -\frac{\delta\chi}{\dot{\overline{\chi}}}$$

Background evolution

- Evolution in the field theory description

$$\begin{aligned}
\ddot{\overline{\chi}} + 3H\dot{\overline{\chi}} + e^{2\alpha}V_{,\chi}(\overline{\chi}) + 2\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}\dot{\overline{\chi}} &= 0 \\
\ddot{\overline{\phi}} + 3H\dot{\overline{\phi}} + U_{,\phi}(\overline{\phi}) + 4e^{4\alpha}\alpha_{,\phi}(\overline{\phi})V(\overline{\chi}) - e^{2\alpha}\alpha_{,\phi}(\overline{\phi})\dot{\overline{\chi}}^2 &= 0\,.
\end{aligned}$$

- Evolution in the fluid description

$$\begin{aligned}
\ddot{\overline{\phi}}\dot{\overline{\phi}} + 3H\dot{\overline{\phi}}^2 + U_{,\phi}(\overline{\phi})\dot{\overline{\phi}} &= \overline{Q}^{(\mathrm{F})} \\
\dot{\overline{\rho}}_m + 3H(\overline{\rho}_m + \overline{p}_m) &= -\overline{Q}^{(\mathrm{F})}\,.
\end{aligned}$$

- Interaction term

$$\overline{Q}^{(\mathrm{F})} = -\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}(\overline{\rho}_m - 3\overline{p}_m) = \alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}e^{2\alpha(\overline{\phi})}\left[\dot{\overline{\chi}}^2 - 4e^{2\alpha}V(\overline{\chi})\right].$$

Evolution of first order perturbations : Fluid description

- Evolution of dark energy scalar field and dark matter fluid

$$
\begin{aligned}
\delta\dot{\rho}_m + 3H(\delta p_m + \delta\rho_m) & \\
+(\overline{p}_m + \overline{\rho}_m)\left[\frac{\nabla^2\delta u^s}{a^2} + \frac{3}{2}\dot{A} + \frac{\nabla^2\dot{B}}{2}\right] &= -\delta Q^{(\mathrm{F})} \\
\dot{\overline{\phi}}\left(\delta\ddot{\phi} - \frac{\nabla^2\delta\phi}{a^2} + U_{,\phi\phi}(\overline{\phi})\delta\phi\right) & \\
+\delta\dot{\phi}\left(\ddot{\overline{\phi}} + 6H\dot{\overline{\phi}} + U_{,\phi}(\overline{\phi})\right) + \frac{\dot{\overline{\phi}}^2}{2}\left(\nabla^2\dot{B} + 3\dot{A}\right) &= \delta Q^{(\mathrm{F})} .
\end{aligned}
$$

Evolution of first order perturbations : Field theory description

- Evolution of the scalar field perturbations

$$\delta\ddot{\chi} - \frac{\nabla^2\delta\chi}{a^2} + e^{2\alpha}V_{,\chi\chi}(\overline{\chi})\delta\chi + \frac{\dot{\overline{\chi}}}{2}\left(\nabla^2\dot{B} + 3\dot{A}\right) + 3H\delta\dot{\chi}$$
$$+2\alpha_{,\phi}(\overline{\phi})\left(\dot{\overline{\phi}}\delta\dot{\chi} + \dot{\overline{\chi}}\delta\dot{\phi}\right) + 2\delta\phi\left[\dot{\overline{\phi}}\dot{\overline{\chi}}\alpha_{,\phi\phi}(\overline{\phi}) + e^{2\alpha}\alpha_{,\phi}(\overline{\phi})V_{,\chi}(\overline{\chi})\right] = 0$$

$$\delta\ddot{\phi} - \frac{\nabla^2\delta\phi}{a^2} + U_{,\phi\phi}(\overline{\phi})\delta\phi + 2e^{2\alpha}\alpha_{,\phi}(\overline{\phi})^2\delta\phi\left[8e^{2\alpha}V(\overline{\chi}) - \dot{\overline{\chi}}^2\right]$$
$$+2e^{2\alpha}\alpha_{,\phi}(\overline{\phi})\left[2e^{2\alpha}V_{,\chi}(\overline{\chi})\delta\chi - \dot{\overline{\chi}}\delta\dot{\chi}\right] + \frac{\dot{\overline{\phi}}}{2}\left(\nabla^2\dot{B} + 3\dot{A}\right)$$
$$+e^{2\alpha}\alpha_{,\phi\phi}(\overline{\phi})\delta\phi\left[4e^{2\alpha}V(\overline{\chi}) - \dot{\overline{\chi}}^2\right] = 0$$

Perturbed interaction term

[JPJ & SS-2020]

- Fluid description

$$\delta Q^{(\mathrm{F})} = -(\delta\rho_m - 3\delta p_m)\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}} - (\overline{\rho}_m - 3\overline{p}_m)\left[\alpha_{,\phi\phi}(\overline{\phi})\dot{\overline{\phi}}\delta\phi + \alpha_{,\phi}(\overline{\phi})\delta\dot{\phi}\right]$$

- Field theory description

$$\begin{aligned} \delta Q^{(\mathrm{F})} &= 2e^{2\alpha}\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}\left[\dot{\overline{\chi}}\delta\dot{\chi} - 2e^{2\alpha}V_{,\chi}(\overline{\chi})\delta\chi\right] \\ &+ e^{2\alpha}\alpha_{,\phi\phi}(\overline{\phi})\dot{\overline{\phi}}\delta\phi\left[\dot{\overline{\chi}}^2 - 4V(\overline{\chi})\right] \\ &+ 2e^{2\alpha}\alpha_{,\phi}(\overline{\phi})^2\dot{\overline{\phi}}\delta\phi\left[\dot{\overline{\chi}}^2 - 8e^{2\alpha}V(\overline{\chi})\right] \\ &+ e^{2\alpha}\alpha_{,\phi}(\overline{\phi})\delta\dot{\phi}\left[\dot{\overline{\chi}}^2 - 4e^{2\alpha}V(\overline{\chi})\right] \end{aligned}$$

Key points

- Interacting dark sector can be obtained from a classical field theory action.
- Field theory action can be obtained from $f(\tilde{R}, \tilde{X})$ via conformal transformation.
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Interacting dark sector models in literature

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Olivares.etal - 2005 [35]	$\frac{d\rho_c}{dt} + 3H\rho_c = 3Hc^2\left(\rho_c + \rho_x\right)$	No
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Caldera-Cabral.etal - 2008 [39]	$\dot{\rho}_c = -3H\rho_c + 3H\left(\alpha_x\rho_x + \alpha_c\rho_c\right)$	No
	$\dot{\rho}_c = -3H\rho_c + 3\left(\Gamma_x\rho_x + \Gamma_c\rho_c\right)$	No

Interacting dark sector models in literature

He & Wang - 2008 [40]	$\dot{\rho}_{DM} + 3H\rho_{DM} - \delta H\rho_{DM} = 0$	No
	$\dot{\rho}_{DM} + 3H\rho_{DM} - \delta H\left(\rho_{DM} + \rho_{DE}\right) = 0$	No
Pettorino & Baccigalupi - 2008 [41]	$\phi'' + 2\mathcal{H}\phi' + a^2 U_{,\phi} = a^2 C_c \rho_c$	Yes
Quartin.etal - 2008 [42]	$\frac{\mathrm{d}\rho_c}{\mathrm{d}N} + 3\rho_c = 3\lambda_x\rho_x + \lambda_c\rho_c$	No
Boehmer.etal - 2009 [43]	$\dot{\rho}_c = -3H\rho_c - \frac{\alpha}{M_0}\rho_\varphi^2$	No
	$\dot{\rho}_c = -3H\rho_c - \frac{\beta}{M_0}\rho_c^2$	No
	$\dot{\rho}_c = -3H\rho_c - \frac{\gamma}{M_0}\rho_\varphi\rho_c$	No
Beyer.etal - 2010 [44]	$\ddot{\varphi} + 3H\dot{\varphi} - \alpha M^3 e^{-\alpha\varphi/M} = \frac{\beta}{M}\rho_\chi$	Yes
Lopez Honorez.etal - 2010 [45]	$\dot{\rho}_{dm} + 3H\rho_{dm} = \beta(\phi)\rho_{dm}\dot{\phi}$	Yes
Avelino & Silva - 2012 [46]	$\dot{\rho}_m + 3H\rho_m = \alpha H a^\beta \rho_w$	No
Pan.etal - 2012 [47]	$\dot{\rho}_m + 3H\rho_m = 3\lambda_m H\rho_m + 3\lambda_d H\rho_d$	No

Interacting dark sector models in literature

Salvatelli.etal - 2013 [48]	$\dot{\rho}_{dm} + 3\mathcal{H}\rho_{dm} = \xi\mathcal{H}\rho_{de}$	No
Chimento.etal - 2013 [49]	$\rho'_{\rm m} + \gamma_{\rm m}\rho_{\rm m} = -\alpha\rho'\rho$	No
Amendola.etal - 2014 [50]	$\dot{\rho}_\alpha + 3H\rho_\alpha = -\kappa\sum_i C_{i\alpha}\dot{\phi}_i\rho_\alpha$	Yes
Marra - 2015 [51]	$\dot{\rho}_m + 3H\rho_m = \nu\delta^n_m\rho_m\dot{\phi}/M_{Pl}$	No
Bernardi & Landim - 2016 [52]	$\dot{\rho}_m + 3H\rho_m = Q\left(\rho_\phi + \rho_m\right)\dot{\phi}$ $\dot{\rho}_m + 3H\rho_m = Q\rho_\phi\dot{\phi}$	No No
Pan & Sharov - 2016 [53]	$\dot{\rho}_{dm} + 3\mathcal{H}\rho_{dm} = 3\lambda_m H\rho_{dm} + 3\lambda_d H\rho_d$	No
Bruck & Mifsud - 2017 [54][2]	$\nabla^\mu T^{DM}_{\mu\nu} = Q\nabla_\nu\phi$ $Q = \frac{C_{,\phi}}{2C}T_{DM} + \frac{D_{,\phi}}{2C}T^{\mu\nu}_{DM}\nabla_\mu\phi\nabla_\nu\phi$ $-\nabla_\mu\left[\frac{D}{C}T^{\mu\nu}_{DM}\nabla_\nu\phi\right]$	Yes if $D=0$
Gonzalez & Trodden - 2018 [55]	$\dot{\rho}_\chi + 3H\rho_\chi = \alpha'\dot{\phi}\rho_\chi$	Yes
Barros.etal - 2018 [56]	$\dot{\rho}_c + 3H\rho_c = -\kappa\beta\dot{\phi}\rho_c$	Yes
Landim - 2019 [57]	$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -Q\rho_m$	Yes

Background evolution with interacting dark sector

Exact background solution

[JPJ & SS-2020]

- Rewrite the interaction term obtained in fluid description as

$$\overline{Q}^{(\mathrm{F})} = -\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}(\overline{\rho}_m - 3\overline{p}_m) = -\dot{\alpha}(\overline{\phi})(\overline{\rho}_m - 3\overline{p}_m)$$

- Evolution equation for the dark matter fluid

$$\dot{\overline{\rho}}_m + 3H\overline{\rho}_m(1+\omega_m) = \dot{\alpha}(\overline{\phi})\overline{\rho}_m(1-3\omega_m)$$

- Evolution of the dark matter energy density

$$\overline{\rho}_m = \overline{\rho}_{m_0} a^{-3(1+\omega_m)} e^{[\alpha(\overline{\phi})-\alpha_0](1-3\omega_m)},$$

Autonomous system for the background evolution

[JPJ & SS-2020, JPJ.etal-2020]

- Define the dimensionless variables

$$x = \sqrt{\frac{C_1}{6}}\frac{\dot{\phi}}{HM_{Pl}}, \quad y = \sqrt{\frac{C_1}{3}}\frac{\sqrt{U}}{HM_{Pl}}$$

$$\lambda = -\frac{M_{Pl}}{\sqrt{C_1}}\frac{U_{,\phi}}{U}, \quad \Gamma = \frac{UU_{,\phi\phi}}{U_{,\phi}^2}$$

$$\alpha = \alpha(\phi), \quad \beta = -\frac{M_{Pl}}{\sqrt{C_1}}\frac{\alpha_{,\phi}}{\alpha}, \quad \gamma = \frac{\alpha\alpha_{,\phi\phi}}{\alpha_{,\phi}^2}$$

- These parameters are evolved as functions of number of e-foldings

$$N \equiv \ln a = \ln\left(\frac{1}{1+z}\right)$$

more details

Autonomous system for the background evolution

$$
\begin{aligned}
x' + \frac{3}{2}x\left(1 - x^2 + y^2 - \frac{\Omega_r}{3}\right) - \sqrt{\frac{3}{2}}\left(\lambda y^2 + \frac{q}{x}\right) &= 0 \\
y' + \frac{3}{2}y\left(\sqrt{\frac{2}{3}}\lambda x - x^2 + y^2 - \frac{\Omega_r}{3} - 1\right) &= 0 \\
\Omega_m' + \Omega_m\left(3y^2 - 3x^2 - \Omega_r\right) + \sqrt{6}q &= 0 \\
\Omega_r' + \Omega_r\left(1 - 3x^2 + 3y^2 - \Omega_r\right) &= 0 \\
\lambda' + \sqrt{6}\lambda^2 x\left(\Gamma - 1\right) &= 0 \\
\beta' + \sqrt{6}\beta^2 x(\gamma - 1) &= 0 \\
\alpha' + \sqrt{6}\alpha\beta x &= 0
\end{aligned}
$$

Cosmological parameters

- Scaled interaction term

$$q \equiv \alpha\,\beta\,x\,\Omega_m = -\frac{\alpha_{,\phi}(\overline{\phi})\dot{\overline{\phi}}\overline{\rho}_m}{3\sqrt{6}H^3M_{Pl}^2} = \frac{\overline{Q}}{3\sqrt{6}H^3M_{Pl}^2}.$$

- Slow-roll parameter

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}\left(x^2 - y^2 + \frac{\Omega_r}{3} + 1\right), \quad \epsilon < 1 \Rightarrow \ddot{a} > 0$$

- Energy density parameter

$$\Omega_\phi = x^2 + y^2, \quad \rho_i = 3H^2M_{Pl}^2\Omega_i$$

- Equation of state of scalar field

$$\omega_\phi = \frac{x^2 - y^2}{x^2 + y^2}$$

Background evolution : A specific example

[JPJ & SS-2020, JPJ.etal-2020]

- Consider the following scalar field potential[1] and interaction function

$$U(\phi) = \frac{8\pi M_{Pl}^2 \kappa}{2}\frac{1}{\phi}, \quad \alpha(\phi) = \frac{C}{\sqrt{2}}\phi$$

κ and coupling strength C are constants.

- Evolution in the redshift range $1500 > z > 0$.
- Two scenarios : $C \geq 0$ and $C \leq 0$.
- Numerically solve the autonomous system with respect to N.
- Initial conditions are chosen in such a way that for $N = 0$, parameter values for $C = 0$ roughly matches with the ΛCDM case.

[1] [Pavlov.etal-2013]

Scenario I : $C \geq 0$

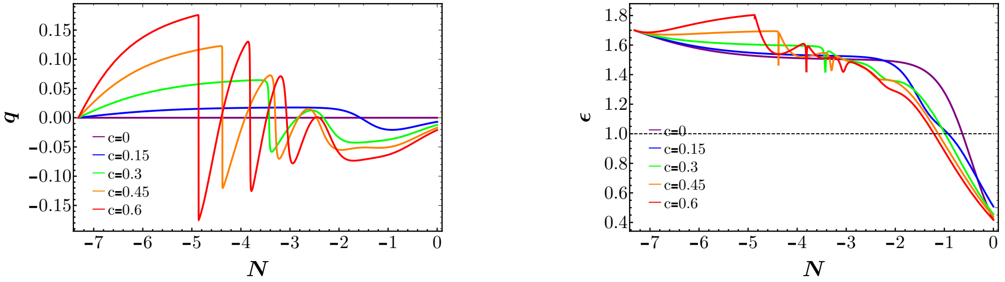


Figure: Left panel: Evolution of interaction term $q \equiv \alpha\beta x\Omega_m$; Right panel: slow-roll parameter ϵ

- Scaled interaction function q oscillates between positive and negative values.
- All the values of C lead to accelerated expansion ($\epsilon < 1$).
- Phase of accelerated expansion begins earlier for higher values of C.

Scenario I : $C \geq 0$; Attractor solution

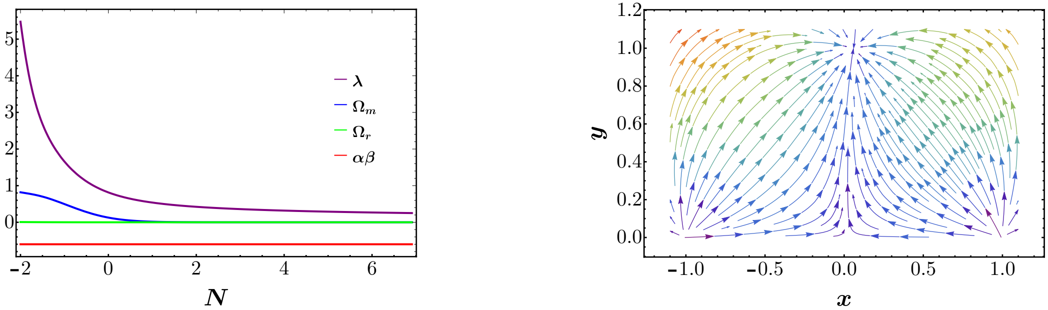


Figure: Left panel: Evolution of various parameters in the future($N > 0$); Right panel: $x - y$ phase space with a dark energy dominated attractor point for $C = 0.6$.

- Cosmological parameters remains nearly constant in the future.
- Interacting dark sector model has a dark energy dominated attractor.

Scenario II : $C \leq 0$

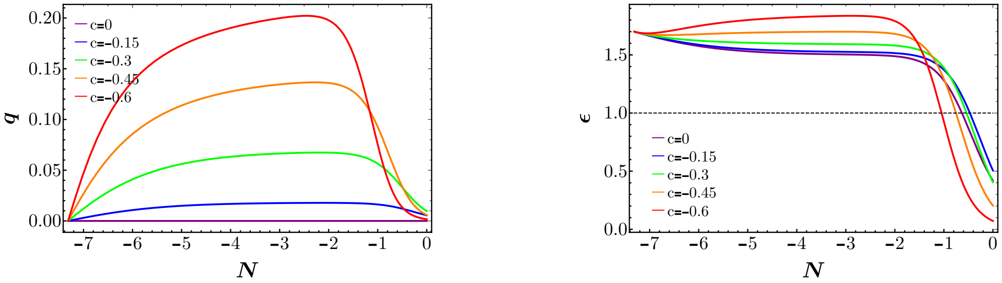
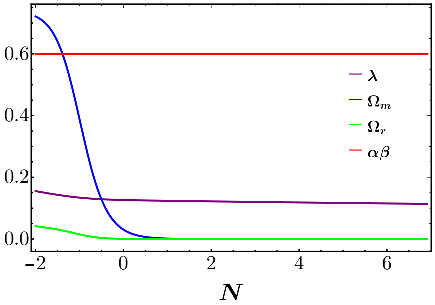


Figure: Left panel: Evolution of interaction term $q \equiv \alpha\beta x\Omega_m$; Right panel: slow-roll parameter ϵ

- Interaction function remains positive during the evolution.
- No oscillatory bahaviour.
- All cases leads to accelerated expansion.

Scenario II : $C \leq 0$; Attractor solution



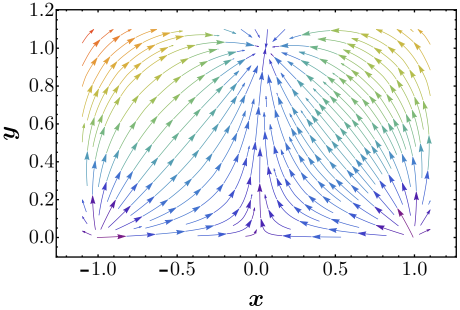


Figure: Left panel: Evolution of various parameters in the future($N > 0$); Right panel: $x - y$ phase space with a dark energy dominated attractor point for $C = -0.6$.

- Cosmological parameters remains nearly constant in the future.
- Interacting dark sector model has a dark energy dominated attractor.

Effect of potentials and interaction functions

[JPJ.etal-2020]

- Study the effect of scalar field potential and interaction functions.

- Consider inverse power law potentials($U(\phi) \propto 1/\phi^m$) and power law interaction function($\alpha(\phi) \propto \phi^n$).

  1. $\alpha(\phi) \propto \phi$, $U(\phi) \propto 1/\phi^m$, $m > 0$
  2. $U(\phi) \propto 1/\phi$, $\alpha(\phi) \propto \phi^n$, $n > 0$

- $C \geq 0$ and $C \leq 0$

Effect of potential $U(\phi) = U_0/\phi^m$, $m > 0$; $C \geq 0$

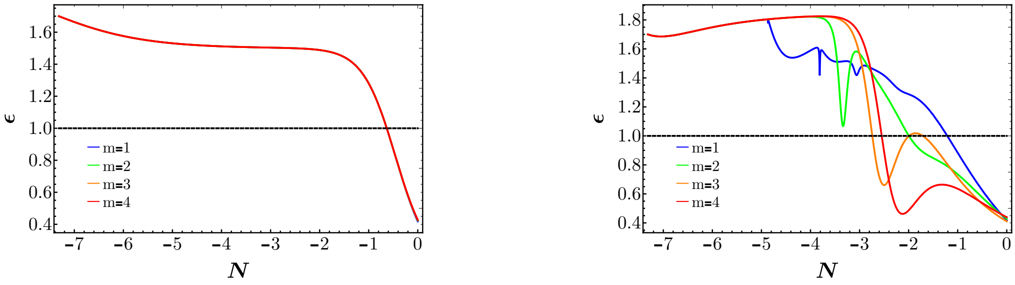
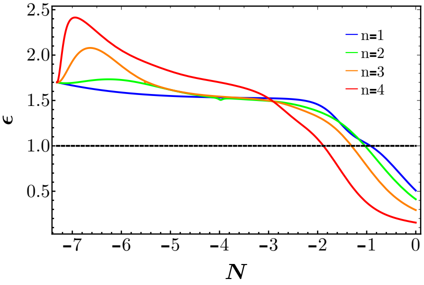


Figure: Evolution of ϵ, Left Panel: $C = 0$; Right Panel: $C = 0.6$

- All values of m considered lead to accelerated expansion.
- For non-onteracting case, evolution is almost identical for different values of m.
- For interacting case evolution is different for different values of m.
- For higher values of m, evolution is smoother.

Effect of interaction function $\alpha(\phi) \propto \phi^n$, $n > 0$; $C \geq 0$



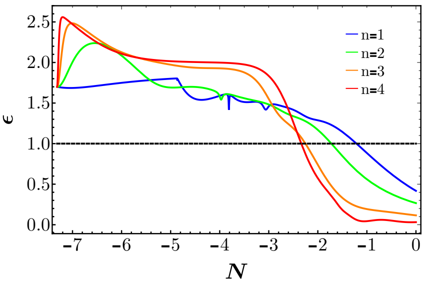


Figure: Evolution of slow-roll parameter for different interaction functions $\alpha(\phi) \propto \phi^n$. Left Panel: C=0.15; Right Panel: C=0.6.

- All the values of n considered lead to accelerated expansion.
- With increasing values of n, accelerated expansion starts early and the evolution is smoother.
- For all values of n, accelerated expansion begins earlier for higher value of coupling strength.

Effect of potential $U(\phi) = U_0/\phi^m$, $m > 0$; $C \leq 0$

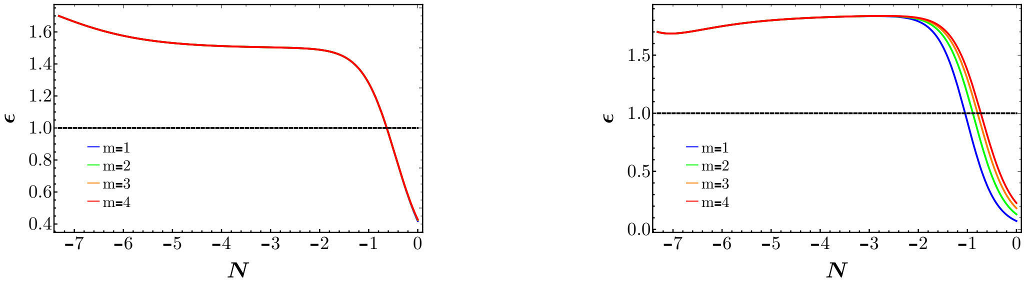


Figure: Evolution of slow-roll parameter for different potentials U_0/ϕ^m. Left Panel: $C = 0$; Right Panel: $C = -0.6$.

- All values of m considered lead to accelerated expansion and there is no oscillatory behaviour.
- For non-interacting case, evolution is almost identical for different values of m.
- Accelerated expansion starts earlier for smaller values of m.

Effect of interaction function $\alpha(\phi) \propto \phi^n$, $n > 0$; $C \leq 0$

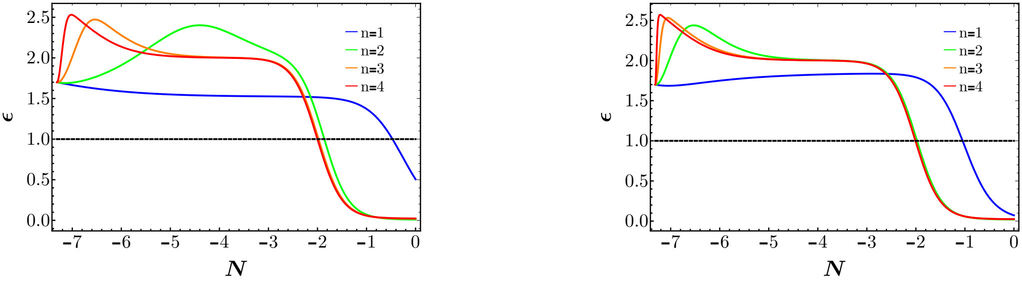


Figure: Evolution of slow-roll parameter for different interaction functions $\alpha(\phi) \propto \phi^n$. Left Panel: C=-0.15; Right Panel: C=-0.6.

- All values n considered lead to accelerated expansion.
- There is a clear difference in the evolution for $n = 1$ and $n > 1$.
- For higher values of n, accelerated expansion begins earlier.

Effect of interaction function $\alpha(\phi) \propto \phi^n$, $n > 0$; $C \leq 0$

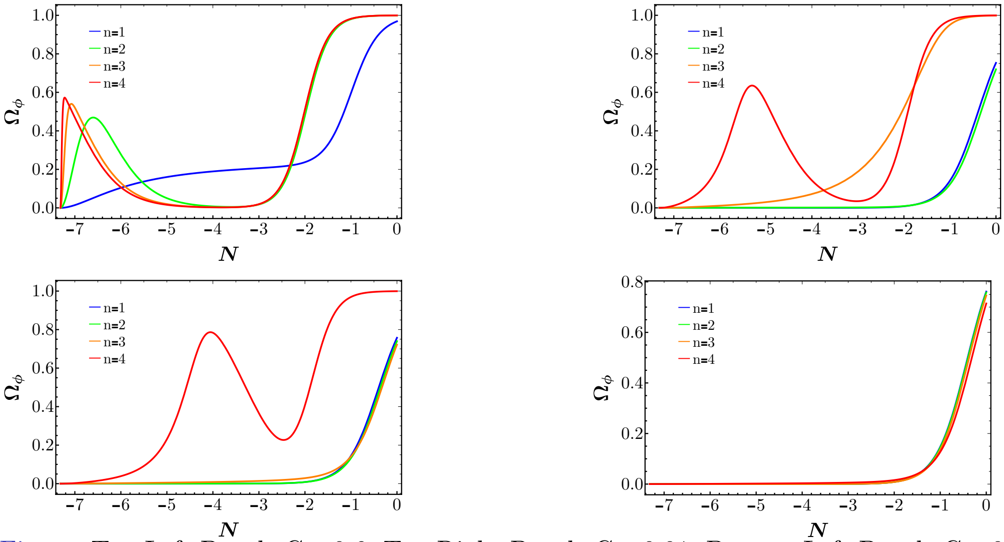


Figure: Top Left Panel: C=-0.6; Top Right Panel: C=-0.01; Bottom Left Panel: C=-0.005; Bottom Right Panel: C=-0.001

Effect of interaction function $\alpha(\phi) \propto \phi^n$, $n > 0$; $C \leq 0$

- There is a brief dark energy dominated phase in the past which is followed by a dark matter dominated phase.
- Initial dark energy dominated phase does not lead to accelerated expansion.
- All values of n considered lead to dark energy dominated phase.
- For $C \sim 10^{-1}$, for all values of n considered, $\Omega_\phi \to 1$ while $N < 0$: Not consistent with the observations.
- Smaller values of coupling strength ($C \sim 10^{-3}$) lead to agreeable parameter values.

Conclusions

- Background evolution with interacting dark sector can be described by an autonomous system.
- For linear coupling function, and inverse potential, the model lead to accelerated expansion.
- Model has a dark energy dominated attractor for both positive and negative coupling strengths.
- For positive coupling strengths, background evolutions shows oscillatory behaviour.
- Effect of difference in the potenital is negligible for non-interacting scenario.
- Higher order interaction function with large negative values of coupling strengths are not favoured by observations.

Ongoing work : Constraining the model parameters

[JPJ.etal-2020]

- Constraints on the parameters using BAO data compiled by *Ryan.etal-2019*.

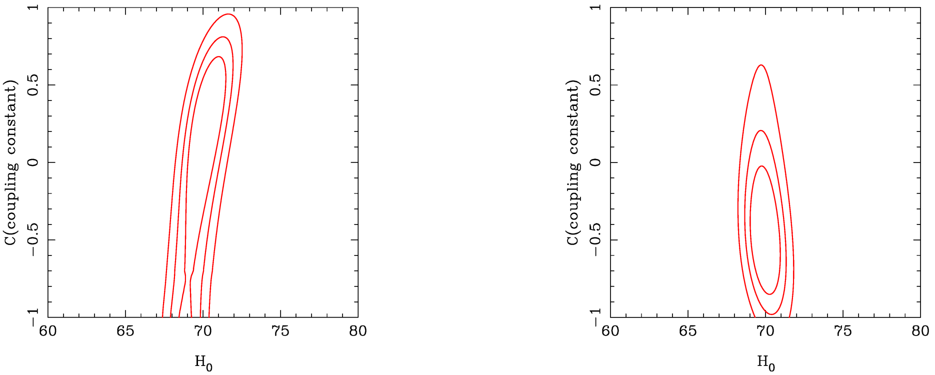


Figure: Constraints on H_0 and C from BAO data for $\alpha(\phi) \propto \phi$ and $U(\phi) \propto 1/\phi^m$. Left Panel: $m = 1$; Right Panel: $m = 2$.

Ongoing work : Constraining the model parameters

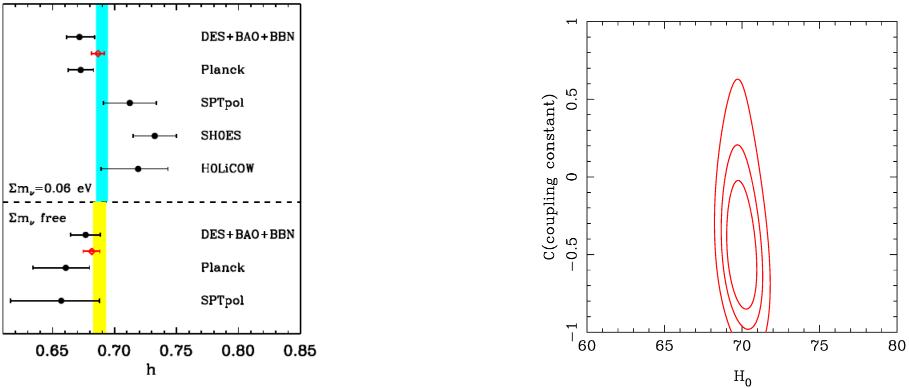


Figure: Comparison with previous estimates of H_0 from BAO data

- $H_0^{(\Lambda CDM)} < H_0^{(intds)}$

Future work

- Constrain the model parameters using observational data.
- Study the evolution of scalar perturbation with interacting dark sector.
- Obtain further constrains on the model parameters using the perturbative analysis.

Thank You

Self interacting sterile neutrino

- Sterile neutrino interacting with a light pseudo-scalar.
- No interaction with standard neutrino after recombination.
- Sterile neutrino strongly couples with the pseudo scalar, forming a non-relativistic fluid.

$f(\tilde{R},\tilde{X})$ model to interacting scalar fields

- Consider $f(\tilde{R},\tilde{X}) = h(\tilde{X})f(\tilde{R})$
- Redefine fields such that

$$\psi = \sqrt{\frac{3}{2\kappa^2}}\ln\, F$$

$$\begin{aligned} e^{2\alpha(\phi)}\left(\frac{\partial\chi}{\partial\psi}\right)^2 + \left(\frac{\partial\phi}{\partial\psi}\right)^2 &= 1 \\ e^{2\alpha(\phi)}\frac{\partial\chi}{\partial\tilde{X}}\frac{\partial\chi}{\partial\psi} + \frac{\partial\phi}{\partial\tilde{X}}\frac{\partial\phi}{\partial\psi} &= \sqrt{\frac{3}{2\kappa^2}}\frac{h_{,\tilde{X}}}{h} \\ e^{2\alpha(\phi)}\left(\frac{\partial\chi}{\partial\tilde{X}}\right)^2 + \left(\frac{\partial\phi}{\partial\tilde{X}}\right)^2 &= \frac{1}{he^{\sqrt{\frac{2\kappa^2}{3}}\psi}} + \frac{3}{2\kappa^2}\frac{h^2_{,\tilde{X}}}{h^2} \end{aligned}$$

return

Background evolution with general interacting dark sector

- Consider the general action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - C_1\left(\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi + U(\phi)\right)\right.$$
$$\left. - C_2\left(\frac{1}{2}e^{2\alpha(\phi)}\nabla^\mu\chi\nabla_\mu\chi + e^{4\alpha(\phi)}V(\chi)\right)\right]$$

- C_1 and C_2 is defined based on the dimensions of ϕ and χ
- Makes calculations simpler for a variety of scalar field potentials.

return

Background evolution with general interacting dark sector

- Background energy density and pressure of the dark matter

$$\rho_m = C_2 e^{2\alpha(\phi)} \left(\frac{\dot{\chi}^2}{2} + e^{2\alpha(\phi)} V(\chi) \right)$$

$$p_m = C_2 e^{2\alpha(\phi)} \left(\frac{\dot{\chi}^2}{2} - e^{2\alpha(\phi)} V(\chi) \right)$$

- Energy conservation equation

$$C_1 \left(\ddot{\phi} + 3H\dot{\phi} + U_\phi(\phi) \right) \dot{\phi} = Q$$

$$\dot{\rho_m} + 3H(\rho_m + p_m) = -Q,$$

- $Q = C_2 \alpha_\phi(\phi)\dot{\phi} \left(e^{2\alpha(\phi)}\dot{\chi}^2 - 4e^{\alpha(\phi)}V(\chi) \right) = -\alpha_\phi(\phi)\dot{\phi}(\rho_m - 3p_m)$

return

Autonomous system for the background evolution

$$
\begin{aligned}
x' + \frac{3}{2}x\left(1 - x^2 + y^2 - \frac{\Omega_r}{3}\right) - \sqrt{\frac{3}{2}}\left(\lambda y^2 + \frac{q}{x}\right) &= 0 \\
y' + \frac{3}{2}y\left(\sqrt{\frac{2}{3}}\lambda x - x^2 + y^2 - \frac{\Omega_r}{3} - 1\right) &= 0 \\
\Omega_m' + \Omega_m\left(3y^2 - 3x^2 - \Omega_r\right) + \sqrt{6}q &= 0 \\
\Omega_r' + \Omega_r\left(1 - 3x^2 + 3y^2 - \Omega_r\right) &= 0 \\
\lambda' + \sqrt{6}\lambda^2 x\left(\Gamma - 1\right) &= 0 \\
\beta' + \sqrt{6}\beta^2 x(\gamma - 1) &= 0 \\
\alpha' + \sqrt{6}\alpha\beta x &= 0 \\
x^2 + y^2 + \Omega_m + \Omega_r - 1 &= 0
\end{aligned}
$$

Scenario I : $C \geq 0$; Energy density parameters

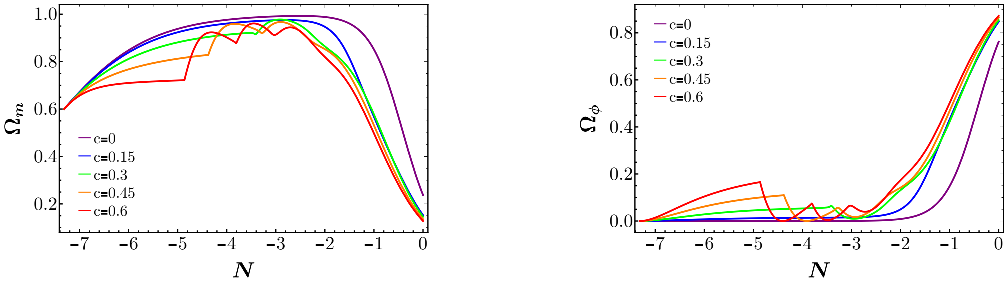


Figure: Evolution of Energy density parameters as function of N; Left panel: Dark matter; Right panel: Dark energy.

- All values of C leads to dark energy dominated phase.
- Oscillations in the evolution of q causes the oscillatory behaviour in the evolution of energy density parameters.

Scenario I : $C \geq 0$; Scaled Hubble parameter

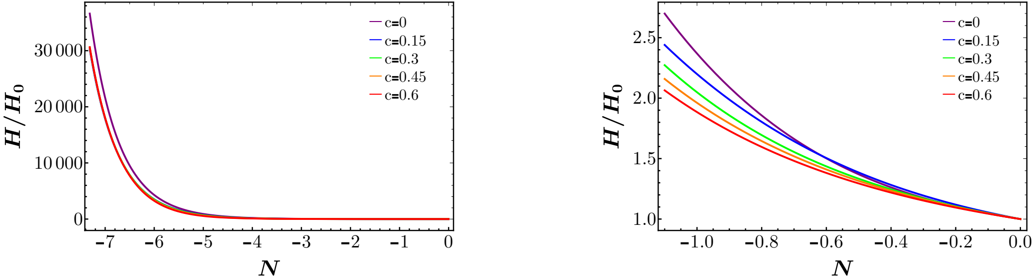
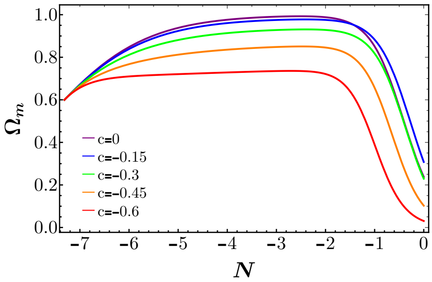


Figure: Evolution of Hubble parameter $h = H/H_0$ as function of N.

- Scaled Hubble parameter has a smooth evolution.
- Hubble parameter takes higher values in the past for non-interacting case.

Scenario II : $C \leq 0$; Energy density parameters



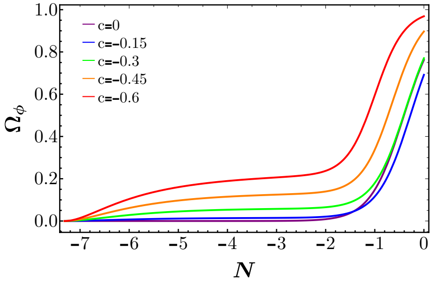


Figure: Evolution of Energy density parameters as function of N; Left panel: Dark matter; Right panel: Dark energy.

- All cases lead to dark energy dominated phase.
- No oscillatory behaviour.
- Value of coupling strength has a stronger effect on the current values of the parameters as compared to $C > 0$ case.

Scenario II : $C \leq 0$; Scaled Hubble parameter

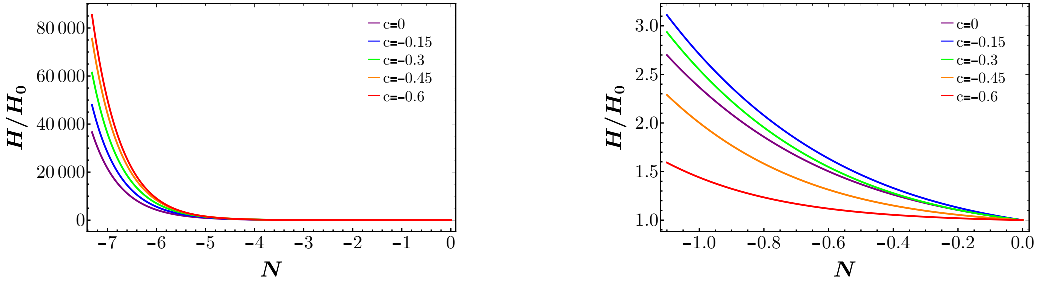


Figure: Evolution of Hubble parameter $h = H/H_0$ as function of N.

- Hubble parameter takes lower values in the past for non-interacting case.

Effect of potential $U(\phi) = U_0/\phi^m$, $m > 0$; $C \geq 0$

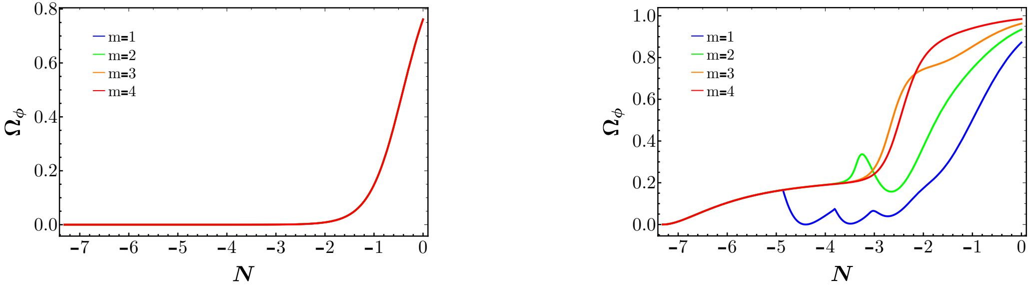


Figure: Evolution of Ω_ϕ; Left Panel: $C = 0$; Right Panel: $C = 0.6$.

- For $C = 0$, evolution is almost identicl for different values of m.
- All values of m lead to dark energy dominated phase.
- Higher values of m lead to higher current values of Ω_ϕ.

Effect of interaction function $\alpha(\phi) \propto \phi^n$, $n > 0$; $C \geq 0$

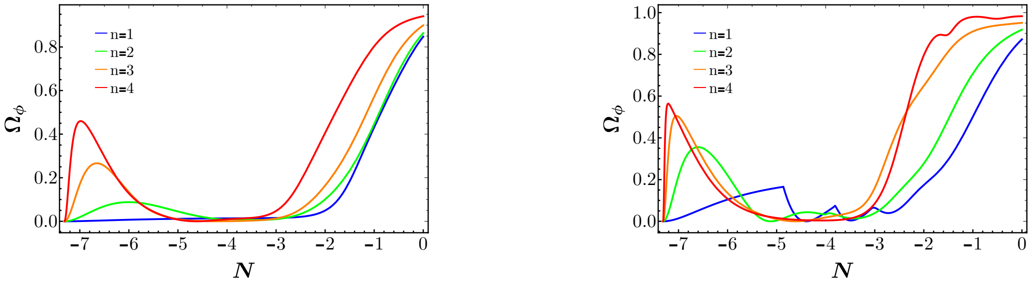


Figure: Evolution of Ω_ϕ for different interaction functions $\alpha(\phi) \propto \phi^n$. Left Panel: C=0.15; Right Panel: C=0.6.

- All values of n lead to dark energy dominated phase.
- Larger values of n lead to larger current value of Ω_ϕ.
- For, larger values of n for $C = 0.6$ there is a brief dark energy dominated phase before the matter dominated phase.

Effect of potential $U(\phi) = U_0/\phi^m$, $m > 0$; $C \leq 0$

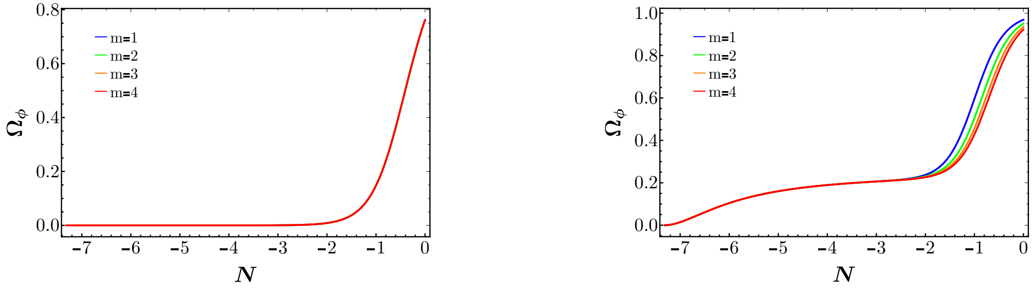


Figure: Evolution of Ω_ϕ as function of N for different potentials U_0/ϕ^m. Left Panel: $C = 0$; Right Panel: $C = -0.6$.

- All values of m lead to dark energy dominated phase.
- For non-interacting case, evolution is almost identical for different values of m.
- Lower values of m lead to higher current values of Ω_ϕ.