Evaporation of four-dimensional dynamical black holes sourced by the quantum trace anomaly

Paolo Meda
Ph.D. student at University of Genoa – INFN Sez. Genova
Paolo.Meda@ge.infn.it

Sixteenth Marcel Grossmann Meeting – MG16
July 5th, 2021

Joint work with N. Pinamonti, S. Roncallo and N. Zanghì
\[ G_{\mu\nu}[g] = 8\pi G \langle \omega | :T_{\mu\nu}[g, \phi] :| \omega \rangle \]

- The Einstein tensor is equated to the expectation value of the quantum stress-energy tensor in physical states \(|\omega\rangle\)

- **Physical applications**
  - *Cosmology in the Early Universe*
  - *Black Hole Physics  ✔*

- When the back-reaction is not considered, a Schwarzschild static black hole emits **thermal radiation** at infinity

  \[ T_H = \frac{\hbar c^3}{8\pi G k_B M} \approx 6.17 \times 10^{-8} \left( \frac{M_\odot}{M} \right) K \]

- In case of **dynamical black holes**:
  - Studying the dynamics of the horizon in a *local way*
  - Using a *semiclassical model of evaporation*

**Semiclassical Einstein equation**

Studying the back-reaction of a quantum matter field \(\phi\) propagating over a classical curved spacetime \((M, g_{\mu\nu})\)
Dynamical black holes

- Spherically symmetric spacetimes

\[ ds^2 = d\gamma^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2) \]

Parametrization of the normal metric in terms of outgoing and ingoing light rays (Kruskal coordinates)

\[ dt^2 = -2A(V,U)dVdU \]

or in the advanced Eddington-Finkelstein coordinates (Bardeen-Vaidya metric)

\[ ds^2 = -e^{2\Phi(\nu,r)} \left( 1 - \frac{2m(\nu,r)}{r} \right) d\nu^2 + 2e^{\Phi(\nu,r)} d\nu dr \]

- Apparent horizon

Local definition of the horizon which reduces to the event horizon in the static case

\[ \mathcal{H} = \{ p \in \mathcal{M} : r - 2m = 0 \} \]

- Hayward (‘98): thermodynamics of dynamical black holes

(see the auxiliary slide...)

Definitions of null congruences and apparent horizon
Negative ingoing flux for black hole evaporation

A negative ingoing flux $\langle T_{\nu\nu} \rangle_\omega$ on the horizon induces a negative rate of evaporation of the mass with respect to the advanced time $v$

$$M \doteq m(r_{\mathcal{H}}) = \frac{r_{\mathcal{H}}^3}{2}, \quad \partial_v M(v) \equiv \Delta M(v) < 0.$$  

Evaporation sourced by the quantum trace anomaly

- The negative flux on the horizon can be induced by the state-independent trace anomaly of a massless conformally coupled scalar field

$$\langle T^\rho_\rho \rangle_\omega = \chi \left( C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} + R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right), \quad \lambda = 1/(2880\pi^2).$$

- The variation of the mass is constrained by the matter content outside the horizon, assuming a vacuum-like matter state in the past

$$\Delta M = - (S + W)$$

Examples of evaporation models

Approximation: The background geometry outside the black hole horizon is not too different from the one obtained in a classical model of gravitational collapse

i. Schwarzschild spacetime is not a solution of the semiclassical equations

ii. Vaidya spacetime for a radiating mass $M(v)$

$$d\gamma^2 = - \left( 1 - \frac{2M(v)}{r} \right) dv^2 + 2dvdr$$

$$\langle T_{\nu\nu} \rangle_\omega \equiv - \frac{3\lambda}{40M(v)^2}$$

$$\partial_v M(v) = - \frac{3\pi \lambda}{10M(v)^2}$$

$$\partial_v S_\gamma(v) = \partial_v \left( \frac{A_{\mathcal{H}}}{4} \right)(v) = - \frac{12\pi^2 \lambda}{5M(v)}$$
THANKS FOR THE ATTENTION

Paolo Meda

University of Genoa – INFN Sez. Genova

• Email: Paolo.Meda@ge.infn.it
• arXiv page
• Personal homepage
Thermodynamic interpretation of the dynamical black hole laws

- **Kodama vector field and surface gravity**

  A notion of time is given by the Kodama vector $K = \text{curl}_r r$. It is divergenceless $\nabla_\mu K^\mu = 0$, and it defines covariantly conserved currents for any stress-energy tensor $T^\mu_{\ \rho}$: $\nabla_\mu (T^\mu_{\ \rho} K^\rho) = 0$. It also yields the surface gravity of the black hole: $\mathcal{L}_K K^\mu = K^\rho (\nabla_\rho K_\mu - \nabla_\mu K_\rho) \to \kappa K_\mu$, where $\kappa = \frac{1}{2} \gamma^{ij} \nabla_i \nabla_j r$.

- **Hayward’s first law of black hole (thermo)dynamics**

  On the apparent horizon:

  $$\partial_k m = \frac{\kappa}{8\pi G} \partial_k A - \frac{1}{2} \gamma^{ij} T_{ij} \partial_k V$$

  Relates the variation of the mass to the change of the area and to the change of the volume

  $$dE = TdS + p dV$$

  Relates the variation of the energy to the change of the entropy and to work

- **Temperature of the apparent horizon $T_H = \kappa_H/2\pi$**

  - It is the temperature of the thermal distribution related to the tunneling probability of quantum particles across the horizon [arXiv: 0707.4422]

  - It is the temperature associated to Hadamard two-point functions in the scaling limit on the horizon [arXiv: 2102.11547]