Evaporation of four-dimensional dynamical black holes sourced by the quantum trace anomaly

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Joint work with **N.Pinamonti**, **S. Roncallo** and **N. Zanghì**

https://arxiv.org/abs/2103.02057

$$G_{\mu\nu}[g] = 8\pi G \langle\omega| {:}T_{\mu\nu}[g,\phi]{:} |\omega\rangle$$

- The Einstein tensor is equated to the expectation value of the quantum stress-energy tensor in physical states $|\omega\rangle$

- **Physical applications**
  - *Cosmology in the Early Universe*
  - *Black Hole Physics* ✓

- When the back-reaction is not considered, a Schwarzschild static black hole emits **thermal radiation** at infinity

$$T_H = \frac{\hbar c^3}{8\pi G k_B M} \approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \; K$$

- In case of **dynamical black holes:**
  - Studying the dynamics of the horizon in a *local way*
  - Using a *semiclassical model of evaporation*

Semiclassical Einstein equation

Studying the back-reaction of a quantum matter field ϕ propagating over a classical curved spacetime $(M, g_{\mu\nu})$

Dynamical black holes

- **Spherically symmetric spacetimes**

$$\mathrm{d}s^2 = \mathrm{d}\gamma^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

Parametrization of the normal metric in terms of **outgoing** and **ingoing light rays** (*Kruskal coordinates*)

$$\mathrm{d}\gamma^2 = -2A(V,U)\mathrm{d}V\mathrm{d}U$$

or in the advanced Eddington-Finkelstein coordinates (*Bardeen-Vaidya metric*)

$$\mathrm{d}\gamma^2 = -\mathrm{e}^{2\Phi(v,r)}\left(1 - \frac{2m(v,r)}{r}\right)\mathrm{d}v^2 + 2\mathrm{e}^{\Phi(v,r)}\mathrm{d}v\mathrm{d}r$$

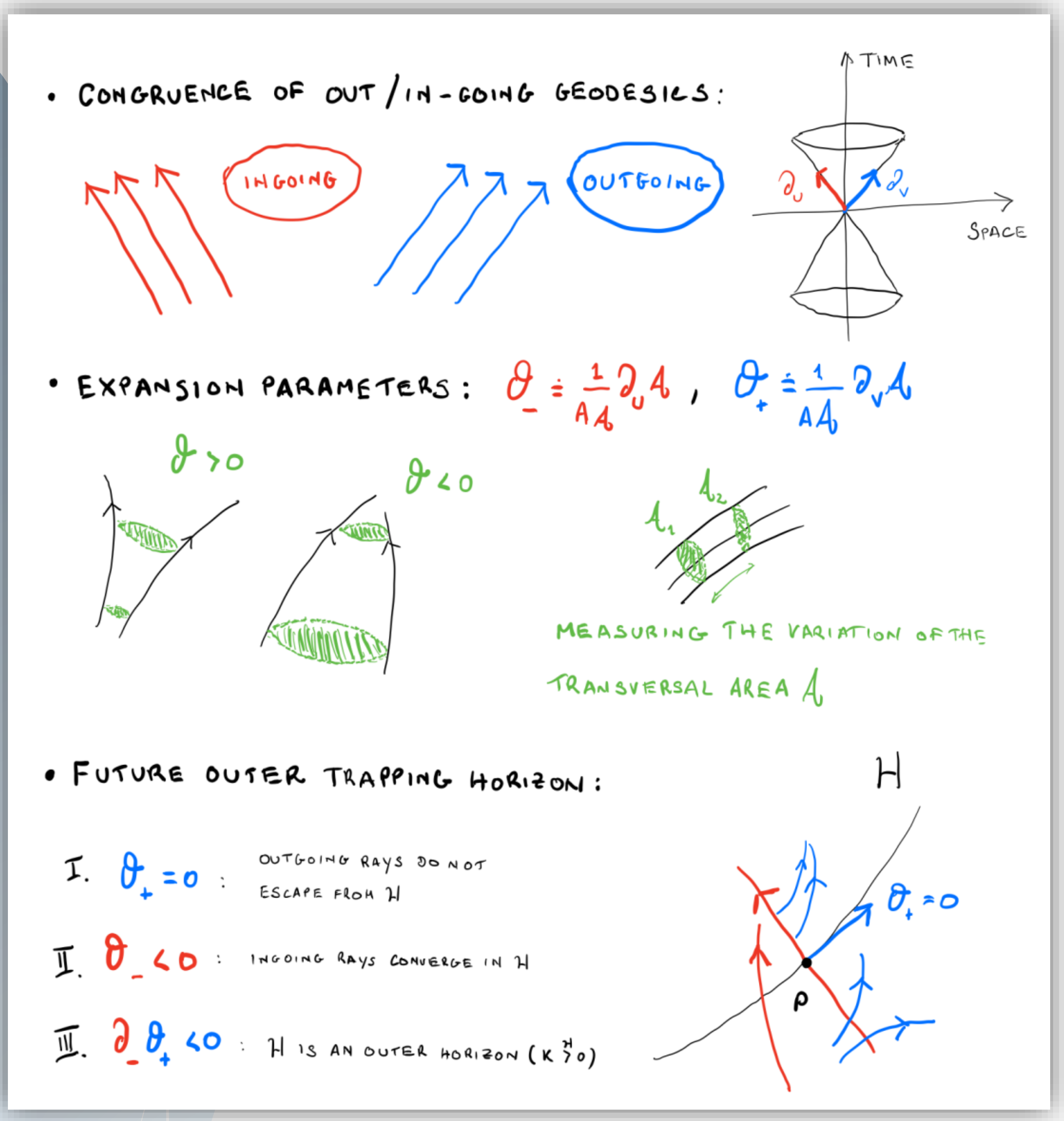
- **Apparent horizon**

Local definition of the horizon which reduces to the *event horizon* in the static case

$$\mathcal{H} = \{p \in \mathcal{M} : r - 2m = 0\}$$

- **Hayward ('98): thermodynamics of dynamical black holes**

(see the auxiliary slide...)



Definitions of null congruences and apparent horizon

Negative ingoing flux for black hole evaporation

A **negative ingoing flux** $\langle :T_{VV}: \rangle_\omega$ on the horizon induces a **negative rate of evaporation** of the mass with respect to the *advanced time* v

$$M \doteq m(r_{\mathcal{H}}) = \frac{r_{\mathcal{H}}}{2}, \qquad \partial_v M(v) \overset{\mathcal{H}}{=} \mathcal{A}_{\mathcal{H}} \langle :T_{VV}: \rangle_\omega < 0 \qquad \langle :T_{\mu\nu}: \rangle_\omega k^\mu k^\nu \big|_{\mathcal{H}} < 0$$

Evaporation sourced by the quantum trace anomaly

- The negative flux on the horizon can be induced by the *state-independent* **trace anomaly** of a massless conformally coupled scalar field

$$\langle :T_\rho{}^\rho: \rangle_\omega = \lambda \left(C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} + R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right), \qquad \lambda = 1/(2880\pi^2).$$

- The variation of the mass is **constrained** by the matter content <u>outside the horizon</u>, assuming a vacuum-like matter state in the past

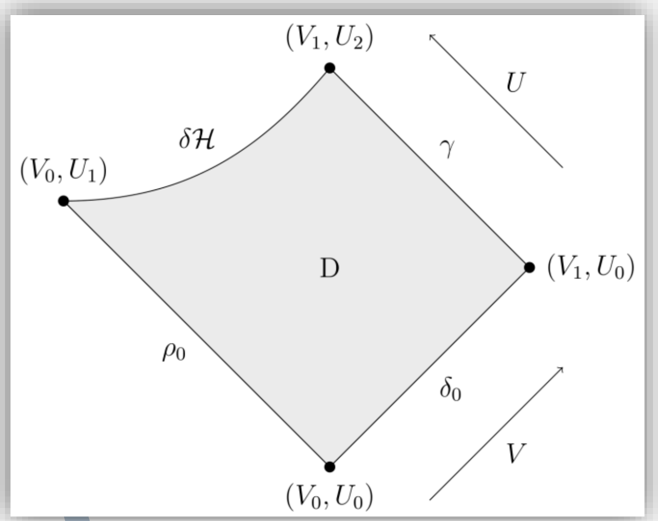
$$\Delta M = -\left(\mathcal{S} + \mathcal{W}\right)$$

Variation of the mass on δH | **Quantum source containing $\langle :T_\rho{}^\rho: \rangle_\omega$** | **Energy flux along γ depending on $\langle :T_{UV}: \rangle_\omega$**

- If the quantum state satisfies the following *averaged energy condition*

$$\mathcal{W}_f = 4\pi \int_\gamma \frac{\langle :T_{UV}: \rangle_\omega r^2}{A} f(V,U) A dU \geq 0$$

for a carefully chosen exponential smooth function $f(V,U)$, then $\Delta M < 0$.



Outer region which intersects a portion of the apparent horizon (image copyright: S. Roncallo)

Examples of evaporation models

<u>Approximation:</u> the background geometry outside the black hole horizon is not too different from the one obtained in a classical model of gravitational collapse

i. *Schwarzschild spacetime* is not a solution of the semiclassical equations

ii. *Vaidya spacetime* for a radiating mass $M(v)$

$$d\gamma^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2 dv dr$$

$$\langle :T_{VV}: \rangle_\omega \overset{\mathcal{H}}{=} -\frac{3\lambda}{40 M(v)^2} \to \begin{cases} \partial_v M(v) = -\dfrac{3\pi\lambda}{10 M(v)^2} \\ \partial_v S_{\mathcal{H}}(v) = \partial_v \left(\dfrac{\mathcal{A}_{\mathcal{H}}}{4}\right)(v) = -\dfrac{12\pi^2\lambda}{5 M(v)} \end{cases}$$

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THANKS FOR THE ATTENTION

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Thermodynamic interpretation of the dynamical black hole laws

- ***Kodama vector field and surface gravity***

A notion of time is given by the Kodama vector $K = curl_\gamma r$. It is **divergenceless**: $\nabla_\mu K^\mu = 0$, and it defines **covariantly conserved currents** for any stress-energy tensor $T^\mu{}_\rho$: $\nabla_\mu(T^\mu{}_\rho K^\rho) = 0$. It also yields the **surface gravity** of the black hole: $\mathfrak{L}_K K_\mu = K^\rho(\nabla_\rho K_\mu - \nabla_\mu K_\rho) \overset{H}{\to} \kappa\, K_\mu$, where $\kappa = \frac{1}{2}\gamma^{ij}\nabla_i\nabla_j r$.

- ***Hayward's first law of black hole (thermo)dynamics***

On the apparent horizon:

Black Hole dynamics	Thermodynamics
$\partial_k m = \frac{\kappa}{8\pi G}\partial_k A - \frac{1}{2}\gamma^{ij}T_{ij}\partial_k V$ Relates the variation of the mass to the change of the area and to the change of the volume	$\mathrm{d}E = T\mathrm{d}S + p\mathrm{d}V$ Relates the variation of the energy to the change of the entropy and to work

- ***Temperature of the apparent horizon $T_H = \kappa_H/2\pi$***
  - It is the temperature of the thermal distribution related to the **tunneling probability** of quantum particles across the horizon [*arXiv: 0707.4425*]
  - It is the temperature associated to **Hadamard two-point functions** in the scaling limit on the horizon [*arXiv: 2102.11547*]

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