Sixteenth Marcel Grossmann Meeting (online conference 5-10/07/2021)

Talk in the parallel session 08/07/2021

«Dark Energy and the accelerating universe» (DE1)

ON THE EVOLUTION OF INHOMOGENEOUS PERTURBATIONS IN THE ΛCDM MODEL AND $f(R)$ MODIFIED GRAVITY THEORIES

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1) Standard ΛCDM Cosmological Model

2) $f(R)$ extended gravity theories and $f(R)$ cosmology in the Jordan frame

3) Lemaitre-Tolman-Bondi metric, inhomogeneous Universe

4) Perturbation approach, solutions in the ΛCDM model and in the Jordan frame

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1) **STANDARD ΛCDM COSMOLOGICAL MODEL**

2) $f(R)$ extended gravity theories and $f(R)$ cosmology in the Jordan frame

3) Lemaitre-Tolman-Bondi metric, inhomogeneous Universe

4) Perturbation approach, solutions in the ΛCDM model and in the Jordan frame

STANDARD ΛCDM COSMOLOGICAL MODEL

COSMOLOGICAL CONSTANT Λ + cold dark matter

FLRW metric (Friedmann-Lemaître-Robertson-Walker)

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-k\,r^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$

$a(t)$: scale factor

k: curvature parameter

$H(t)$: Hubble function

FLRW dynamics
Homogeneous and isotropic Universe

GR + cosmological constant Λ

- I Friedmann equation

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\chi\,\rho + \Lambda}{3} - \frac{k}{a^2}$$

- II Friedmann equation (acceleration equation)

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{\chi}{6}\,(\rho + 3\,p) + \frac{\Lambda}{3}$$

$$\left(\frac{\ddot{a}}{a}\right) > 0$$

$$p_{DE}(\rho_{DE}) = w\,\rho_{DE}$$

$$w < -1/3 \quad \text{Dark Energy}$$

$$w = -1 \quad \rightarrow \quad \Lambda$$

Stress-energy tensor of cosmological fluid

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

$\rho = \rho(t)$ energy density
$p = p(t)$ pressure

STANDARD ΛCDM COSMOLOGICAL MODEL

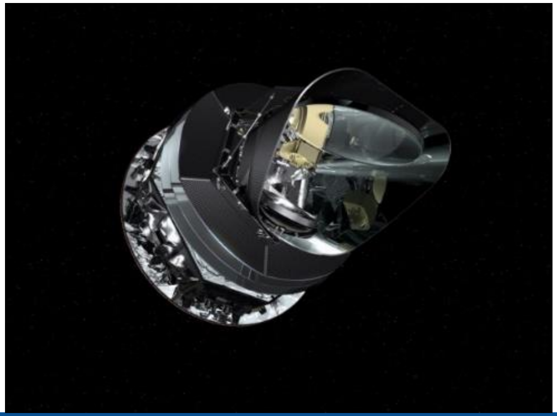
COSMOLOGICAL CONSTANT Λ + cold dark matter

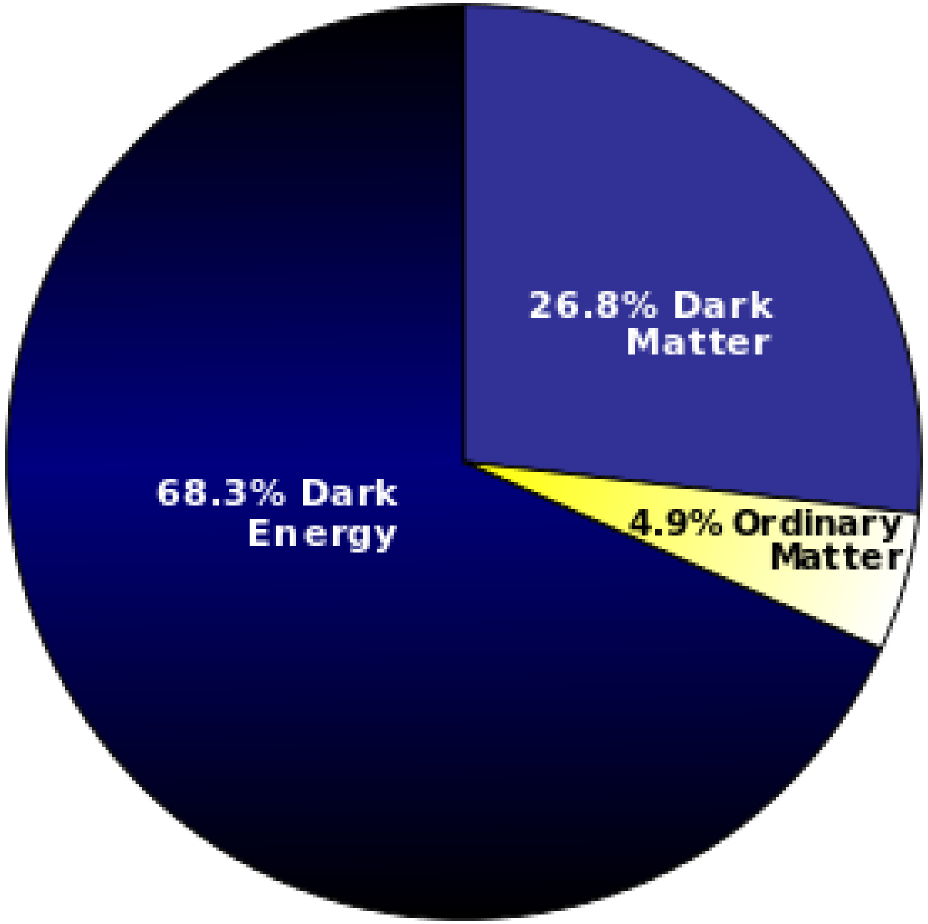
ΛCDM
«CONCORDANCE MODEL»

- **Ordinary Matter**
  barionic matter and radiation)

- **Dark Matter**
  WIMP? (*Weakly Interacting Massive Particles*)
  Axions?

- **Dark Energy**
  Cosmological constant Λ?





Planck collaboration, *Planck 2015 result, XIII. Cosmological parameters*, A&A 594, A13 (2016).

CONTENTS

GENERAL RELATIVITY

$$G_{\mu\nu} = \chi\, T_{\mu\nu}$$

Dark energy → modified source

$f(R)$ MODIFIED GRAVITY

$$G_{\mu\nu} = \chi\, T_{\mu\nu}$$

Geometrical modification of gravity theory

$\mathcal{L}_{EH} = R$ Einstein-Hilbert	Gravitational Lagrangian density	$\mathcal{L}_g = f(R)$ Extra degree of freedom
$S_{EH} = -\frac{1}{2\chi}\int_\Omega d^4x\,\sqrt{-g}\,R$	Gravitational action	$S_g = -\frac{1}{2\chi}\int_\Omega d^4x\,\sqrt{-g}\,f(R)$
$G_{\mu\nu} = \chi\, T_{\mu\nu}$	Gravitational field equations	$f'(R)\,R_{\mu\nu} - \frac{1}{2}\,f(R)\,g_{\mu\nu} + g_{\mu\nu}\,g^{\rho\sigma}\,\nabla_\rho\nabla_\sigma f'(R) - \nabla_\mu\nabla_\nu f'(R) = \chi\,T_{\mu\nu}$

SOTIRIOU, T.P., & FARAONI, V.,
Rev. Mod. Phys. 82, 451 (2010)

R: Ricci scalar

$$f'(R) \equiv \frac{df}{dR}$$

∇_μ: covariant derivative

$f(R)$ MODIFIED THEORIES OF GRAVITY

Geometrical modification of gravity theory

$$\mathcal{L}_g = f(R) = \mathrm{R} + \mathrm{F}(R)$$

Deviation from Einstein-Hilbert theory

Modified gravitational field equations

$$f'(R)\, R_{\mu\nu} - \frac{1}{2}\, f(R)\, g_{\mu\nu} + g_{\mu\nu}\, g^{\rho\sigma}\, \nabla_\rho \nabla_\sigma f'(R) - \nabla_\mu \nabla_\nu f'(R) = \chi\, T_{\mu\nu}$$

$$G_{\mu\nu} = \chi\left(T_{\mu\nu} + T_{\mu\nu}^{[F]}\right)$$

Explicit modification of Einstein-Hilbert equations

No-Einsteinian geometrical contribution

can be recast as an effective source

$$T_{\mu\nu}^{[F]} = -\frac{1}{\chi}\left[F'(R)\, R_{\mu\nu} - \frac{1}{2}\, F(R)\, g_{\mu\nu} + g_{\mu\nu}\, g^{\rho\sigma}\, \nabla_\rho \nabla_\sigma F'(R) - \nabla_\mu \nabla_\nu F'(R)\right]$$

JORDAN FRAME $f(R)$ GRAVITY

Dynamically equivalent action to $f(R)$ theories

Jordan Frame (JF)

$$S_g = -\frac{1}{2\chi}\int_\Omega d^4x\,\sqrt{-g}\;[\phi\, R - V(\phi)]$$

Scalar field $\phi \;= f'(R)$

Potential

$$V(\phi) = R(\phi)\phi - f(R(\phi))$$

- The extra degree of freedom of $f(R)$ is replaced by a scalar field
- Non-minimal coupling between scalar field and metric
- Corresponding field equations:

$$G_{\mu\nu} = \frac{\chi}{\phi}\,T_{\mu\nu} - \frac{1}{2\,\phi}\,g_{\mu\nu}\,V(\phi) + \frac{1}{\phi}\,[\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\,\Box\phi]$$

II order diff. eqs. in $g_{\mu\nu}$

$$3\,\Box\phi + 2\,V(\phi) - \phi\,V'(\phi) = \chi\,T$$

$f(R)$ COSMOLOGY

- Extra degree of freedom in the parameterization, functional form of $f(R)$
- Mimic ΛCDM in the high-redshift regime, well-tested by the CMB
- Cosmic accelerated expansion with an effective cosmological constant
- Phenomenology of ΛCDM as a limiting case

Hu-Sawicki

$$f(R) = R - m^2 \frac{c_1 \left(\frac{R}{m^2}\right)^n}{c_2 \left(\frac{R}{m^2}\right)^n + 1}$$

c_1, c_2 parameters; $n > 0$ $\qquad m^2 \equiv \frac{\chi \rho_{0m}}{3}$

Starobinsky

$$f(R) = R - \mu R_c \left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right]$$

Tsujikawa

$$f(R) = R - \mu R_c \tanh\left(\frac{|R|}{R_c}\right)$$

$n, \mu, R_c > 0$

Hu, W., & Sawicki, I., Phys. Rev. D ,76, 064004 (2007)

AMENDOLA, L., TSUJIKAWA, S., *Dark Energy: Theory and Observations*, Cambridge University Press (2010)

STAROBINSKY, A.A., Jetp Lett. 86, 157 (2007)

TSUJIKAWA, S., Phys. Rev. D, 77, 023507 (2008)

$f(R)$ HU-SAWICKI MODEL

Metric $f(R)$ formalism

$$n = 1 \qquad f(R) = R - m^2 \frac{c_1 \frac{R}{m^2}}{c_2 \frac{R}{m^2} + 1}$$

$\Longleftrightarrow$

Jordan frame (equivalent scalar-tensor formalism)

$$V(\phi) = \frac{m^2}{c_2}\left[c_1 + 1 - \phi - 2\sqrt{c_1\,(1-\phi)}\right]$$

c_1, c_2 parameters; $n > 0$ $\quad m^2 \equiv \frac{\chi\,\rho_{0m}}{3} = H_0^2\,\Omega_{0m}$

- Cosmological constant for $R \gg m^2$

  $f(R) \approx R - 2\Lambda_{eff}$ with $\Lambda_{eff} = \frac{c_1}{c_2}\,m^2$

- Constraining parameters, considering ΛCDM as a limiting case

  $\frac{c_1}{c_2} \approx 6\,\frac{\Omega_{0\Lambda}}{\Omega_{0m}}$ and $F_R(z=0) = \left(\frac{dF}{dR}\right)_{z=0} = -\frac{c_1}{c_2^2}\left[3\left(1 + 4\frac{\Omega_{0\Lambda}}{\Omega_{0m}}\right)\right]^{-2}$ with $|F_R(z=0)| \lesssim 10^{-7}$

Liu, T., Zhang, X., & Zhao, W., Phys. Lett. B, 777, 286 (2018)

$f(R)$ HU-SAWICKI MODEL

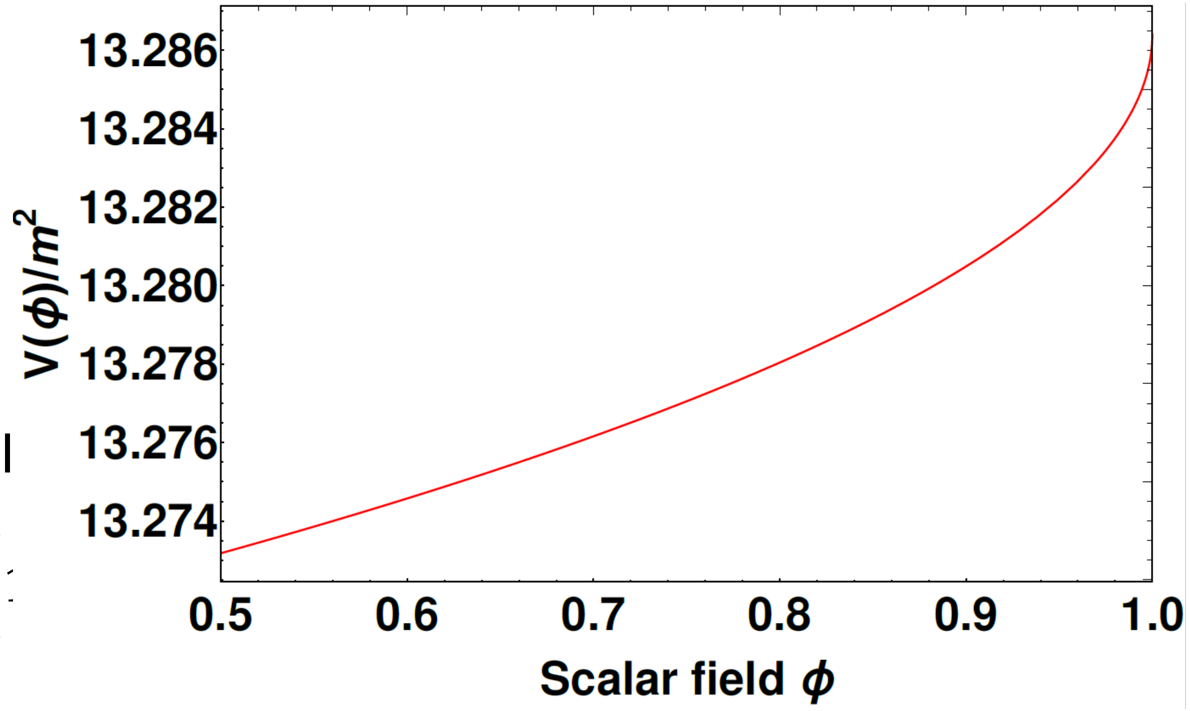
Metric $f(R)$ formalism

$n = 1$

$$f(R) = R - m^2 \frac{c_1 \frac{R}{m^2}}{c_2 \frac{R}{m^2} + 1}$$

Jordan frame (equivalent scalar-tensor formalism)

$$V(\phi) = \frac{m^2}{c_2}\left[c_1 + 1 - \phi - 2\sqrt{c_1\,(1-\phi)}\right]$$



c_1, c_2 parameters; $n > 0$ $\quad m^2 \equiv \frac{\chi\,\rho_{0m}}{3} = H_0^2\,\Omega_{0m}$

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3) **LEMAITRE-TOLMAN-BONDI METRIC, INHOMOGENEOUS UNIVERSE**

4) Perturbation approach, solutions in the ΛCDM model and in the Jordan frame

LEMAITRE-TOLMAN-BONDI (LTB) METRIC
SPHERICALLY SIMMETRIC SOLUTION

LTB line element

$$ds^2 = dt^2 - e^{2\alpha}\, dr^2 - e^{2\beta}\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Inhomogeneous and isotropic Universe

Evolution of a cosmological dust

2 metric functions

$$\alpha = \alpha(t,r)$$

$$\beta = \beta(t,r)$$

Einstein field equation 01 in GR:

$$\frac{\dot{\beta}'}{\beta'} = \dot{\alpha} - \dot{\beta} \longrightarrow \beta' = g(r)e^{\alpha-\beta}$$

$$(\dots)' = \partial_r(\dots)$$

$$(\dot{\dots}) = \partial_t(\dots)$$

Parameterization

$$e^{\beta} = r\, a(t,r)$$

$$g(r) = \sqrt{1 - r^2 K^2(r)}$$

Using the parameterization on the right, the LTB metric becomes:

$$ds^2 = dt^2 - \frac{[(ar)']^2}{1 - r^2 K^2(r)}\, dr^2 - (ar)^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$a(t,r)$: generalized scalar factor

ΛLTB COSMOLOGICAL MODEL

GR + COSMOLOGICAL CONSTANT IN THE LTB METRIC

LTB line element in GR

$$ds^2 = dt^2 - \frac{[(ar)']^2}{1 - r^2\,K^2(r)}\,dr^2 - (ar)^2(d\theta^2 + \sin^2\theta\,d\varphi^2)$$

Generalization of the FLRW line element. If $a(t,r)$ and $K^2(r)$ do not depend on the radial coordinate r, the LTB line element ≡ FLRW line element.

$a(t,r)$: generalized scalar factor

Einstein field equations in the LTB metric:

$(\dots)' = \partial_r(\dots)$

$\dot{(\dots)} = \partial_t(\dots)$

Eq.00 $$(\chi\rho + \Lambda)\,[(a\,r)^3]' = 3\,[\dot{a}^2\,a\,r^3 + a\,r^3\,K^2]'$$

Eq.11 $$2\,\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K^2}{a^2} = \Lambda$$

Dynamics includes inhomogeneities

PEEBLES, P.J.E., *Principles of Physical Cosmology* (1993, Princeton University Press)

MONTANI, G., BATTISTI, M.V., BENINI, R., IMPONENTE, G., *Primordial Cosmology* (2011, World Scientific Publishing Company)

LTB METRIC IN THE JORDAN FRAME GRAVITY

Different dynamics respect to GR due to non-minimally coupled scalar field. No potential $V(\phi)$ in Eq.01

$\alpha = \alpha(t,r)$, $\beta = \beta(t,r)$ Metric functions

$\rho = \rho(t,r)$ Energy density

$\phi = \phi(t,r)$ Scalar field

Eq.01

$$\frac{\dot{\beta}'}{\beta'} = \dot{\alpha} - \dot{\beta} - \frac{1}{2\phi\beta'}\left(\dot{\phi}' - \dot{\alpha}\,\phi'\right)$$

Eq.00

$$\dot{\beta}^2 + 2\dot{\alpha}\dot{\beta} + e^{-2\beta} - e^{-2\alpha}[2\beta'' + 3\beta'^2 - 2\alpha'\beta'] = \frac{\chi}{\phi}\rho - \frac{V(\phi)}{2\phi} - \frac{1}{\phi}\left[(\dot{\alpha} + 2\dot{\beta})\dot{\phi} - e^{-2\alpha}(\phi'' - \alpha'\phi' - 2\beta'\phi')\right]$$

Eq.11

$$2\ddot{\beta} + 3\dot{\beta}^2 + e^{-2\beta} - e^{-2\alpha}\beta'^2 = -\frac{V(\phi)}{2\phi} - \frac{1}{\phi}\left[\ddot{\phi} + 2\dot{\beta}\,\dot{\phi} - 2\,e^{-2\alpha}\beta'\phi\right]$$

Scalar field equation

$$3\ddot{\phi} + 3(\dot{\alpha} + 2\dot{\beta})\dot{\phi} - 3e^{-2\alpha}[\phi'' - \alpha'\phi' + 2\beta'\phi'] + 2V(\phi) - \phi\frac{dV}{d\phi} = \chi\rho$$

$V(\phi)$ affects the other field equations

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4) **PERTURBATION APPROACH, SOLUTIONS IN THE ΛCDM MODEL AND IN THE JORDAN FRAME**

INHOMOGENEOUS PERTURBATIONS

LTB metric inhomogeneities → Small deviations from a flat FLRW background metric

We adopt this perturbative method for both the LCDM and f(R) cosmological models

Linear perturbation approach	Separation of variables method
$a(t,r) = \bar{a}(t) + a_p(t,r)$	$a_p(t,r) = A_p(t)\,\mathcal{A}_p(r)$
$\alpha(t,r) = \bar{\alpha}(t) + \delta\alpha(t,r)$	$\delta\alpha(t,r) = \delta A(t)\,\delta\mathcal{A}(r)$
$\beta(t,r) = \bar{\beta}(t,r) + \delta\beta(t,r)$	$\delta\beta(t,r) = \delta B(t)\,\delta\mathcal{B}(r)$
$\rho(t,r) = \bar{\rho}(t) + \delta\rho(t,r)$	$\delta\rho(t,r) = \delta\mathrm{P}(t)\,\delta\varrho(r)$
$\phi(t,r) = \bar{\phi}(t) + \delta\phi(t,r)$	$\delta\phi(t,r) = \delta\Phi(t)\,\delta\varphi(r)$

$\overline{(\dots)}$: background term

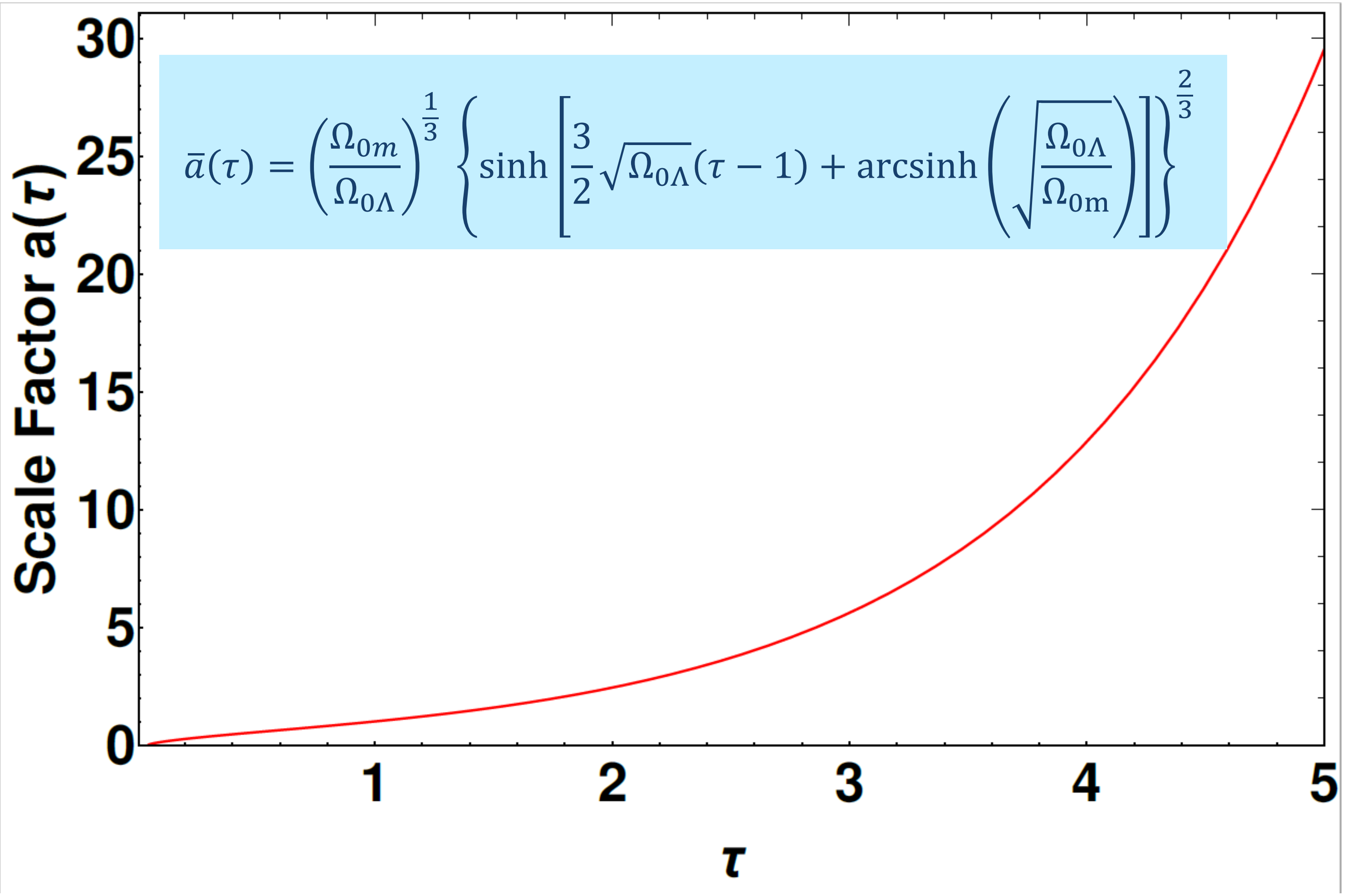
$\delta(\dots)$: perturbative term

$\delta(\dots) \ll \overline{(\dots)}$

SCHIAVONE, T., MONTANI, G., (in prep.)
P. MARCOCCIA, G. MONTANI, *Weakly Inhomogeneous models for the Low-Redshift Universe*, arXiv:1808.01489v2 [gr-qc].

BACKGROUND ΛCDM SOLUTION

Analytical solution, solving Einstein field eqs.



Background metric functions → FLRW metric

$\bar{\alpha} = \ln \bar{a}$ and $\bar{\beta} = \ln(r\,\bar{a})$

Radiation component negligible in the late Universe: $\Omega_{0r} \sim 0$

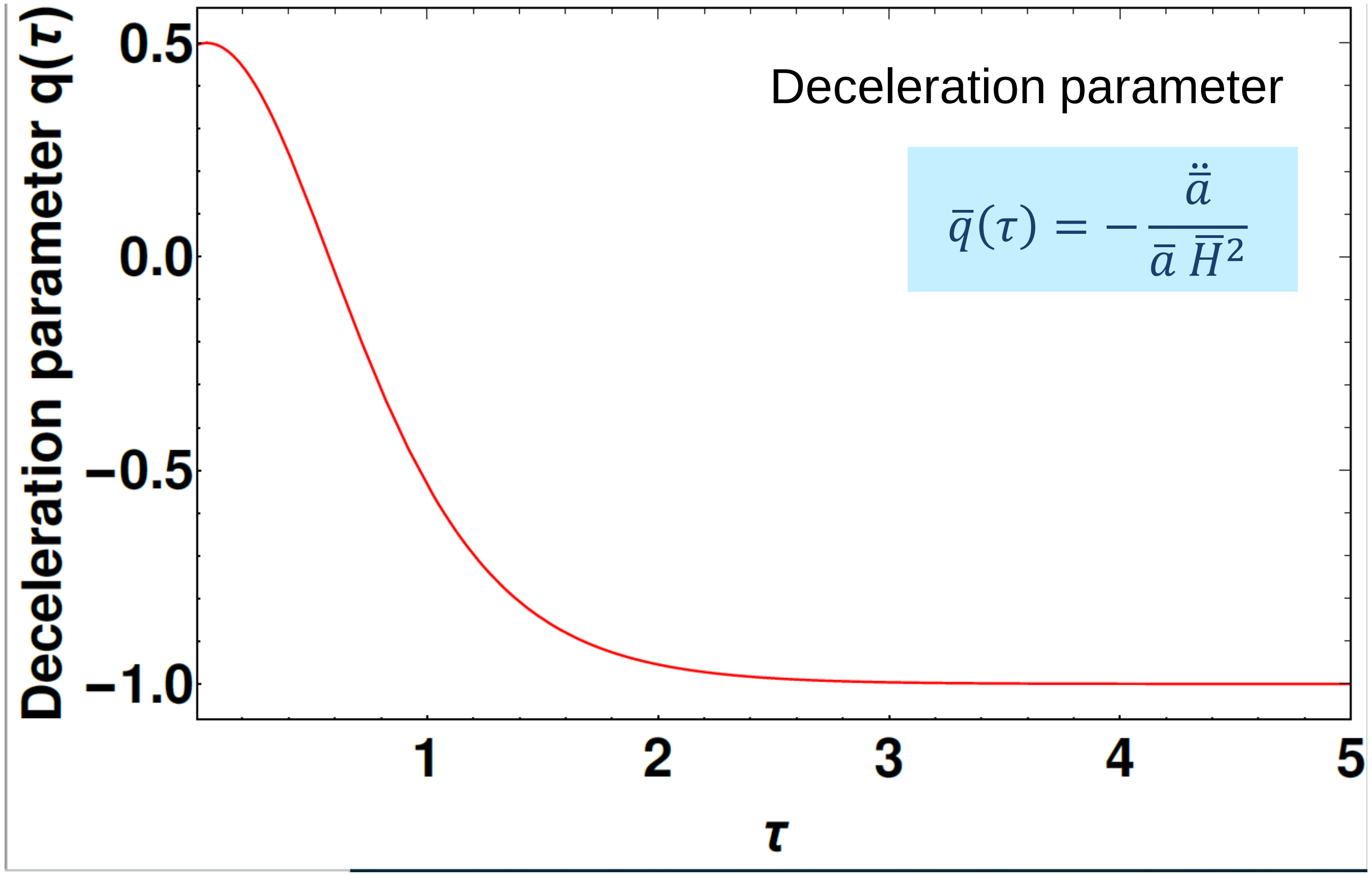
Dimensionless time variable $\tau \equiv \frac{t}{t_0}$

t_0: today cosmic time ($\tau = 1$)

Condition: $\bar{a}(\tau = 1) \equiv 1$

BACKGROUND ΛCDM SOLUTION

Analytical solution, solving Einstein field eqs.



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Dimensionless time variable $\tau \equiv \frac{t}{t_0}$

t_0: today cosmic time ($\tau = 1$)

$\bar{q}(\tau \to \infty) \to -1$

FIRST ORDER PERTURBATION - ΛLTB SOLUTION
TIME EVOLUTION

First order perturbations

$$a(t,r) = \bar{a}(t) + a_p(t,r) \qquad a_p(t,r) = A_p(t)\,\mathcal{A}_p(r)$$

$$\rho(t,r) = \bar{\rho}(t) + \delta\rho(t,r) \qquad \delta\rho(t,r) = \delta\mathrm{P}(t)\,\delta\varrho(r)$$

Solving the linearized Einstein field eq.11 and using the background solution $\bar{a}$

$$\ddot{A}_p + \dot{A}_p\,\frac{\dot{\bar{a}}}{\bar{a}} - A_p\left[\frac{\ddot{\bar{a}}}{\bar{a}} + \left(\frac{\dot{\bar{a}}}{\bar{a}}\right)^2\right] = 0$$

No analytical solution. It can be solved numerically

Initial conditions:

$$\begin{cases} A_p(\tau = 1) = 10^{-5} \\ \dot{A}_p(\tau = 1) = 0 \end{cases}$$

FIRST ORDER PERTURBATION - ΛLTB SOLUTION

TIME EVOLUTION

Solving the linearized Einstein field eq.11 and using the background solution $\bar{a}$

$$\ddot{A}_p + \dot{A}_p \frac{\dot{\bar{a}}}{\bar{a}} - A_p \left[\frac{\ddot{\bar{a}}}{\bar{a}} + \left(\frac{\dot{\bar{a}}}{\bar{a}} \right)^2 \right] = 0$$

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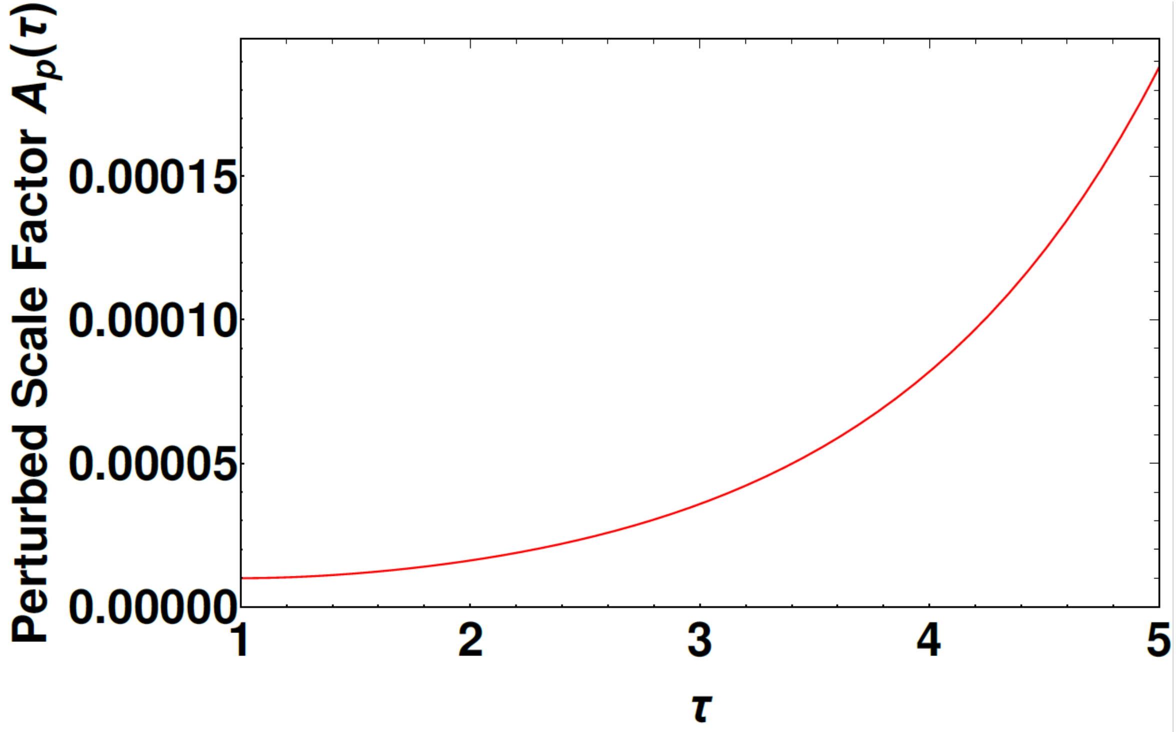
First order perturbations

$$a(t,r) = \bar{a}(t) + a_p(t,r) \qquad a_p(t,r) = A_p(t)\,\mathcal{A}_p(r)$$

$$\rho(t,r) = \bar{\rho}(t) + \delta\rho(t,r) \qquad \delta\rho(t,r) = \delta\mathrm{P}(t)\,\delta\varrho(r)$$



FIRST ORDER PERTURBATION - ΛLTB SOLUTION

TIME EVOLUTION

Solving the linearized Einstein field eq.11 and using the background solution $\bar{a}$

$$\ddot{A}_p + \dot{A}_p \frac{\dot{\bar{a}}}{\bar{a}} - A_p \left[\frac{\ddot{\bar{a}}}{\bar{a}} + \left(\frac{\dot{\bar{a}}}{\bar{a}}\right)^2\right] = 0$$

No analytical solution. It can be solved numerically

Initial conditions:

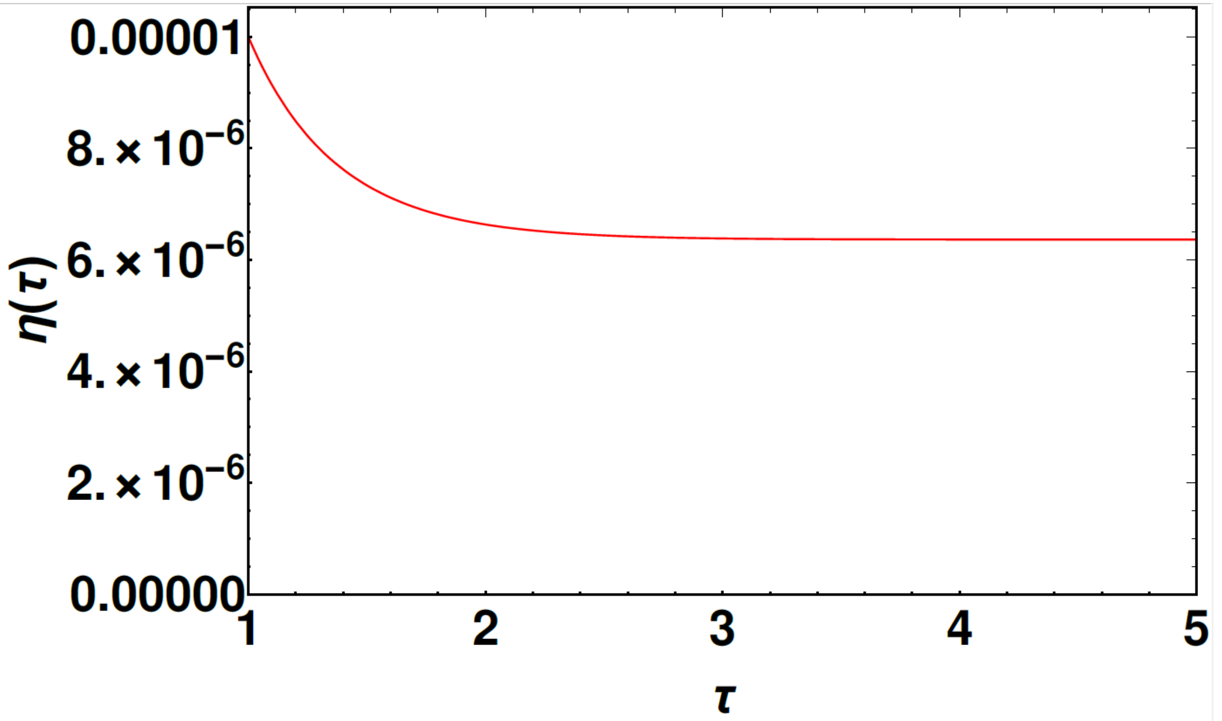
$$\begin{cases} A_p(\tau = 1) = 10^{-5} \\ \dot{A}_p(\tau = 1) = 0 \end{cases}$$

Ratio: $\eta(\tau) = \frac{A_p(\tau)}{\bar{a}(\tau)} \ll 1$ $\quad \frac{Perturbation\ term}{Background\ term}$

First order perturbations

$$a(t,r) = \bar{a}(t) + a_p(t,r) \qquad a_p(t,r) = A_p(t)\,\mathcal{A}_p(r)$$

$$\rho(t,r) = \bar{\rho}(t) + \delta\rho(t,r) \qquad \delta\rho(t,r) = \delta\mathrm{P}(t)\,\delta\varrho(r)$$



FIRST ORDER PERTURBATION - ΛLTB SOLUTION
RADIAL DEPENDENCE

First order perturbations

$$a(t,r) = \bar{a}(t) + a_p(t,r) \qquad a_p(t,r) = A_p(t)\,\mathcal{A}_p(r)$$

$$\rho(t,r) = \bar{\rho}(t) + \delta\rho(t,r) \qquad \delta\rho(t,r) = \delta\mathrm{P}(t)\,\delta\varrho(r)$$

Starting from the linearized Einstein field eq.11 and using the separation of variables method:

$$\mathcal{A}_p(r) = K^2(r) \longrightarrow a_p(t,r) = A_p(t)\,K^2(r)$$

From the linearized Einstein field eq.00 in r:

$$r\,\mathcal{A}_p'(r) + 3\mathcal{A}_p(r) = 0 \longrightarrow \mathcal{A}_p(r) \propto \frac{1}{r^3}$$

Inhomogeneous perturbations go asymptotically to zero as the radial coordinate increases, according to the cosmological principle

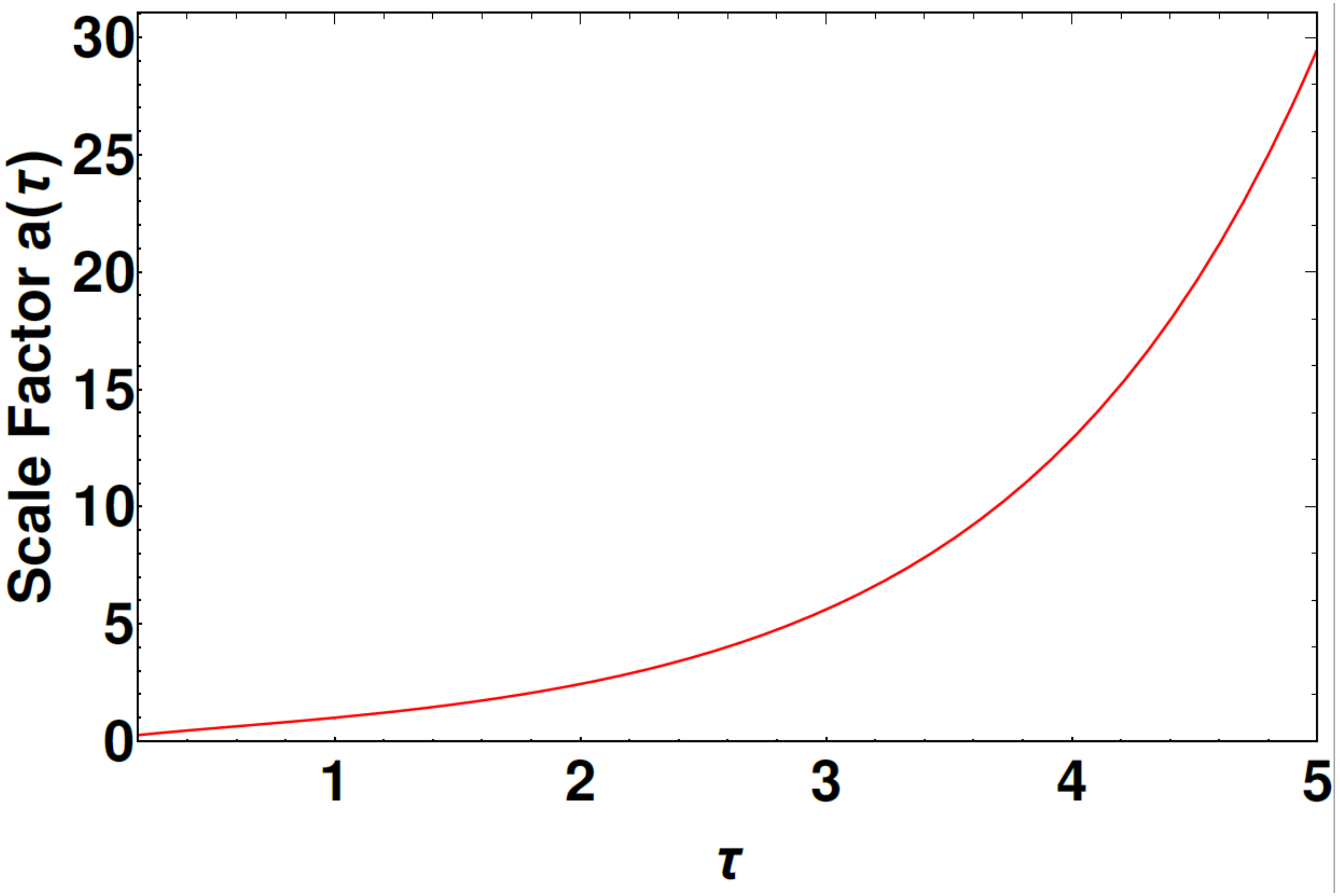
Matter contribution

$\delta\varrho(r)$ Radial part of energy density perturbation

Free function
It is not constrained by the linearized continuity equation

BACKGROUND HU-SAWICKI SOLUTION IN THE JF

Numerical solution, integrating modified gravity field eqs. in the Jordan frame



Radiation component negligible in the late Universe: $\Omega_{0r} \sim 0$

Conditions:

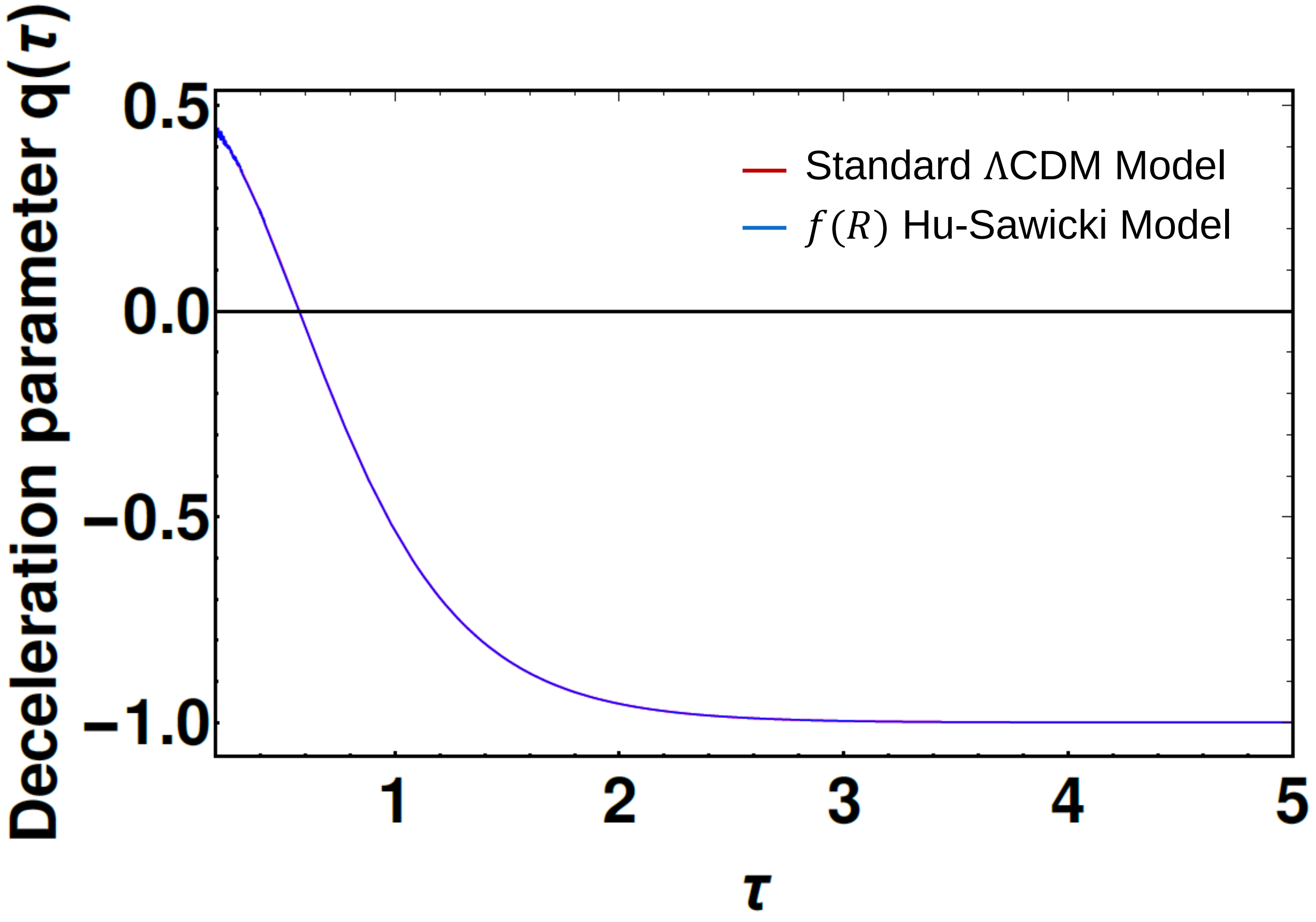
$$\begin{cases} \bar{a}(\tau = 1) = 1 \\ \bar{\phi}(\tau = 1) = 1 - |F_{R0}| \\ \frac{d\bar{\phi}}{d\tau}(\tau = 1) = 0 \end{cases}$$

We set $|F_{R0}| = 10^{-7}$

Background evolution close to the ΛCDM scenario

BACKGROUND HU-SAWICKI SOLUTION IN THE JF

Numerical solution, integrating modified gravity field eqs. in the Jordan frame



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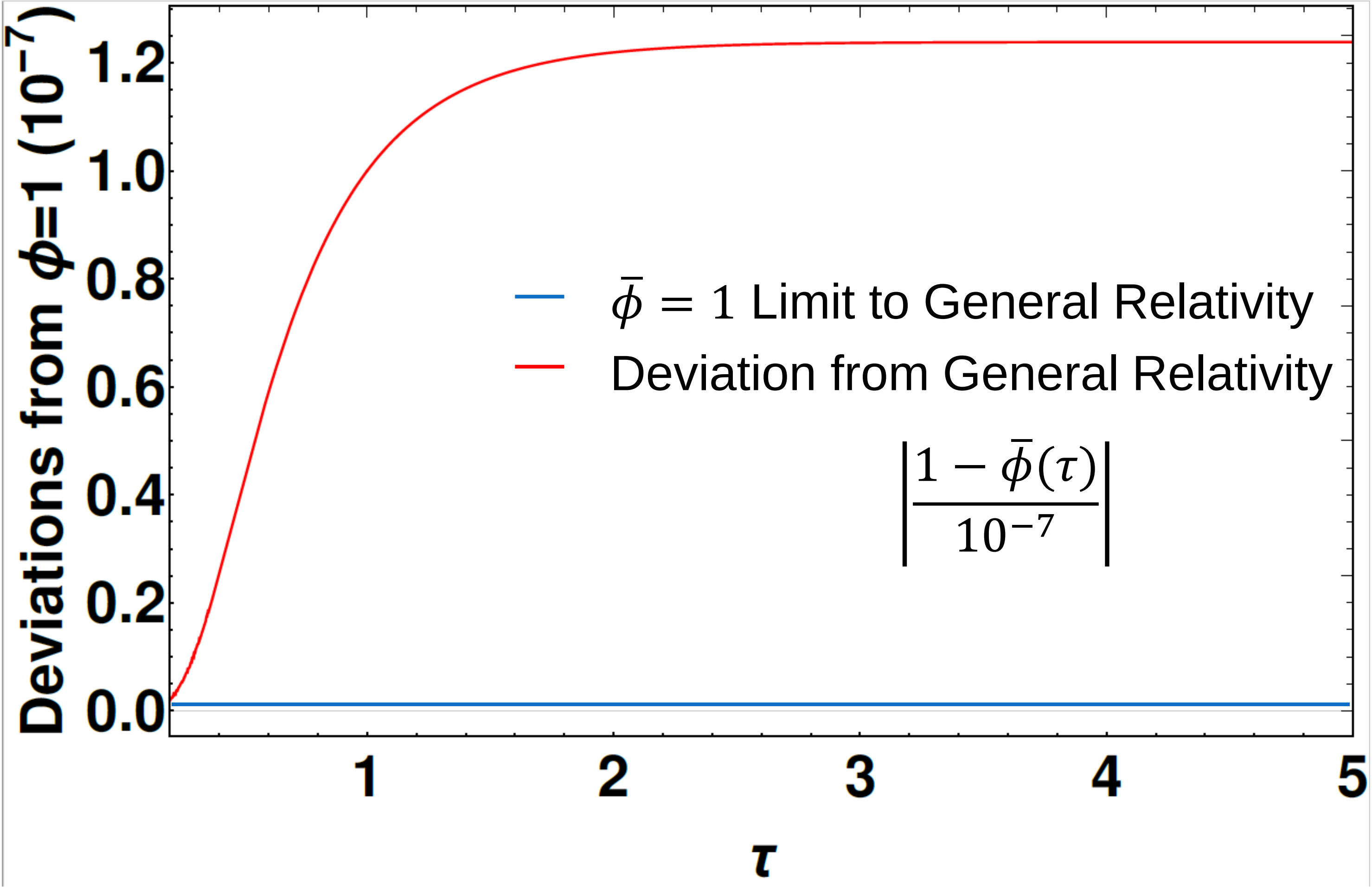

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$$\frac{d\bar{\phi}}{d\tau}(\tau = 1) = 0$$

We set $|F_{R0}| = 10^{-7}$

Background evolution close to the ΛCDM scenario

FIRST ORDER PERTURBATION – JF SOLUTION

Starting from the field eqs. in the JF, we obtain linearized eqs. and we study separately time and radial evolution of inhomogeneous solutions.

First order perturbations

$\alpha(t,r)=\bar{\alpha}(t)+\delta\alpha(t,r)$	$\delta\alpha(t,r)=\delta A(t)\,\delta\mathcal{A}(r)$
$\beta(t,r)=\bar{\beta}(t,r)+\delta\beta(t,r)$	$\delta\beta(t,r)=\delta B(t)\,\delta\mathcal{B}(r)$
$\rho(t,r)=\bar{\rho}(t)+\delta\rho(t,r)$	$\delta\rho(t,r)=\delta\mathrm{P}(t)\,\delta\varrho(r)$
$\phi(t,r)=\bar{\phi}(t)+\delta\phi(t,r)$	$\delta\phi(t,r)=\delta\Phi(t)\,\delta\varphi(r)$

It can be shown that the potential affects only the time evolution of the linearized field equations system

$$V(\phi)=V(\bar{\phi}+\delta\phi)\approx V(\bar{\phi})+\left.\frac{dV}{d\phi}\right|_{\bar{\phi}}\delta\phi$$

Analytical solution for the radial part; numerical integration of the system for time solutions.

FIRST ORDER PERTURBATION – JF SOLUTION
RADIAL DEPENDENCE

From the separation of variable method, we obtain:

$$\delta\varrho(r) \propto \delta\varphi(r)$$

Combining the field eqs., we obtain a Yukawa-like solution (screening mechanism). Inhomogeneities go to zero faster than ones in GR.

Solution independent of $V(\phi)$
It applies to any $f(R)$ model

$$\delta\varphi(r) = \frac{C}{r}\, e^{-\frac{r}{r_C}}$$

Yukawa-like solution

$$\delta\varrho(r) = \frac{\lambda_3\, C}{r}\, e^{-\frac{r}{r_C}}$$

Constants: $C, r_C, \lambda_1, \lambda_3,\ \mu_1, \mu_2, \mu_3$
with $\lambda_4 = [r_C^2\,(3\,\mu_1 + \mu_2\,\lambda_3)]^{-1}$

$$\delta\mathcal{A}(r) = \frac{C}{\lambda_1}\left(\frac{2\,\mu_1 + \mu_2\,\lambda_3}{r} + \frac{4}{\lambda_4\, r_c\, r^2} + \frac{4}{\lambda_4\, r^3}\right) e^{-\frac{r}{r_C}}$$

$$\delta\mathcal{B}(r) = -\frac{C}{\lambda_4}\left(\frac{\mu_1\,\lambda_4}{r} + \frac{2}{r_C\, r^2} + \frac{2}{r^3}\right) e^{-\frac{r}{r_C}}$$

FIRST ORDER PERTURBATION – JF SOLUTION
TIME EVOLUTION

Linearized field eqs. system can be numerically solved

Eq. 01 $\delta\dot{A} = \lambda_1\,\delta\dot{B}$ $\delta\dot{B} = \frac{1}{2\,\bar{\phi}\,\mu_1}\left(\delta\dot{\Phi} - \frac{\dot{\bar{a}}}{\bar{a}}\delta\Phi\right)$

Conditions:
$$\eta_\rho(\tau = 1) = 10^{-5}$$
$$\delta\Phi(\tau = 1) = 10^{-12} \ll |F_{R0}|$$
$$\frac{d\,\delta\Phi}{d\tau}(\tau = 1) = 0$$

Continuity equation $\dot{\eta}_\rho + \tau_1\delta\dot{B} = 0$ with $\eta_\rho(\tau) = \frac{\delta\mathrm{P}(\tau)}{\bar{\rho}(\tau)}$ density contrast

Constants: $\lambda_1, \lambda_3,\ \tau_1, \mu_1, \mu_3$

Scalar field eq.
$$3\,\delta\ddot{\Phi} + 9\,\frac{\dot{\bar{a}}}{\bar{a}}\,\delta\dot{\Phi} + \left[\Omega_{0m}\frac{1}{m^2}\left(\frac{dV}{d\phi}\Big|_{\bar{\phi}} - \bar{\phi}\,\frac{d^2V}{d\phi^2}\Big|_{\bar{\phi}}\right) - \frac{\mu_3}{\bar{a}^2}\right]\delta\Phi + 3\tau_1\,\lambda_3\,\dot{\bar{\phi}}\,\delta\dot{B} - 3\,\lambda_3\,\frac{\Omega_{0m}}{\bar{a}^3}\,\eta_\rho = 0$$

CONCLUSIONS

- Inhomogeneities in the Lemaitre-Tolman-Bondi metric regarded as small deviations from a flat FLRW background metric
- One possible method to discriminate between GR and f(R) modified gravity consists in studying the evolution of inhomogeneous perturbations
- Different solutions at first-order perturbation theory
- The radial evolution of inhomogeneous perturbations within the framework of Jordan frame gravity is independent of the scalar field potential $V(\phi)$: the distinctive radial solution is a feature of any $f(R)$ model

Thank you
for the attention

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