KING'S College LONDON Founded 1829

Gravitational Anomalies, Dark Matter and Leptogenesis

Sarben Sarkar

Dept. of Physics, King's College London, UK

References:

M de Cesare, N E Mavromatos, S Sarkar, Eur. Phys. J C **75** 514 (2015)
T Bossingham, N E Mavromatos, S Sarkar, Eur Phys J C **78** 113 (2018)
T Bossingham, N E Mavromatos and S Sarkar, Eur. Phys. J. C 79 50 (2019)
N E Mavromatos and S Sarkar, Universe 5, no. 1:5 https://doi.org/10.3390/5010005
N E Mavromatos and S Sarkar, Eur. Phys. C **80** 558 (2020)
J Ellis, N E Mavromatos and S Sarkar, Phys. Lett. B **725** 407 (2013)

Science & Technology Facilities Council

Tie together Bary(Lepto)ogenesis (BAU), dark matter and neutrino mass from a string-inspired perspective

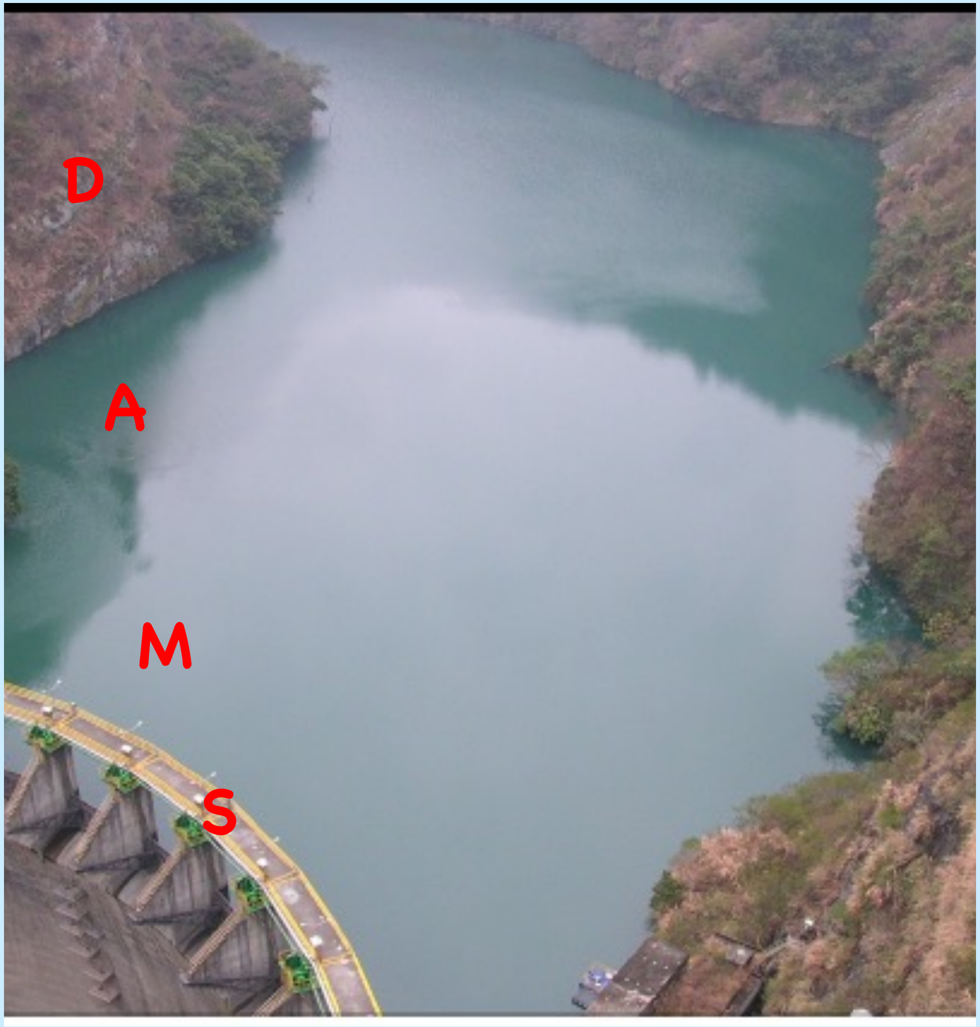
Our recipe:

D Dark matter
(gravitational axion)

A Anomalies
(gravitational + gauge)

M ordinary matter
(SM)

S strings
(gravity with torsion)



Components of the Universe's energy density:

- ρ_0 component energy density
- ρ_c critical energy density

- $\Omega = \frac{\rho_0}{\rho_c}$, $\Omega_{matter} = 0.27 \pm 0.04$, $\Omega_B = 0.044 \pm 0.004$
- Ω_B is the ratio for baryonic matter

- n_B baryon number density, n_γ photon number density
- $\frac{n_B}{n_\gamma} = 6.1 \pm 0.3 \times 10^{-10}$ at $\mathrm{T} \sim 1GeV$
- If Universe had been matter-antimatter symmetric $\frac{n_B}{n_\gamma} = \frac{n_{\overline{B}}}{n_\gamma} \approx 10^{-18}$
- Leptogenesis: creation of primordial lepton-antilepton asymmetry (M Fukugita and T Yanagida, Phys. Lett. B174 45 (1986))

Sakharov's Baryogenesis recipe (A D Sakharov, JETP Lett. **5** 24 (1967))

Baryon number violating processes (sphaleron) in Standard Model

C and CP violation

$$L(X \to \lambda + p) \neq L(\underline{X} \to \underline{\lambda} + \underline{p})$$

$$X \to Y + b \quad \Delta B = 1 \qquad \bar{X} \to \bar{Y} + \bar{b} \quad \Delta B = -1$$

CPT invariance assumed

Out of equilibrium: first order electroweak phase transition

$$\Gamma(X \to Y + b) \neq \Gamma(Y + b \to X)$$

CPT symmetry

C is charge conjugation
P is parity (space reflection)
T is time-reversal

$$\Theta = CPT$$

CPT theorem:
For any Lorentz invariant Hermitian local Lagrangian

$$\Theta L(x) \Theta^{-1} = L^{\dagger}(-x)$$

Anomalies here there and everywhere: (literally the spice of life)

- **Electroweak interactions chiral**
- $J_{\mu}^{B}=\frac{1}{3}\sum_{f}\left(\overline{q_{fL}}\gamma_{\mu}q_{fL}+\overline{u_{R}}\gamma_{\mu}u_{R}+\overline{d_{R}}\gamma_{\mu}d_{R}\right)$

  $J_{\mu}^{L}=\sum_{f}\left(\overline{l_{fL}}\gamma_{\mu}l_{fL}+\overline{e_{fR}}\gamma_{\mu}e_{fR}\right)$

- **Triangle (ABJ) anomaly** **(S L Adler, Phys. Rev 177 2426 (1969))**
- $\partial^{\mu}J_{\mu}^{L}=\partial^{\mu}J_{\mu}^{B}=\frac{N_f}{32\pi^2}\left(-g^2W_{\mu\nu}\widetilde{W}^{\mu\nu}+g'^2B_{\mu\nu}\widetilde{B}^{\mu\nu}\right)$

  where the $W_{\mu\nu}$ and $B_{\mu\nu}$ are the $SU(2)$ and $U(1)$ gauge field two forms
- $B-L$ **is conserved** ⇨ **Leptogenesis leads to baryogenesis**

- Sakharov recipe difficulties: insufficient CP violation in the Standard Model, electroweak phase transition required to be 1st order

- DAMS ingredients beyond SM

Bosonic gravitational multiplet of strings consists of a
- graviton, $g_{\mu\nu}$
- spin 0 scalar field Φ the dilaton
- spin 1 antisymmetric gauge field $B_{\mu\nu}$ with gauge invariant field strength

axion

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} \qquad e^{-2\Phi} \varepsilon_{bac}{}^{\mu} H^{abc} = 4\partial^{\mu} b(x) = 4B_{\mu}(x)$$

- N right-handed sterile neutrino with large Majorana mass
- A CPTV background with non-zero $B_{\mu}(x)$

Strings ⇒ Gravity with torsion

- The Kalb-Ramond field tensor in the low energy string gravitational action appears as:

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R-\frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)=\int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}\overline{R}\right)$$

where $\overline{R}$ is the curvature for the *affine* connection

$$\overline{\Gamma}^{\mu}_{\nu\rho}=\Gamma^{\mu}_{\nu\rho}+\frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho}\neq\overline{\Gamma}^{\mu}_{\rho\nu}$$

(D J Gross and J H Sloan, Nucl. Phys. B 291 41 (1987)

contorsion

- Torsion tensor $T^{\kappa}_{\lambda\mu}=\Gamma^{\kappa}_{\lambda\mu}-\Gamma^{\kappa}_{\mu\lambda}$

- In a generlisation of Einstein's theory: add torsion to get Einstein-Cartan theory

- Torsion is necessary to incorporate fermions

DAMS $\Rightarrow$ Leptogenesis effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \mathrm{i}\,\overline{N}\gamma^{\mu}\partial_{\mu}N - \frac{m_N}{2}\left(\overline{N^c}N + \overline{N}N^c\right) - \overline{N}\gamma^{\mu}B_{\mu}\gamma^{5}N - \sum_k y_k \overline{L}_k \tilde{\phi} N + h.c.$$

CPTV and LIV background from torsion

- Model leads to acceptable leptogenesis at the tree level, owing to Majorana nature of N which is taken to be very heavy
- Freeze out of *N at* $T = T_D$.
- *B-L* conservation converts ΔL to ΔB
- $B_{\mu} = \partial_{\mu} b$
- Non-zero B_0 background
- The *underlying torsion* connects this model to dark matter

Details of leptogenesis

- Stick to $k = 1$ for simplicity
- Background:

$$B_i = 0, \qquad i = 1, 2, 3$$
$$B_0(z) = \Phi f(z)$$

where $z = \frac{m_N}{T}$, and $f(z) = 1$ or $f(z) = z^{-3}$.

- Gravitational background

$g_{00} = 1$, $g_{ij} = -a^2(t)\delta_{ij}$, space Cartesian

- Generation of lepton asymmetry due to CP and CPTV tree-level decays:
- $N \to l^- h^+, \upsilon h^0$ channel 1
- $N \to l^+ h^-, \overline{\upsilon} h^0$ channel 2

where $l^\pm$ are charged leptons, $\upsilon\ (\overline{\nu})$ are active (anti)neutrinos

- For $\Phi \neq 0$ the decay rates of N differ for channels 1 and 2 leading to a lepton asymmetry which freezes out at T_D.

Results for leptogenesis

$\frac{\Delta L^{tot}}{s} \simeq .017 \ \frac{B_0}{m_N}$ at freezeout temperature $T = T_D$

$$\frac{m_N}{T_D} \simeq 1.6$$

For $y \sim 10^{-5}$ and $m_N = \mathrm{O}(100) TeV$ and $B_0 \sim 0.1 MeV$ for phenomenologically acceptable leptogenesis to occur at

$$T_D \sim 60 TeV$$

Gravitational axions as Dark Matter I

- **Peccei and Quinn solution of strong CP problem**

$$\mathcal{L} \supset \vartheta(x)\frac{\alpha_s}{2\pi}Tr(G \wedge G)$$

where the ϑ vacuum parameter becomes a field and G is the QCD gauge field 2-form (R D Peccei and H R. Quinn, Phys Rev. Lett 38 1440 (1977))

Standard procedure: select a vacuum with $\langle\vartheta(x)\rangle = 0$ with a fluctuation a(x)

- ***Move away from Goldstone axion a(x)*** **to gravitational axion b(x) whose couplings are determined by the Planck scale**
- **Classical torsionful spacetime manifold characterised by:**

$\underline{e}^a = e^a_\mu \, dx^\mu$

$\underline{\omega}^{ab} = \omega^{ab}_\mu dx^\mu$

torsion 2-from

$$d\underline{e}^a + \underline{\omega}^a{}_b \wedge \underline{e}^b = \mathfrak{I}^a,$$

curvature 2-from

$$d\underline{\omega}^a{}_c + \underline{\omega}^a{}_b \wedge \underline{\omega}^b{}_c = \underline{R}^a{}_c$$

where $\underline{e}^a$ is the vierbein and $\underline{\omega}^a{}_b$ is the spin-connection 1-forms

Gravitational axions as Dark Matter II

The action of fermions of SM on a torsionful manifold is given by

$$S = S_{gravity} - \frac{1}{2}\sum_f \int \left(\overline{\Psi}_f \underline{\gamma} \wedge * \mathcal{D}\Psi_f - \mathcal{D}\overline{\Psi}_f \wedge * \underline{\gamma}\Psi_f\right) - \frac{1}{2}\int \mathcal{F} \wedge * \mathcal{F} - \int Tr[G \wedge * G]$$

where for any fermionic field Ψ

$$S_{gravity} = \frac{1}{4\kappa^2}\int \varepsilon_{abcd}\, \underline{\mathcal{R}}^{ab} \wedge \underline{e}^c \wedge \underline{e}^d$$

$$\mathcal{D}\Psi = d\Psi + \frac{1}{4}\underline{\omega}^{ab}\gamma_{ab}\Psi + ie\underline{\mathcal{A}}\Psi + ig\underline{\mathcal{B}}\,\Psi, \qquad \kappa^2 = 8\pi M_{Pl}^2$$

with $\underline{\mathcal{A}}$ and $\underline{\mathcal{B}}$ the gauge connection 1-forms for $U(1)_{em}$ and $SU(3)_c$, $\underline{\gamma} = \gamma_a \underline{e}^a$ and $\gamma_{ab} = \frac{1}{2}[\gamma_a, \gamma_b]$.

Equation of motion with respect to $\underline{\omega}^{ab}$ gives that the torsion satisfies

$$\mathcal{T}_{abc} = -\frac{\kappa^2}{2}\varepsilon_{abcd}\sum_f J_f^{5\,d}$$

where $J_f^{5\,d} = i\overline{\Psi}_f\gamma^d\gamma^5\Psi_f$. (See also R Utiyama, Phys Rev 101, 1597 (1956); T W B Kibble, J Math Phys 2 212 (1961);D W Sciama, Monthly Not. Roy, Astr. Soc. 113 34 (1953); F W Hehl, Gen Rel Grav 4 333 (1973); E Cartan, Riemannian Geometry in an Orthogoanl Frame (World Scientific, 2001), O Casillo-Feliso;a et al. Phys Rev D 91 085017 (2015))

Anomalies

(cf S Basilakos et al. PRD **101 045001 (2020)**
M J Duncan, N Kaloper, K A Olive, Nucl. Phys. B387 215 (1992))

Writing $\underline{J}^5 = \sum_f (J_f^5)_a \underline{e}^a$ as a 1-form the axial anomaly implies

$$d * \underline{J}^5 = -\frac{\alpha_{em}\overline{Q}^2}{\pi}\mathcal{F}\wedge\mathcal{F} - \frac{\alpha_s N_q}{2\pi} Tr[G\wedge G] - \frac{N_f}{8\pi^2}\widetilde{R}^{ab}\wedge\widetilde{R}^{ab}$$

which leads to the following *quantised* torsion-field contribution to the action

$$-\frac{\alpha_{em}\overline{Q}^2}{\pi f_b}\int b\,\mathcal{F}\wedge\mathcal{F} - \frac{1}{2}\int d\mathbf{b}\wedge * \,db - \frac{1}{8\pi^2}\int\left(\Theta + \frac{N_f}{f_b}b\right)\widetilde{R}^{ab}\wedge\widetilde{R}_{ab} - \frac{\alpha_s}{2\pi}\int\left(\theta + \frac{N_q}{f_b}b\right)Tr[G\wedge G]$$

The parameter $f_b = \kappa^{-1}\sqrt{8/3} = 4\times 10^{18}$ GeV.

Θ and θ terms label Pontryagin densities, are allowed in the theory, play the role of counterterms and do not affect the equations of motion.

From the last term see that *b* solves the strong CP problem and $m_b \sim 10^{-12} eV$ and so can be regarded as the QCD axion.

Concluding remarks

- The gravitational anomaly (GA) may not be cancelled in the early universe in a quantum theory of gravity.

- The gauge anomaly though can be designed to cancel for a suitable GUT theory. Anomaly cancellation is necessary for the theories to be renormalizable.

- GA implies breakdown of diffeomorphism Invariance and hence local Lorentz invariance (LIV)

- LIV allows a non-zero Φ required for our leptogenesis model

- The Kalb-Ramond axion could play the role of a QCD axion and so be axionic dark matter (M Lattanzi and S Mercuri, Phys Rev D 81 125015 (2010))