Gravitational Anomalies, Dark Matter and Leptogenesis

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References:
N E Mavromatos and S Sarkar, Universe 5, no. 1:5 https://doi.org/10.3390/5010005
Tie together
Bary(Lepto)ogenesis (BAU),
dark matter and neutrino
mass from a string-inspired
perspective

Our recipe:

D Dark matter
  (gravitational axion)

A Anomalies
  (gravitational + gauge)

M ordinary matter
  (SM)

S strings
  (gravity with torsion)
Components of the Universe’s energy density:

- $\rho_0$ component energy density
- $\rho_c$ critical energy density

$$\Omega = \frac{\rho_0}{\rho_c}, \; \Omega_{\text{matter}} = 0.27 \pm 0.04, \; \Omega_B = 0.044 \pm 0.004$$

- $\Omega_B$ is the ratio for baryonic matter

- $n_B$ baryon number density, $n_\gamma$ photon number density

$$\frac{n_B}{n_\gamma} = 6.1 \pm 0.3 \times 10^{-10} \; \text{at} \; T \sim 1\,\text{GeV}$$

- If Universe had been matter-antimatter symmetric $\frac{n_B}{n_\gamma} = \frac{n_B}{n_\gamma} \approx 10^{-18}$

Sakharov's Baryogenesis recipe

(A D Sakharov, JETP Lett. 5 24 (1967))

Baryon number violating processes (sphaleron) in Standard Model

C and CP violation

\[ L(\chi \rightarrow \chi + p) \neq L(\bar{\chi} \rightarrow \bar{\chi} + p) \]

\[ X \rightarrow Y + b \quad \Delta B = 1 \quad \bar{X} \rightarrow \bar{Y} + \bar{b} \quad \Delta B = -1 \]

CPT invariance assumed

Out of equilibrium: first order electroweak phase transition

\[ \Gamma(X \rightarrow Y + b) \neq \Gamma(Y + b \rightarrow X) \]

CPT symmetry

C is charge conjugation
P is parity (space reflection)
T is time-reversal

\[ \Theta = CPT \]

CPT theorem:
For any Lorentz invariant Hermitian local Lagrangian

\[ \Theta L(x) \Theta^{-1} = L^\dagger(-x) \]
Anomalies here there and everywhere: (literally the spice of life)

- Electroweak interactions chiral

\[ J^B_\mu = \frac{1}{3} \sum_f (\bar{q}_{fL} \gamma_\mu q_{fL} + \bar{u}_{R} \gamma_\mu u_{R} + \bar{d}_{R} \gamma_\mu d_{R}) \]

\[ J^L_\mu = \sum_f (\bar{l}_{fL} \gamma_\mu l_{fL} + \bar{e}_{fR} \gamma_\mu e_{fR}) \]

- Triangle (ABJ) anomaly (S L Adler, Phys. Rev 177 2426 (1969))

\[ \partial^\mu J^L_\mu = \partial^\mu J^B_\mu = \frac{N_f}{32\pi^2} (-g^2 W_{\mu\nu} \tilde{W}^{\mu\nu} + g'^2 B_{\mu\nu} \tilde{B}^{\mu\nu}) \]

where the \( W_{\mu\nu} \) and \( B_{\mu\nu} \) are the \( SU(2) \) and \( U(1) \) gauge field two forms

- \( B - L \) is conserved  

Leptogenesis leads to baryogenesis

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Sakharov recipe difficulties: insufficient CP violation in the Standard Model, electroweak phase transition required to be 1st order

DAMS ingredients beyond SM

Bosonic gravitational multiplet of strings consists of a
- graviton, $g_{\mu\nu}$
- spin 0 scalar field $\Phi$ the dilaton
- spin 1 antisymmetric gauge field $B_{\mu\nu}$ with gauge invariant field strength

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$$

$$e^{-2\Phi} \varepsilon_{b\mu c} H^{abc} = 4 \partial_{\mu} b(x) = 4 B_{\mu}(x)$$

- $N$ right-handed sterile neutrino with large Majorana mass
- A CPTV background with non-zero $B_{\mu}(x)$
Strings \rightarrow Gravity with torsion

The Kalb–Ramond field tensor in the low energy string gravitational action appears as:

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} \bar{R} \right) \]

where \( \bar{R} \) is the curvature for the affine connection

\[
\bar{\Gamma}_\mu^{\nu\rho} = \Gamma_\mu^{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^\mu_{\nu\rho} \neq \bar{\Gamma}_\rho^{\mu\nu}
\]


• Torsion tensor \( T^K_{\lambda\mu} = \Gamma^K_{\lambda\mu} - \Gamma^K_{\mu\lambda} \)

• In a generalisation of Einstein’s theory: add torsion to get Einstein–Cartan theory

• Torsion is necessary to incorporate fermions
DAMS

Leptogenesis effective Lagrangian

\[ \mathcal{L} = \mathcal{L}_{SM} + i \bar{N} \gamma^\mu \partial_\mu N - \frac{m_N}{2} (\bar{N}^c N + \bar{N} N^c) - \bar{N} \gamma^\mu B_\mu \gamma^5 N - \sum_k y_k \bar{L}_k \tilde{\Phi} N + h.c. \]

CPTV and LIV background from torsion

- Model leads to acceptable leptogenesis at the tree level, owing to Majorana nature of \( N \) which is taken to be very heavy.
- Freeze out of \( N \) at \( T = T_D \).
- \( B-L \) conservation converts \( \Delta L \) to \( \Delta B \).
- \( B_\mu = \partial_\mu b \).
- Non-zero \( B_0 \) background.
- The underlying torsion connects this model to dark matter.
Details of leptogenesis

- **Stick to** $k = 1$ **for simplicity**
- **Background:**
  \[ B_i = 0, \quad i = 1, 2, 3 \]
  \[ B_0(z) = \Phi f(z) \]
  where $z = \frac{m_N}{T}$, and $f(z) = 1$ or $f(z) = z^{-3}$.

- **Gravitational background**
  \[ g_{00} = 1, \quad g_{ij} = -a^2(t)\delta_{ij}, \text{ space Cartesian} \]
- **Generation of lepton asymmetry due to CP and CPTV tree-level decays:**
  - $N \to l^- h^+, \nu h^0$ channel 1
  - $N \to l^+ h^-, \overline{\nu} h^0$ channel 2
  where $l^\pm$ are charged leptons, $\nu$ ($\overline{\nu}$) are active (anti)neutrinos
- **For** $\Phi \neq 0$ **the decay rates of** $N$ **differ for channels 1 and 2 leading to a lepton asymmetry which freezes out at** $T_D$. 

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Results for leptogenesis

\[ \frac{\Delta L_{\text{tot}}}{s} \approx 0.017 \frac{B_0}{m_N} \quad \text{at freezeout temperature} \quad T = T_D \]

\[ \frac{m_N}{T_D} \approx 1.6 \]

For \( y \sim 10^{-5} \) and \( m_N = 0(100)\text{TeV} \) and \( B_0 \sim 0.1\text{MeV} \) for phenomenologically acceptable leptogenesis to occur at

\[ T_D \sim 60\text{TeV} \]
Gravitational axions as Dark Matter

- Peccei and Quinn solution of strong CP problem
  \[ \mathcal{L} \supset \vartheta(x) \frac{\alpha_s}{2\pi} Tr(G \wedge G) \]
  where the \( \vartheta \) vacuum parameter becomes a field and \( G \) is the QCD gauge field 2-form (R D Peccei and H R. Quinn, Phys Rev. Lett 38 1440 (1977))

Standard procedure: select a vacuum with \( \langle \vartheta(x) \rangle = 0 \) with a fluctuation \( a(x) \)

- \textit{Move away from Goldstone axion} \( a(x) \) to gravitational axion \( b(x) \) whose couplings are determined by the Planck scale
- Classical torsionful spacetime manifold characterised by:
  \[ e^a = e^a_\mu \, dx^\mu \]
  \[ \omega^{ab} = \omega^{ab}_\mu \, dx^\mu \]

  where \( e^a \) is the vierbein and \( \omega^{ab}_a \) is the spin-connection 1-forms

  \[ de^a + \omega^a_b \wedge e^b = \zeta^a, \]
  \[ d\omega^a_c + \omega^a_b \wedge \omega^b_c = R^a_c \]
Gravitational axions as Dark Matter II

The action of fermions of SM on a torsionful manifold is given by

\[ S = S_{\text{gravity}} - \frac{1}{2} \sum_f \int \left( \overline{\Psi}_f \gamma^a \mathcal{D} \Psi_f - \mathcal{D} \overline{\Psi}_f \gamma^a \Psi_f \right) - \frac{1}{2} \int \mathcal{F}^a \wedge \mathcal{F}^a - \int \text{Tr}[G^a \wedge G^a] \]

where for any fermionic field \( \Psi \)

\[ S_{\text{gravity}} = \frac{1}{4 \kappa^2} \int \varepsilon_{abcd} \mathcal{R}^{ab} \wedge \mathcal{E}^c \wedge \mathcal{E}^d \]

\[ \mathcal{D} \Psi = d\Psi + \frac{1}{4} \omega^{ab} \gamma_{ab} \Psi + i e \mathcal{A} \Psi + ig \mathcal{B} \Psi, \]

with \( \mathcal{A} \) and \( \mathcal{B} \) the gauge connection 1-forms for \( U(1)_{em} \) and \( SU(3)_c \),

\[ \gamma = \gamma_a e^a \text{ and } \gamma_{ab} = \frac{1}{2} [\gamma_a, \gamma_b]. \]

Equation of motion with respect to \( \omega^{ab} \) gives that the torsion satisfies

\[ \mathcal{T}_{abc} = -\frac{\kappa^2}{2} \varepsilon_{abcd} \sum_f J_f^{5d} \]

where \( J_f^{5d} = i \overline{\Psi}_f \gamma^a \gamma^d \gamma^5 \Psi_f \).

Anomalies

(cf S Basilakos et al. PRD 101 045001 (2020)

Writing $J_5^f = \sum_f (J_5^f)_a e^a$ as a 1-form the axial anomaly implies

$$d \ast J_5 = -\frac{\alpha_{em}Q^2}{\pi} \mathcal{F} \wedge \mathcal{F} \frac{\alpha_s N_q}{2\pi} Tr[G \wedge G] - \frac{N_f}{8\pi^2} \tilde{R}^{ab} \wedge \tilde{R}^{ab}$$

which leads to the following quantised torsion-field contribution to the action

$$-\frac{\alpha_{em}Q^2}{\pi f_b} \int b \mathcal{F} \wedge \mathcal{F} - \frac{1}{2} \int d b \wedge \ast d b - \frac{1}{8\pi^2} \int \left(\Theta + \frac{N_f}{f_b} b\right) \tilde{R}^{ab} \wedge \tilde{R}_{ab} - \frac{\alpha_s}{2\pi} \int \left(\Theta + \frac{N_q}{f_b} b\right) Tr[G \wedge G]$$

The parameter $f_b = \kappa^{-1} \sqrt{8/3} = 4 \times 10^{18}$ GeV.

$\Theta$ and $\theta$ terms label Pontryagin densities, are allowed in the theory, play the role of counterterms and do not affect the equations of motion.

From the last term see that $b$ solves the strong CP problem and $m_b \sim 10^{-12}$ eV and so can be regarded as the QCD axion.
Concluding remarks

• The gravitational anomaly (GA) may not be cancelled in the early universe in a quantum theory of gravity.

• The gauge anomaly though can be designed to cancel for a suitable GUT theory. Anomaly cancellation is necessary for the theories to be renormalizable.

• GA implies breakdown of diffeomorphism invariance and hence local Lorentz invariance (LIV).

• LIV allows a non-zero $\Phi$ required for our leptogenesis model.

• The Kalb-Ramond axion could play the role of a QCD axion and so be axionic dark matter (M Lattanzi and S Mercuri, Phys Rev D 81 125015 (2010))