Hyper-Fast Positive Energy Warp Drives

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arXiv: 2006.07125v2

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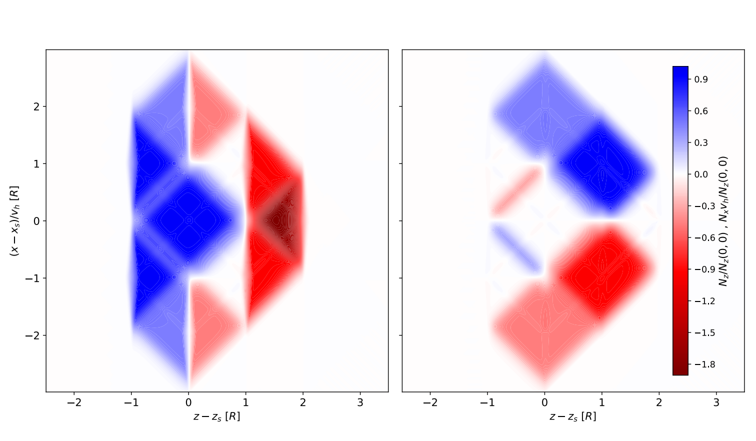
Main Take-Away

General relativity permits superluminal warp drives that obey the weak energy condition, removing the need for exotic sources with negative energy density.



Outline

- > Natário-class Space-times
- > Warp Drive Energy Problems
- > Hyperbolic Shift Vector Potential
- > Positive Energy Rules
- > Example Soliton
- > Further Challenges
- > Prospects

What is a Warp Drive?

No formal definition, will take as curvature + stress-energy soliton with arbitrary speed. Describe with "Natário warp drive" space–times with metric

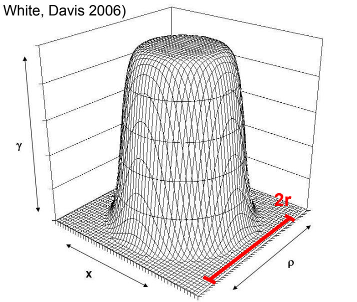
$$ds^2 = -\left(N^2 - N^i N_i\right) dt^2 - 2N_i dx^i dt + h_{ij} dx^i dx^j, \text{ where } h_{ij} = \delta_{ij}$$

The Alcubierre (1994) warp drive is the first and simplest example ($N = 1$, $N_x = N_y = 0$):

$$N_z = v_s f(r_s)$$

(Image Credit: White, Davis 2006)

$f(r_s) =$



Energy Problems

- The first warp drives required negative energy (Alcubierre 1994, Natário 2002)
- More than exists in the visible universe, $E_{tot} \sim -6 \times 10^{62} v_s^2/c^2$ kg for $r = 100$ m (Pfenning & Ford 1997)
- Proofs put forth state negative energy is required (Olum 1999, Lobo & Crawford 2003)

General Energy Density:

$$E = \frac{1}{16\pi}\left({}^{(3)}R - Tr(\mathbf{K}\cdot\mathbf{K}) + Tr(\mathbf{K})^2\right)$$

Alcubierre (1994):

$$E_{\mathrm{Alc}} = \frac{-1}{32\pi}\left(\left(\partial_x N_z\right)^2 + \left(\partial_y N_z\right)^2\right)$$

Natário (2002):

$$E_{\mathrm{Nat}} = \frac{-1}{16\pi} Tr(\mathbf{K}\cdot\mathbf{K})$$

K is the extrinsic curvature of the hypersurface in space-time, also known as the second fundamental form. ${}^{(3)}R$ is the intrinsic Ricci curvature of the hypersurface.

An Opening in Solution Space

The energy density viewed by an Eulerian (time-like, free falling, $\mathbf{v} = -N\nabla t$) observer for a space–time with general shift vector has a mixture of terms

$$E \propto \underbrace{2\partial_x N_x \partial_y N_y + 2\partial_x N_x \partial_z N_z + 2\partial_z N_z \partial_y N_y}_{\text{Indeterminant}}$$

$$\underbrace{-\frac{1}{2}\left(\partial_x N_y + \partial_y N_x\right)^2 - \frac{1}{2}\left(\partial_x N_z + \partial_z N_x\right)^2 - \frac{1}{2}\left(\partial_z N_y + \partial_y N_z\right)^2}_{\text{Negative Definite}}$$

The last three terms are negative definite,
the first three are of indeterminant type giving a
potential island of opportunity for positive energies.

Hyperbolic Shift Vector Potential

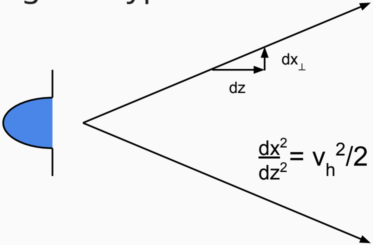
A potential ϕ simplifies the geometry to a single real function

$$N_i = \partial_i \phi$$

Hyperbolic relations within **N** had not been explored, are investigated here by a wave equation

$$\partial_x^2 \phi + \partial_y^2 \phi - \frac{2}{v_h^2} \partial_z^2 \phi = \rho$$

Disturbances in the potential (shift vector) will 'propagate' along the hypersurface with 'speed' $v_h / \sqrt{2}$



Energy under Hyperbolic Potential

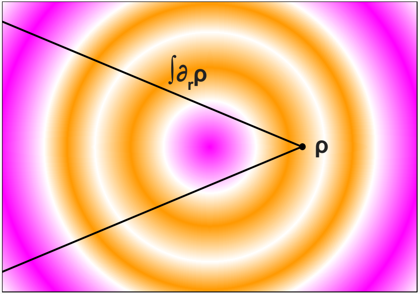
The energy simplifies and can be bounded from below

$$E = \frac{1}{16\pi}\left(2\partial_z^2\phi\left(\rho + \frac{2}{v_h^2}\partial_z^2\phi\right) - 4\left(\partial_z\partial_x\phi\right)^2\right) \quad (1)$$

$$\geq \frac{1}{8\pi}\rho \times \partial_z^2\phi \quad (2)$$

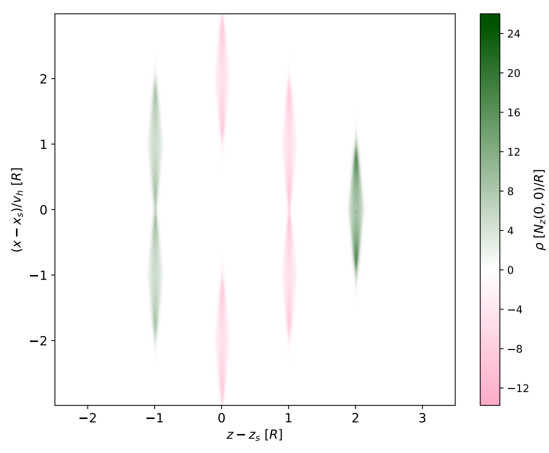
$$= \frac{1}{8\pi}\rho \times \frac{1}{2v_h}\int dx'\partial_r\rho(r, |x'| + |y|)|_{r=z-|\Delta x|/v_h}, \quad (3)$$

and with additional rules, sufficient conditions for non-negative energy density can be made



Constructing a Soliton ($v_h = 10$)

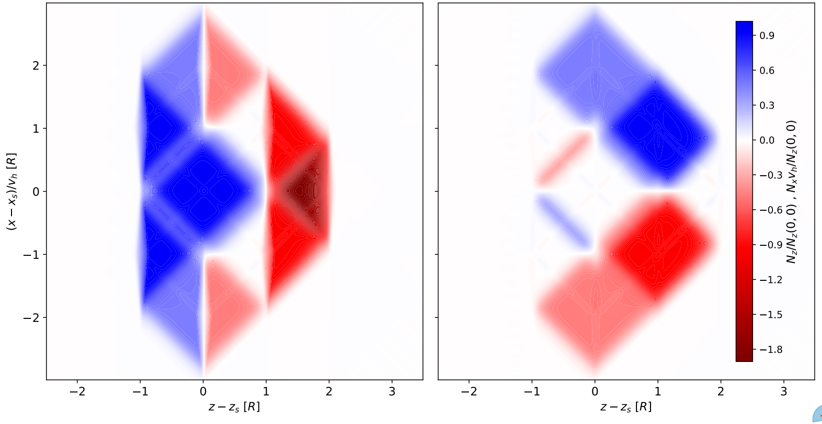
- Boundaries of sources are nearly vertical in z-x plane
- Shift vectors propagate outwards along 'future' wave cone
- Integrated source should vanish outside soliton to ensure asymp. vacuum



Soliton Shift Vector

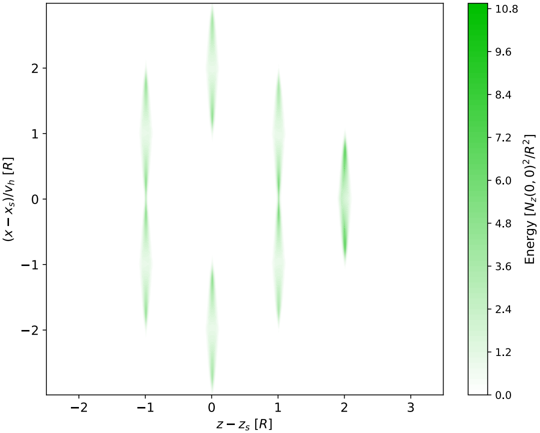
> Want a flat central region that matches the soliton motion (to screen right)

> Has multiple domains
> Integrated shift is zero



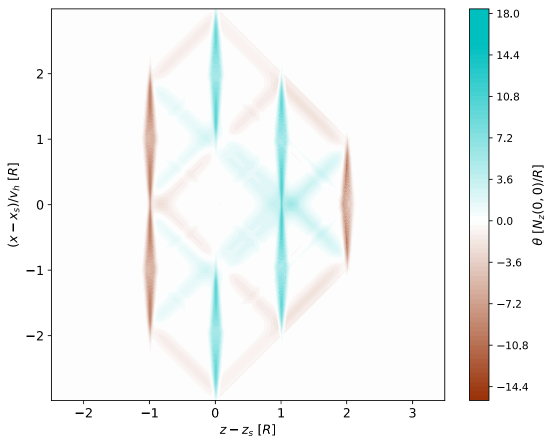
Soliton Energy

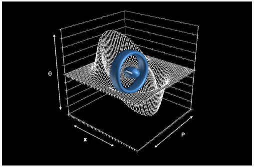
> Mostly confined to hyperbolic source regions
> Momentum flux is zero (consequence of potential)
> Total energy for bubble of $R = 100$ m and bndry. thickness 1 m is $\sim 0.1 M_{\odot} v_s^2/c^2$.



Hypersurface Volume Exp. Factor θ

- Contains expansions and contractions on all sides
- Largest factors coincide with stress-energy sources
- For Alcubierre soln., θ and E are out of phase

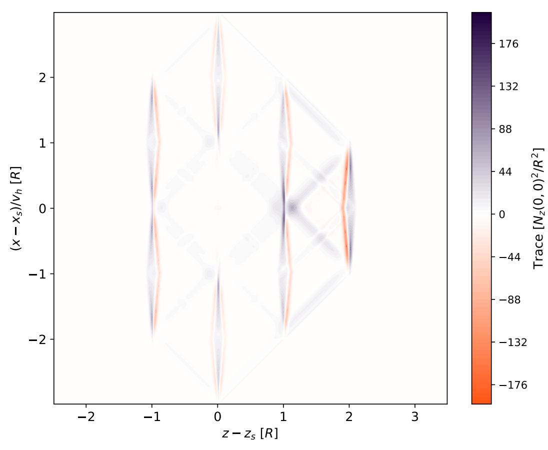




(Alcubierre Drive Image Credit: White 2013)

Trace Condition

- Has positive and negative regions (breaks trace 'energy' condition)
- Requires large pressures, but no net motions
- EOS range matches quasi-relativistic massive fluid
- Trace can be used to place bounds on stress principal values, complete WEC



Next: Improving Energy Requirements

- Energies are still very high for a modest warp bubble ($\sim 0.1 M_{\odot} v_s^2 / c^2$)
- Several techniques have lowered the Alcubierre energy requirement by tens of orders of magnitude
- Can a form of this be applied to the new solitons without using exotic matter?

(Figure Credit: Van Den Broeck 1999)

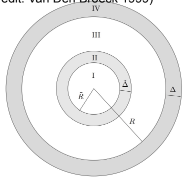
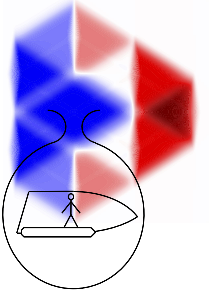
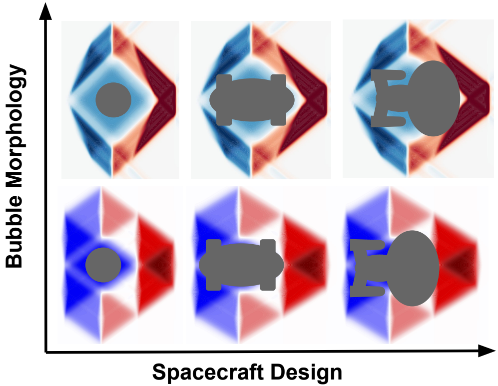


Figure 1: Region I is the 'pocket', which has a large inner metric diameter. II is the transition region from the blown–up part of space to the 'normal' part. It is the region where B varies. From region III outward we have the original Alcubierre metric. Region IV is the wall of the warp bubble; this is the region where f varies. Spacetime is flat, except in the shaded regions.



Further Steps

- Incorporate plasma stress-energy dynamics
- Explore morphologies; move beyond the hyperbolic potential
- Insert a functional craft
- Formation/acceleration of the soliton
- Low-energy experiment
- Explore horizon problems



Summary

> General relativity permits superluminal warp drives that obey the weak energy condition and are sourced by conventional physics, contrary to conventional wisdom

> The next step is to determine how far the energy requirements can be reduced

> Followed by identifying creation/acceleration mechanisms and preparation for experimental verification

Acknowledgements

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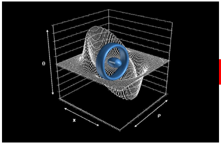
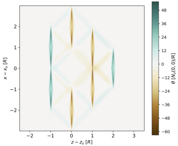
Backup Slides

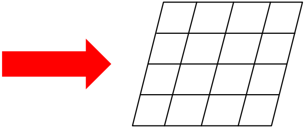
Comparison with Previous Literature

- Literature before 2020 required exotic matter
- Olum (1999), Lobo & Crawford (2003) claim proof that this is unavoidable
- Proofs are point-like, collapse interior to a point
- Positive energy soliton is too complex to be collapsed, is destroyed by point-like limit



(Alcubierre Drive Image Credit: White 2013)





Comparison with Recent Literature

> Recent papers by Santiago, Schuster, and Visser[a] state the soliton total energy must vanish (divergence theorem)

> The soliton structure has a patchwork smoothness that requires a similarly patch-worked usage of the divergence theorem

[a] arXiv:2106.05002v1, arXiv:2105.03079v1