TIDAL HEATING ON THE BRANE

Implications for Black Holes and Exotic Compact Objects

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OUTLINE

- Tidal heating for compact objects in the presence of extra dimensions will be discussed.
- Nature of these compact objects will be explored.
- From the phase of the GW waveform, possible constraint on the extra dimension will be presented.

Reference

SC, Datta and Sau, arXiv: 2103.12430.

*Dey, Biswas and SC, PRD **103**, 084019 (2021) [arXiv: 2010.07966]*

*Dey, SC and Afshordi, PRD **101**, 104014 (2020) [arXiv: 2001.01301]*

WHAT IS TIDAL HEATING?

- As two compact objects in-spiral around each other, the tidal field of one affects the other, thereby introducing gravitational perturbations.
- A part of these perturbations travel to infinity, while a part travels within the horizon, changing the horizon area, mass and angular momentum.
- These modifications in geometrical properties of compact objects is referred to as tidal heating.

[Alvi, PRD 64, 104020 (2001)]

[Chatziioannou, Poisson and Yunes, PRD 87, 044022 (2013)]

[Datta, Brito, Bose, Pani and Huges, PRD 101, 044004 (2020)]

BLACK HOLES OR EXOTIC COMPACT OBJECTS?

- Observation of GW has sparked serious debate as to whether these arise from merger of black holes or, some exotic compact objects.

[Cardoso, Franzin and Pani, PRL 116, 171101 (2016)]

[Abedi, Dykaar and Afshordi, PRD 96, 082004 (2017)]

[Cardoso, Franzin, Maselli, Pani and Raposo, PRD 95, 084014 (2017)]

- The exotic compact objects are often introduced by using ad-hoc boundary conditions in the near-horizon regime.
- Such boundary conditions are more natural for black holes on the brane.

[Dey, SC and Afshordi, PRD 101, 104014 (2020)]

[Dey, Biswas and SC, PRD 101, 104014 (2020)]

BLACK HOLE ON THE BRANE

- The gravitational field equations on the vacuum brane becomes

[Shiromizu, Maeda and Sasaki PRD 62, 024012 (2000)]

$$ {}^{(4)}G_{ab} + E_{ab} = 0 \ . $$

- The rotating solution arising out of the above field equations become,

[Aliev and Gumrukcuoglu, PRD 71, 104027 (2005)]

$$ ds^2 = -\left(1 - \frac{2Mr + Q}{\rho^2}\right) dt^2 - \left(\frac{2a\,(2Mr + Q)\sin^2\theta}{\rho^2}\right) d\phi dt + \left(r^2 + a^2 + \frac{(2Mr + Q)\,a^2 \sin^2\theta}{\rho^2}\right) \sin^2\theta d\phi^2 $$

$$ + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \ , $$

- The functions ρ^2 and Δ are:

$$ \rho^2 = r^2 + a^2\cos^2\theta \ , \quad \Delta = r^2 + a^2 - 2Mr - Q \ . $$

BOUNDARY CONDITIONS

- The gravitational perturbation will satisfy Teukolsky-like equation, which separates into radial and angular parts.
- Large distance from the binary system the perturbation must coincide with the tidal field

$$\mathcal{E}_{\hat{\phi}\hat{\phi}} - \mathcal{E}_{\hat{\theta}\hat{\theta}} - 2i\mathcal{E}_{\hat{\theta}\hat{\phi}} = 8\pi \frac{\sqrt{6}M_2}{5b^3} \sum_{m=-2}^{2} {}_2Y_{2m}(\theta,\phi) Y^*_{2m}(\theta_0,\phi_0) \ ,$$

[Alvi, PRD 64, 104020 (2001)]

- In the near horizon, for BH we have purely ingoing modes, while for ECO we have both ingoing and outgoing modes.

RATE OF CHANGE OF AREA

- Perturbations modify the null generators and hence the shear.

$$\frac{dA_{\rm BH1}}{dt} = \frac{2}{\kappa_{+1}}\int |\sigma_{\rm HH}|^2 \sqrt{h^{(2)}}\, d^2x = \frac{2(r_{+1}^2 + a_1^2)}{\kappa_{+1}}\int |\sigma_{\rm HH}|^2 d\Omega\ ,$$

[Hawking and Hartle, CMP 27, 283 (1972)]

- The shear is related to the gravitational perturbation.

$$\frac{dA_{\rm BH1}}{dt} = \frac{64\pi M_1^5 M_2^2}{5b^6}\frac{\chi_1^2 \sin^2\theta_0}{\sqrt{1-\chi_1^2+\frac{Q}{M_1^2}}}\left[\mathcal{A}+\mathcal{B}\sin^2\theta_0\right]\ .$$

$$\mathcal{A} \equiv 1 - \frac{3}{4}\chi_1^2 + \frac{Q}{M_1^2}\left\{2 - \frac{3}{4}\bar{\chi}_1^2 + \frac{Q}{M_1^2}\right\}\ ;\quad \mathcal{B} \equiv \frac{15}{4}\chi_1^2\left(1+\frac{Q}{M_1^2}\right)\ .$$

[SC, Datta and Sau, arXiv: 2103.12430]

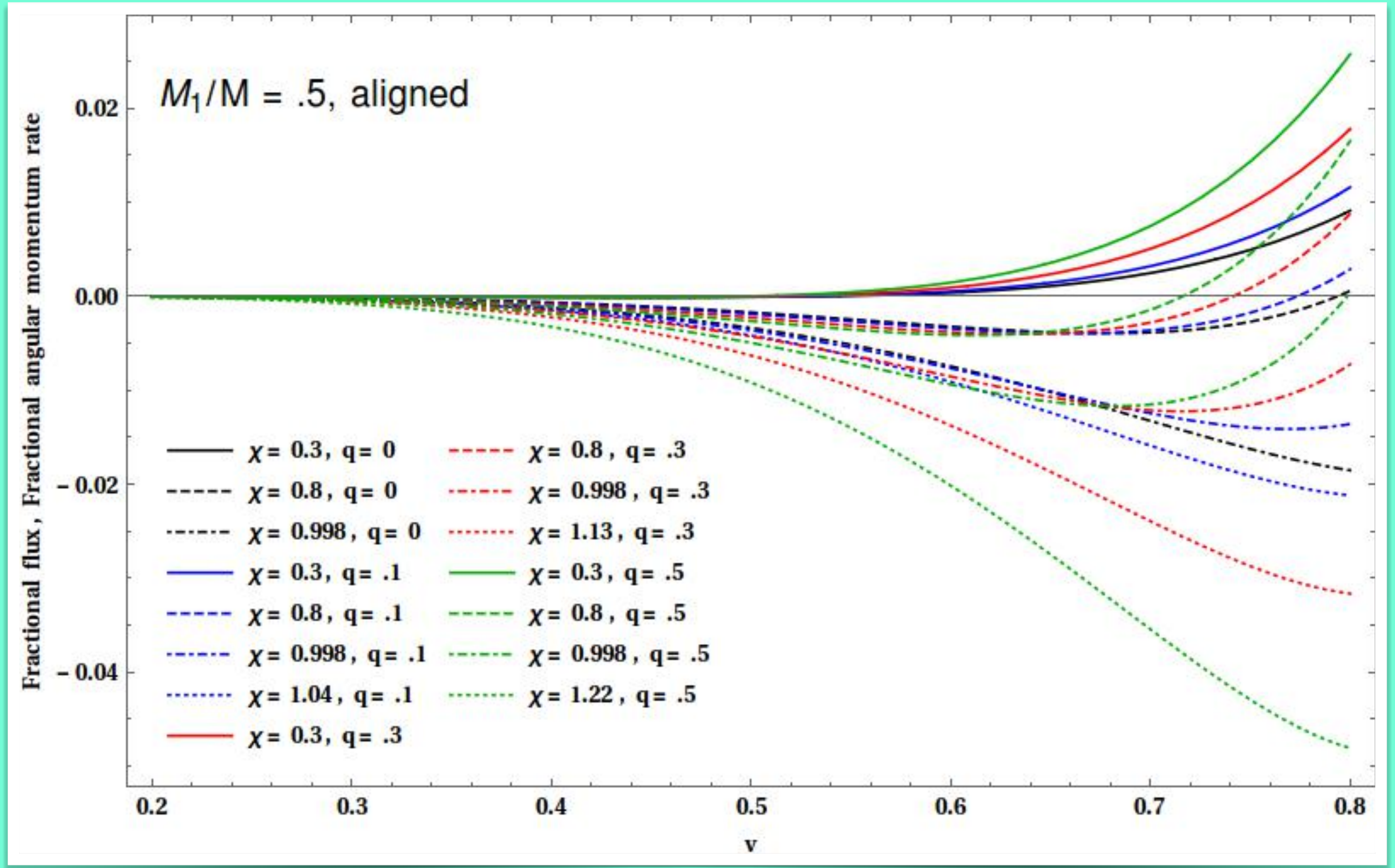
IMPLICATIONS FOR MASS AND ANGULAR MOMENTUM

- The mass and angular momentum also gets modified.
- From first law of black hole mechanics [Alvi, PRD 64, 104020 (2001)]

$$\frac{\kappa_{+1}}{8\pi}\frac{dA_{\rm BH1}}{dt} = (\Omega - \Omega_{\rm H1})^2\, I(\theta_0)\ ,$$

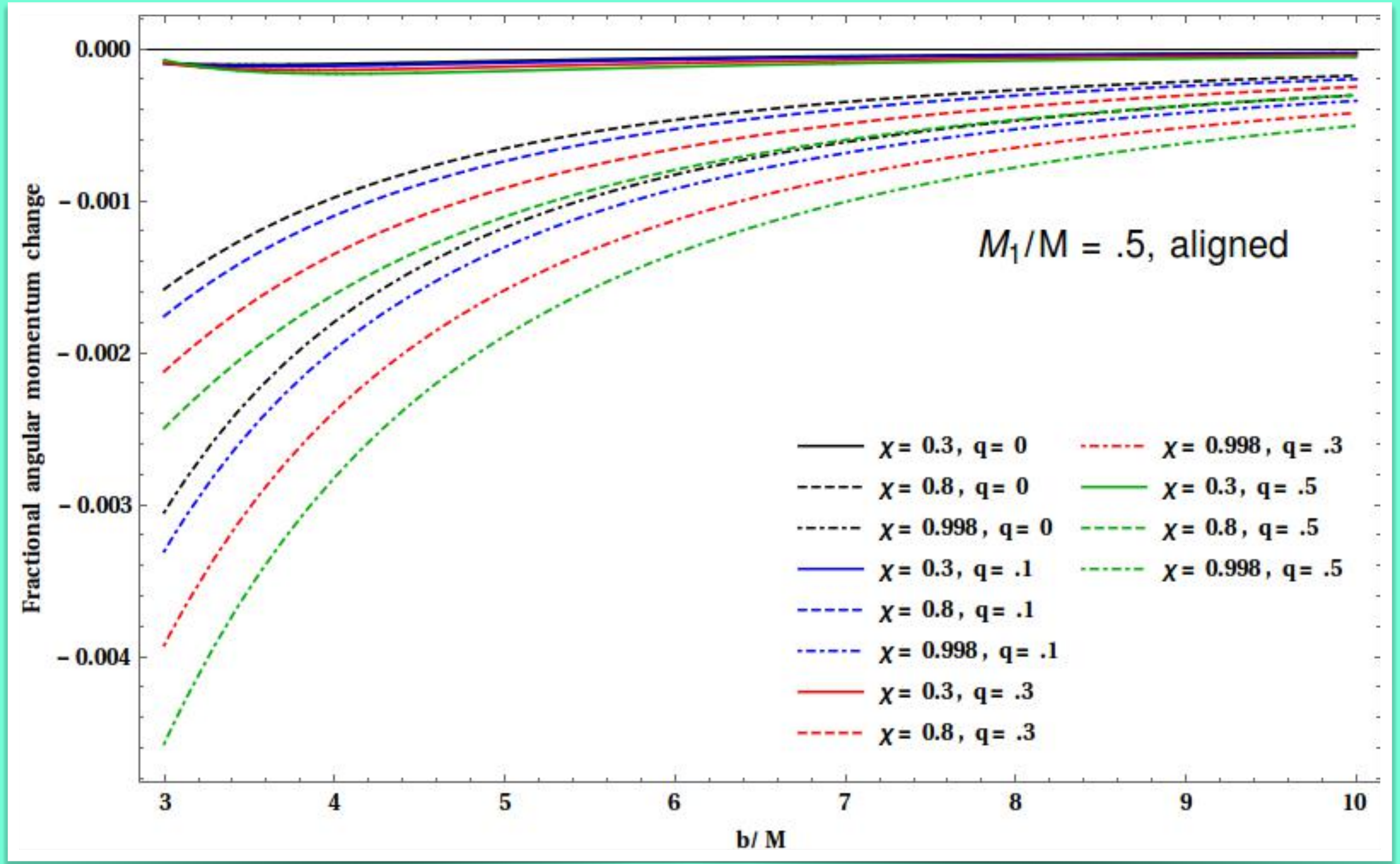
$$\frac{dM_1}{dt} = \Omega\,(\Omega - \Omega_{\rm H1})\, I(\theta_0)\ ; \qquad \frac{dJ_1}{dt} = (\Omega - \Omega_{\rm H1})\, I(\theta_0)\ .$$

- Using $(dA_{\rm BH1}/dt)$, the rate of change of mass and angular momentum can be determined.



$$\frac{(dJ_1/dt)}{(dJ/dt)_{\rm N}} = \frac{(dM_1/dt)}{(dE/dt)_{\rm N}} \left(\hat{\mathbf{L}}_{\rm orb} \cdot \hat{\mathbf{J}}_1\right) \ .$$

[SC, Datta and Sau, arXiv: 2103.12430]



Presence of Q *reduces the angular momentum. Possible avenue to lose angular momentum.*

[SC, Datta and Sau, arXiv: 2103.12430]

CONSTRAINTS ON TIDAL CHARGE

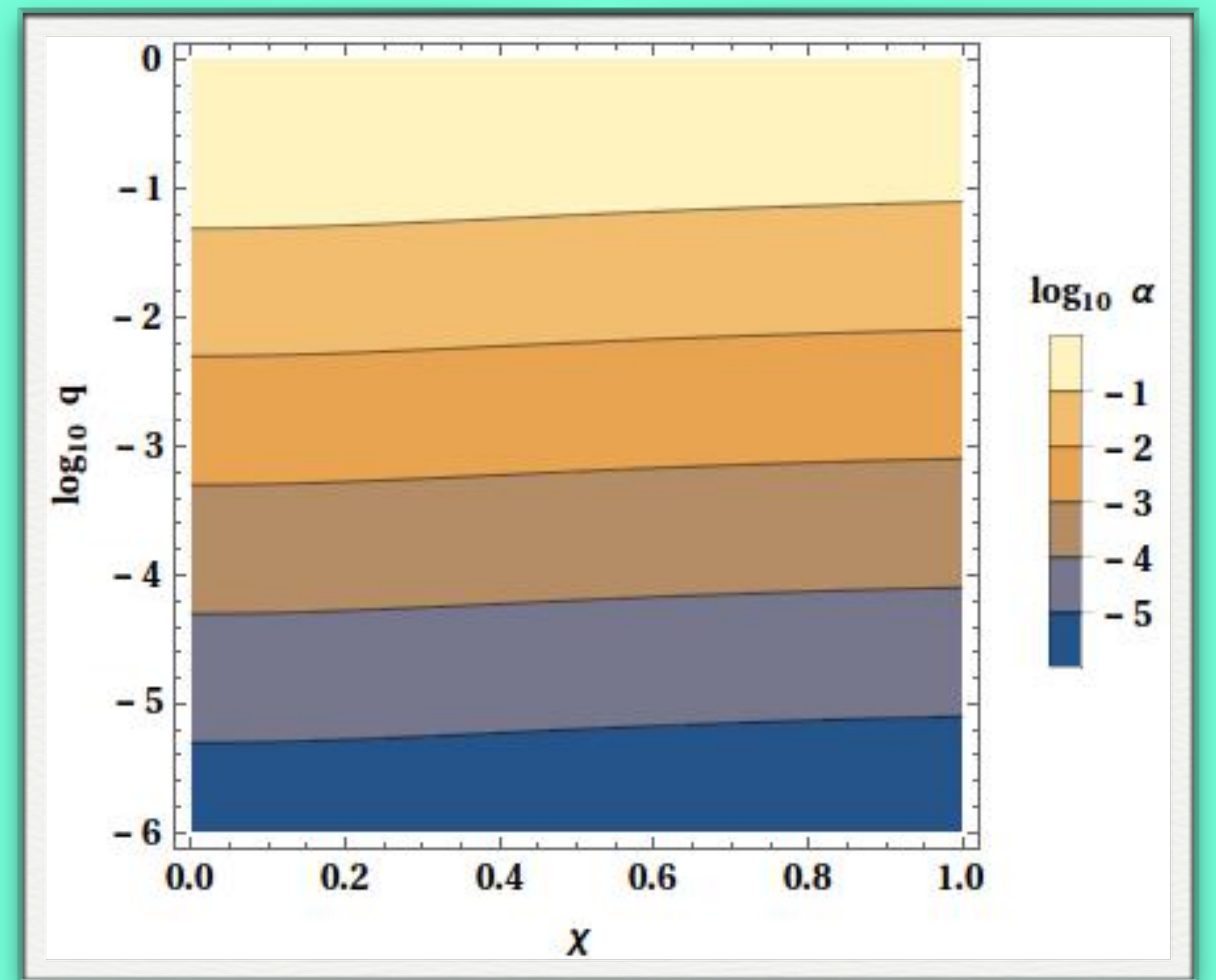
- The rate of change of mass is affected by the presence of extra dimension. $[q = (Q/M^2)]$ [SC, Datta and Sau, arXiv: 2103.12430]

$$H \equiv \frac{\dot{M}_{\rm BH}^{\rm BW}}{\dot{M}_{\rm BH}^{\rm GR}} \equiv 1 + \alpha \ . \qquad = \frac{1 + 3\chi^2 + q(2 + 3\chi^2 + q)}{1 + 3\chi^2} \ .$$

- A similar phenomenon occurs in the case of exotic compact objects.

$$\dot{M}_{\rm ECO} = \dot{M}_{\rm BH} - |\mathcal{R}|^2 \dot{M}_{\rm BH} \ .$$

- Thus possible bound on $|\mathcal{R}|^2$ can be translated to bound on q



From EMRI, future observations can constrain $|\mathcal{R}|^2$ *to* $\mathcal{O}(10^{-5})$.

Hence $q \lesssim \mathcal{O}(10^{-6})$.

WHAT HAPPENS FOR ECO?

- For exotic compact objects, the rate of change of mass, angular momentum and area gets corrected.
- These can be expanded as a power series in $\epsilon = (r_{\rm s} - r_+)/r_+$

$$\dot{A}_{\rm ECO} = |\mathcal{T}|^2 \dot{A}_{\rm BH} + \mathcal{O}(\epsilon) \ .$$

- The second order term depends on the reflectivity $|\mathcal{R}|^2$ through $\ln \epsilon$.
- Hints towards quantum nature of BH.

CONCLUSIONS

- Extra dimensions affect the tidal heating in a binary system significantly.
- Strong constraint on the extra dimension can be derived from future GW experiments involving EMRI.
- The rate of change of area of an ECO may hide quantum signatures of BH.

THANK YOU

OTHER OBSERVABLES

- Given the rates (dM_1/dt) and (dJ_1/dt), along with the trajectory $b(t) = b_0[1-(t/\tau_0)]^{1/4}$, total change in the hairs and the area can be determined. [SC, Datta and Sau, arXiv: 2103.12430]

$$\frac{\Delta J_1}{M_1^2} = \frac{\eta M_1}{4M}\left\{-\frac{\chi_1}{4}\left(\frac{M}{b}\right)^2 + \frac{2}{7}\left(\hat{\mathbf{L}}_{\rm orb}\cdot\hat{\mathbf{J}}_1\right)\frac{M_1}{M}\left[1+\sqrt{1-\chi_1^2+\frac{Q}{M_1^2}}+\frac{Q}{2M_1^2}\right]\left(\frac{M}{b}\right)^{7/2}\right\}$$
$$\times\left[1+3\chi_1^2+\frac{Q}{M_1^2}\left(2+3\chi_1^2+\frac{Q}{M_1^2}\right)\right] ,$$

- These expressions for non-equatorial planes also do exist.
- The charge Q has significant influence on the rates as well as on the total change.