Article

Using Lorentz violation for early universe GW generation due to black hole destruction in the early universe as by Freeze

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Abstract: **We are using information from a paper deriving** a Lorentz-violating energy-momentum relation entailing an exact mo_mentum cutof as stated by G. Salesi . Salesi in his work allegedly defines Pre Planckian physics, whereas we restrict our given application to GW generation and DE formation in the first 10^-39s to 10^-33s or so seconds in the early universe. This procedure is inacted due to an earlier work whereas referees exhibited puzzlement as to the physical mechanism for release of Gravitons in the very early universe.

1. Introduction

What we are doing here is to utilize having the results of Salesi [1] as to a given actual Lorentz-violating energy-momentum relationship which we utilize to elucidate graviton contributions to an early universe derivation of DE, and the cosmological constant. The idea for this is based upon a referee and academic editor who felt puzzled as to the work done earlier [2] which postulated the existence of a breakup of primordial black holes as by itself contributing to DE, and this publication is intended to fill in the actual conceptual gaps which lead to [2] having such a rocky reception. [2] was initiated specifically because work done in [3] as to a multiverse, was not well received, for reasons the author was told as in the eight equation of [3] one reviewer made the statement that the author was modeling the Universe as a harmonic oscillator which is a canard. See [5],[6], and [7] for further background

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This will lead us to utilize the statement given by [8] linking the cosmological constant and massive gravity as given by

$$m_g = \frac{\hbar\sqrt{\Lambda}}{c} \tag{1}$$

This release of conditions for massive gravity should be in line with what is in Freeze [7]

$$\rho_{BH-breakup-density} = \frac{M_P^4}{32\pi}\cdot\left(\frac{M_P^4}{m^4}\right)\cdot\frac{1}{\left|1+3\omega_Q\right|} \tag{2}$$

If the conditions of an early universe, are greater than this value, for Eq.(2)[6] then according to Freeze, et.al. primordial black holes would break apart. We state that this break up of primordial black holes would be enough to create an initial "sea" of gravitons, due to Eq. (1) which would then add up to be in effect a value for a sufficient number of early universe gravitons, which would be added up per unit volume, to in fact sum up to an energy density equivalent to Eq. (1)

so we have massive gravitons and DE. Hence we will be adding up the number of gravitons which may be released due to Eq. (2) and [9] which states the number of gravitons which may be emitted due to a black hole as given in it's page 47 is .1 percent of emitted energy from a nonrotating black hole. Keep in mind that this is for black holes, as given in [9] with mass.

$$M_{primordial-black-hole} \approx 10^{15} \cdot \left(\frac{t}{10^{23} s} \right) grams \qquad (3)$$

For a **10^-5 gram** black hole, t would have to be about **10^-43** seconds, and according to inflation expands space by a factor of 10^26 over a time of the order of **10^-43** to **10^-32** seconds. Meaning we had 10^-5 gram black holes at the start of inflation, and at the time the density of space would be greater than Eq. (2) we would have a breakup of black holes if we had space-time density greater than or equal to about Eq. (2) then we have .1 % of the mass of the broken BH contributing to gravitons, which after we review it may be relevant to Eq. (1) above. One Planck mass is about 10^-5 grams. And it is worth noting in our development when we go past inflation, that we have Black holes growing to the value of about 1 grams, after 10^-40 seconds which is for a radii of approximately 1 centimeter , whereas we can and will define Black holes of 10^-5 grams which would be for less than a centimeter radii just after the start of inflation.

It is important to keep in mind

$$\frac{1}{2} \cdot \sum_i \omega_i \equiv V(volume) \cdot \int_0^{\hat{\lambda}} \sqrt{k^2 + m^2} \frac{k^2 dk}{4\pi^2} \approx \frac{\hat{\lambda}^4}{16\pi^2}$$
$$\xrightarrow[\hat{\lambda} = M_{Planck}]{} \rho_{boson} \approx 2 \times 10^{71} GeV^4 \approx 10^{119} \cdot \left(\rho_{DE} = \frac{\Lambda}{8\pi G} \right) \qquad (4)$$

In stating this we have to consider that we set the DE via $\rho_{DE} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{DE}{}^4}$, so then that the equation we have to consider is a wavelength $\lambda_{DE} \approx 10^{30} \ell_{Planck}$ which is about 10^{30} times a Plank radius of a space-time bubble which we discuss in [2] as a start point for a nonsingular expansion point for Cosmology, at the start of inflation with the space-time bubble of about a Plank length radius we have to consider is a wavelength $\lambda_{DE} \approx 10^{30} \ell_{Planck}$ which is about 10^{30} times a Plank length radius of a space-time bubble [3] as a nonsingular expansion point for Cosmology,

$$\lambda_{DE} \approx 10^{30} \ell_{Planck} \qquad (4a)$$

And then we can write up having

$$\rho_{DE} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{DE}{}^4} \qquad (4b)$$

This is going to create difficulties which is going to lend us to utilize [1] directly and more so we have a way to refine the argument given in Eq. (4), Eq. (4a) and Eq. (4b).

2. Making Use of [1] and the use of Lorentz violation, as in [1] to define more precisely the contribution of Gravitons to both DE, due to the breakup of Black Holes

To do this we need to review the Lorentz violating energy-momentum relationship. In short we have that

$$E^2 = p^2 + m^2 - \tilde{\lambda} p^3 \tag{5}$$

Where the positive LV parameter $\tilde{\lambda}$ is usually assumed of the order of Planck mass, λ ~ 1/M(Planck.mass). This Lorentz violating energy-momentum relationship leads to, according to [1]

$$d\rho = \frac{8\pi p^3 c}{h} \cdot \frac{\sqrt{1+\frac{m^2}{p^2} - \tilde{\lambda} p}}{\exp\left(\left[\frac{cp}{k_B T_{temp}}\right] \cdot \sqrt{1+\frac{m^2}{p^2} - \tilde{\lambda} p}\right) - 1} \cdot dp \tag{6}$$

$p \approx 1/\tilde{\lambda}$ is used if we integrate, Eq. (6) and if we use the first order Romberg numerical integration as given in [10], page 695, so then for high temperature

$$\rho \approx \frac{8\pi M_P{}^3 c}{h} \cdot \frac{\sqrt{\frac{m^2}{M_P{}^2}}}{\exp\left(\left[\frac{cp}{k_B T_{temp}}\right] \cdot \sqrt{\frac{m^2}{M_P{}^2}}\right) - 1}$$
$$\approx \frac{4M_P^2 c}{\hbar} \cdot \left(\left[\frac{cm}{k_B T_{temp}}\right] - \frac{1}{2}\left(\frac{cm}{k_B T_{temp}}\right)^2\right) \tag{7}$$
$$\xrightarrow[k_B = c = \hbar = M_P = 1]{} 4 \cdot \left(\left[\frac{m}{T_{temp}}\right] - \frac{1}{2}\left(\frac{m}{T_{temp}}\right)^2\right)$$

We will then in the next section interpret Eq. (7) when we set

$$m \approx m_g \cdot N_g \tag{7a}$$

3. Interpreting Eq. (7) when Eq. (7a) is used, so as to ascertain the number of Gravitons

We are then looking at [1]

$$\rho \approx \frac{4M_P^2 c}{\hbar} \cdot \left(\left[\frac{c m_g N_g}{k_B T_{temp}} \right] - \frac{1}{2} \left(\frac{c m_g N_g}{k_B T_{temp}} \right)^2 \right) \tag{7b}$$

Using the Planck units renormalized such that $k_B = c = \hbar = M_P = 1$, we have that we are looking at resetting Eq. (7b) so that the above will be roughly

$$\rho \approx 4 \cdot \left(\left[\frac{10^{-65} N_g}{\left(T_{temp} / T_P\right)} \right] - \frac{1}{2} \left(\frac{10^{-65} N_g}{\left(T_{temp} / T_P\right)} \right)^2 \right) \approx 10^{-60} \tag{7c}$$

We then can up to a modeling round off make the following approximation.

$$\left(\frac{10^{-57} N_g}{\left(T_{temp} / T_P\right)} \right)^2 - 2 \cdot \left[\frac{10^{-57} N_g}{\left(T_{temp} / T_P\right)} \right] + 4 \cdot 10^{-60} \approx 0 \tag{7d}$$

'This value of Eq. (7d) will lead to approximately if $\left(T_{temp} / T_P\right)_{\sim 1}$

$$N_g \approx 10^{57} \cdot \left(1 \pm \left(1 - 2 \cdot 10^{-60}\right)\right) \approx 2 \cdot 10^{57} - 2 \cdot 10^{-3} \tag{8}$$

What is Eq. (8) saying? We ascertain Eq. (8) especially if Eq. (7c) is set to reflect upon the number of Gravitons which may give us Dark Energy. In evaluating Eq. (8) we have that $T_{temp} \leq T_P$

4. Interpreting Eq. (7) and Eq. (8) in terms of Dark Energy, if Gravitons produce DE and how this ties in with the Freeze suggestion as to the breakup of Black holes for gravitons in first 10^-27 seconds.

Roughly put, it means that there is about 1000 to 10,000 mini black holes , in between 10^-43 seconds to 10^-32 seconds which would be destroyed so as to release about 10^57 gravitons, equivalent to about 1 Planck mass. In terms of space-time this would be commensurate to a density of $7\times10^{-30} g / cm^3$, or having a radii of 1000 kilometers for a volume of space for about $7\times10^{-6} g / (1000 km)^3$ of density for 10^57 gravitons to have one Planck mass of gravitons released in a volume of space for a radii of about 1000 kilometers after 10^-32 seconds

This would be about 1000 to 10,000 destroyed mini black holes in less than 1.057 times 10^-10 of a light year in radial distance for 1 Planck mass of radiated gravitons, in far less than 10^-1 seconds in cosmological expansion

The Universe was once just the radius of the Earth-to-the-Sun, which happened when the Universe was about a trillionth (10^–12) of a second old, ,i.e. a sphere of 149.6 million km and the region for about 10^-43-10^-40 seconds was about 10^3 kilometers in radial size

It is important to keep these figures in line and visualize how a sphere of about 10^3 kilometers in radial size would be able to have 10^57 gravitons released in order to have DE formed and possibly then the Cosmological constant

5. What 10^57 gravitons in a radius of 1000 kilometers means in terms of DE and a Cosmological constant calculation

We will first of all refer to an early universe treatment of the uncertainty principle is, in the startup of inflationary cosmology [11]

Set $$a(t) = a_{initial} t^{\gamma} \quad (9)$$

$$\rho \approx \frac{\dot{\phi}^2}{2} + V(\phi) \equiv \frac{\gamma}{8\pi G} \cdot t^2 + V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma - 1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \quad (10)$$

$$V = V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma - 1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \quad (11)$$

And set $$\hbar = G = \ell_P = m_P = k_B = 1, \quad (12)$$

The value of time t will be set as t ~ (10^-32 s/ t(Planck)) whereas we can utilize the ideas of having Planck time set ~ 5 x 10^-44 seconds, hence, t ~ 10^12, in Planck Units, so then we will have, in this situation , Eq.,(10) as reset as [11]

$$\rho \approx \frac{\gamma \cdot (10^{24})}{8\pi} + V_0 \left\{ \frac{8\pi \cdot V_0 \cdot (10^{12})}{\gamma \cdot (3\gamma - 1)} \right\}^{\frac{1}{2} \cdot \left(\sqrt{\frac{\gamma}{\pi}} - 4 \cdot \sqrt{\frac{\pi}{\gamma}} \right)} \approx \frac{E_{effective}}{(1000 Km)^3} \approx \frac{E_{effective}}{(6.25 \times 10^{40} (\ell_P = 1))^3} \quad (13)$$

The interesting thing, is that the factor of roughly 10^-120 shows up in this situation so as to imply that there may be some linkage between setting the effective energy as roughly some proportional power value of Planck Mass.

$$10^{2} m_P (gravitons) \Rightarrow 10^{67} gravitons \quad per \quad 10^{5} m_P (black-hole) \quad (14)$$

6. Making DE equivalent to a sea of initial gravitons, in regime 10^-43 to 10^-32 seconds

Roughly put, one hydrogen atom is about 1.66 times 10^-24 grams. The weight of a Massive graviton is about 10^-65 grams [8][9], hence we are talking about 10^-22 grams, or about 10^44 gravitons, with each graviton about 6×10^{-32} **eV**/c^2 **After 10^-27 seconds, the following in the set of equations given below are Equivalent, and that these together will lead to a cos mological Constant,** Λ of the sort which we will be able to refer to later

$$1 \quad graviton \approx 10^{-65} g \qquad (14a)$$

$$M_{PBH}\left(10^{-32}\right) \approx 10^{15} \times \left(\frac{10^{-32} s}{10^{-23} s}\right) g \approx 10^{6} g \approx 10^{11} M_P \qquad (14b)$$

$$M_{PBH}\left(10^{-38}\right) \approx 10^{15} \times \left(\frac{10^{-38} s}{10^{-23} s}\right) g \approx 10^{0} g \approx 10^{5} M_P \qquad (14c)$$

If so then, using Eq. (14c), a 10^5 Planck mass sized black hole if it has 1/1000 of its mass converted into gravitons, would have 10^2 time Planck mass = 10^63 gravitons, i.e. for 10^63 gravitons which would occur for 10^-38 seconds.To match the restrictions as given in Eq. (7d) and Eq.(8) we need to look at what would allow for 10^57 gravitons, instead of 10^63 gravitons. This would entail having

$$M_{PBH}\left(10^{-43}\right)\approx 10^{15}\times\left(\frac{10^{-43}s}{10^{-23}s}\right)g\approx 10^{-5}g\approx 1M_{P} \quad (14d)$$

Eq.(14d) is in fidelity with having , if 1/1000 of its mass converted to gravitons, a situation where there would be 10^57 gravitons. At a time of 10^-43 seconds. We are then Examining what happens at the end of inflation.

The supposition I have is that one can use 1/1000 of the mass as given in Eq. (14d) for Gravitons and to thereby have 10^-8 g for Gravitons, per black hole of mass 10^-5 g If one has say 10^57 gravitons , for a 1000 kilometer regime as say in the first 10^-32 . Seconds we then have for 10^-65 g per graviton, we are then having for gravitons a value of to be diverted to 10^-8 g per black hole

Table 1, summary of infiationary and post inflationary information

Number of black holes,	**Mass of black hole of size 1 Planck mass set aside for gravitons**	**Mass of black hole for 10^8 gravitons**	**Radii of proto universe**
10^8	10^-8 grams	10^-5 g = 1 Planck mass	1000 Kilometers
Volume of Universe is 10^3 kilometers, cubed	Starting range for Mass of black hole for Gravitons	Assumed to be starting range of BH masses, at about 10^-43 seconds	From less than a meter to 1000 Kilometers for constructing black holes which may be torn asunder by Karen Freeze's criteria

Assuming that gravitons contribute to the Dark Energy value will lead to us using the Karen Freeze model, with gravitons being released in the early universe by the breakup of early universe black holes which have a maximum value of about 1 g, as opposed to the value of the Sun which has about 10^33 grams, in first 10^-32 seconds.

The divergence from the standard model can be seen in postulating a non singular start to the universe and a simple way to do it, as follows Assume at the beginning, one has a spherical shell defined by a volume in the regime of radial space defined by $a - \in \leq r \leq a + \in$ [12]

$$V_{\in} = \frac{8\pi \in}{3} \cdot \left(3a^2 + \in^2\right) \qquad (15)$$

And then for when one has if one has a heat strength of A, for this radial 'shell' $S_{\in}$

$$\delta_{A,\in}(r) = \begin{cases} A/V_{\in} & if\ r \varepsilon S_{\in} \\ 0 & otherwise \end{cases} \qquad (16)$$

Then one has the following "integration" in the region of 'space-time'

$$\int_0^{2\pi}\int_0^{\pi}\int_{a-\in}^{a+\in} \delta_{A,\in}(r)\,\tilde{r}^2 \sin\phi\, d\tilde{r}\, d\phi\, d\theta = A \qquad (17)$$

Following the line of reasoning, we will be examining briefly how this bubble-shell start to cosmology could commence, and how to interpret both Eq. (16) and Eq. (17)

7. What 10^57 gravitons in a radius of 1000 kilo meters means if we Go to the Rosen early universe cosmology.

Let us first recall the Shalyt-Margolin and Tregubovich (2004, p.73) [13] , Shalyt-Margolin (2005, p.62) [14], [15]

$$\Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_P^2 \frac{\Delta E}{\hbar} \Rightarrow \left(\Delta E\right)^2 - \frac{\hbar \Delta t}{\gamma t_P^2}\left(\Delta E\right)^1 + \frac{\hbar^2}{\gamma t_P^2} = 0$$

$$\Rightarrow \Delta E = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 + \sqrt{1 - \frac{4\hbar^2}{\gamma t_P^2 \cdot \left(\frac{\hbar \Delta t}{2\gamma t_P^2}\right)^2}} \right) = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \sqrt{1 - \frac{16\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2}} \right) \quad (18)$$

For sufficiently small γ The above could be represented by[3] [14]

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \left(1 - \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2} \right) \right)$$
$$\Rightarrow \Delta E \approx either \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2}, or \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(2 - \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2} \right) \quad (19)$$

This would lead to a minimal relationship between change in E and change in time as represented by Eq. (19), so that we could to first order, say be looking at something very close to the traditional Heisenberg uncertainty principle results of approximately

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2} \equiv \frac{4\hbar}{\Delta t} \quad (20)$$

$$\Delta E \Delta t \approx 4\hbar \tag{21}$$

Having brought this up, let us then go to the Rosen [16] version of cosmology, and this needs explanation due to its rescaling of the values of the cosmology time and the temperatures involved.

The key point of this mini chapter will be to summarize derivation of the space-time temperature [16]

$$T = \left(\rho_F / \sigma\right)^{1/4} \cdot \frac{\breve{a} r^7}{\left(\breve{a}^4 + r^4\right)^2} \tag{22}$$

With $\breve{a} = 10^{-3} cm$, $\left(\rho_F / \sigma\right)^{1/4} = 1.574 \times 10^{32} K\left(kelvin\right)$

Then according to [2], the initial temperature is

$$T_{initial} = 2.65 \times 10^{-180} K\left(kelvin\right) \tag{23}$$

Whereas the temperature where one has the breakup of Primordial black holes starting is at

$$T_{black-hole-breakup-starts} = 7.41 \times 10^{31} K\left(kelvin\right) \tag{24}$$

Whereas we start the derivation of Eq. (22) in reference [2], and the extreme value of the temperature T for breaking up black holes, again by [2] leads to how black holes may contribute to the grown of DE if we have graviton production, so we consider when a body of mass m and radius R break apart. As given in [2] we have then that if R(radius) is between 1 meter to say 1000 Kilometers this mass m breaks up for m as given by [2]

$$m \approx \frac{8\pi R\left(radius\right)^{3} \cdot \rho}{3} \quad (25)$$

Here, the density function is given by Eq. (12) and Eq.(13), for our application and also we obtain for black holes a break up criteria for mass m Black holes if

$$m \approx -\left(\frac{4\pi\rho}{3}\right) \cdot \left(1 + 3 \cdot \frac{\rho_G}{p_G}\right) \cdot \frac{8m^3}{M_P^6} \quad (26)$$

So we can have the start of breakup of black holes, if we have gravitons from 1/1000 of the mass of given black holes, and if black holes contribute DE according to when pressure is approximately equal to the negative value of the density which would lead to a Black hole contribution of Eq. (27) to DE. As given below.

$$DE - from - black - holes = 7 \times 10^{-30} g / cm^3 = 7 \times 10^{-6} g / (1000 km) \quad (27)$$

This rough value of DE, as given in Eq. (27) will be directly compared to what we can expect as far as applying in Eq. (21) as to comment directly on the Δt time interval for the active generation of DE in the early cosmos.

Keep in mind that J. W. Moffat in [17] postulated in the initial phases of cosmology a situation for which we have no conservation of energy, and in fact this is exactly the situation we could be portraying here , that is if [2] and the description of the Rosen cosmology as in [16] are not wrong

Quote, from [17]

The spontaneous symmetry broken phase will induce a violation of conservation of energy and explain the generation of matter in the very early universe

End of quote

This would be doable if the initial phases of creation of the Universe follow [16] and if we utilize, initially a near zero temperature start regime in the early universe, as in [16]. The early universe will have an energy input via thermal inputs of a value commensurate with [2]

$$E(thermal - energy) = \frac{d(\dim) \cdot k_B \cdot T_{universe}}{2} \tag{27}$$

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