



Sixteenth Marcel Grossmann Meeting
on Recent Developments in Theoretical and Experimental General Relativity,
Gravitation, and Relativistic Field Theories
Virtual Meeting, 5-10 July 2021

The Model of Dark Energy Based on the Quantum-Mechanical Uncertainty Relation

[*Grav. & Cosmol.*, v.25, p.169 (2019); v.26, p.259 (2020); v.27, in press (2021)]

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Introduction

- It is commonly believed now that the effective cosmological constant (*i.e.*, the Lambda-term in Einstein equation) is of crucial importance both
 - in the early Universe (where it is responsible for the inflation) and
 - in the modern Universe (where it manifests itself as the Dark Energy).

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- However, little is known about its physical nature and origin till now.
- Moreover, its magnitudes in the early and modern Universe are absolutely different.
- It was often assumed in the first inflationary models (developed in the late 1980's and 1990's) that the effective Lambda-term is produced by the Bose condensate of the Higgs field, which is an important constituent of the theory of elementary particles.
 - Unfortunately, the detailed calculations did not support this hypothesis.
 - So, the contemporary inflationary models are based on the quite arbitrary Lagrangians, where the additional terms are introduced *a priori*.
- **Is it possible to propose a model of the effective Lambda-term that:**
 - (i) is based on the well-established physical facts and**
 - (ii) can explain its existence and variable magnitude throughout the entire cosmological evolution?**

Formulation of the Model - 1

- The standard Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3c^2} \rho - kc^2 \frac{1}{R^2} + \frac{c^2}{3} \Lambda$$

- The Lambda-term is associated with the vacuum energy density:

$$\Lambda = \frac{8\pi G \rho_v}{c^4}$$

- Our main assumption is that the vacuum energy in the Planck volume is derived from the quantum-mechanical uncertainty relation between the time and energy in the Mandelstam–Tamm form [*Izv. Akad. Nauk SSSR (Ser. Fiz.)*, v.9, p.122 (1945)]:

$$\Delta E \Delta t \approx \frac{C_{UR}}{2} \hbar,$$

where $\Delta E = \rho_v l_P^3$, $l_P = \sqrt{G\hbar/c^3}$ is the Planck length, and

$C_{UR} = 1$ in the Heisenberg case (which is relevant to the problem of measurements) or

$C_{UR} = \pi$ in the Mandelstam–Tamm case (which refers to the long-term evolution of quantum systems) [Deffner & Campbell, *J. Phys. A*, v.50, p.453001 (2017)].

Formulation of the Model - 2

- The resulting effective Lambda-term:

$$\Lambda(t) = \frac{4\pi C_{UR}}{c l_P} \frac{1}{t}$$

- **Master equation** of our cosmological model:

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3c^2} \rho - kc^2 \frac{1}{R^2} + \frac{4\pi C_{UR}}{3\tau} \frac{1}{t},$$

where $\tau = l_P/c = \sqrt{G\hbar/c^5}$ is the Planck time.

- A very unusual feature of this equation is the explicit dependence on time.
- The **simplest solution**, obtained under assumption that the matter density is ignored ($\rho = 0$) and the 3D space is flat (*i.e.*, $k = 0$):

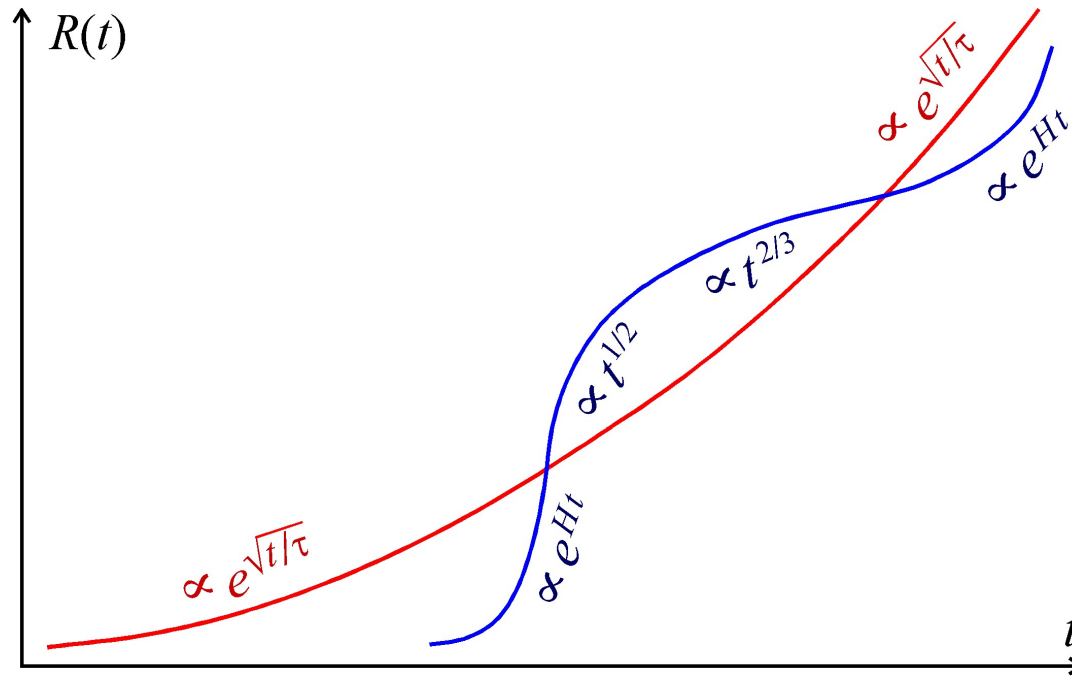
$$R(t) = R^* \exp \left[\sqrt{\frac{16\pi C_{UR}}{3}} \sqrt{\frac{t}{\tau}} \right]$$

- The corresponding Hubble parameter decays with time as

$$H(t) \propto 1/\sqrt{t}$$

Properties of the Model - 1

- The entire cosmological evolution is described by the same universal function, as distinct from the “standard” cosmology, where it is composed of the four absolutely different stages (dominated by the Lambda-term, radiation, non-relativistic matter, and the Lambda-term again).
 - The puzzle of two very different Lambda-terms becomes naturally resolved.



- On the other hand, the age of the Universe in this model becomes much greater:

$$T \approx (T^*/\tau) T^*, \text{ where } T^* \approx 1/H_0$$

$$T^* \approx 4 \cdot 10^{17} \text{ s}, \quad \tau = 5 \cdot 10^{-44} \text{ s} \Rightarrow (T^*/\tau) \approx 10^{61} !$$

- In other words, the Universe turns out to be “quasi-perpetual”.

Properties of the Model - 2

- Does our model resolve also the well-known problems of the early Universe, such as:
 - the absence of singularity;
 - causal connectivity between the remote spatial subregions (and the associated problems of homogeneity and isotropy of the observed space, the absence of topological defects, etc.)
 - a possibility of formation of the approximately flat 3D space?

I. **The problem of singularity** is naturally absent for the obtained solution

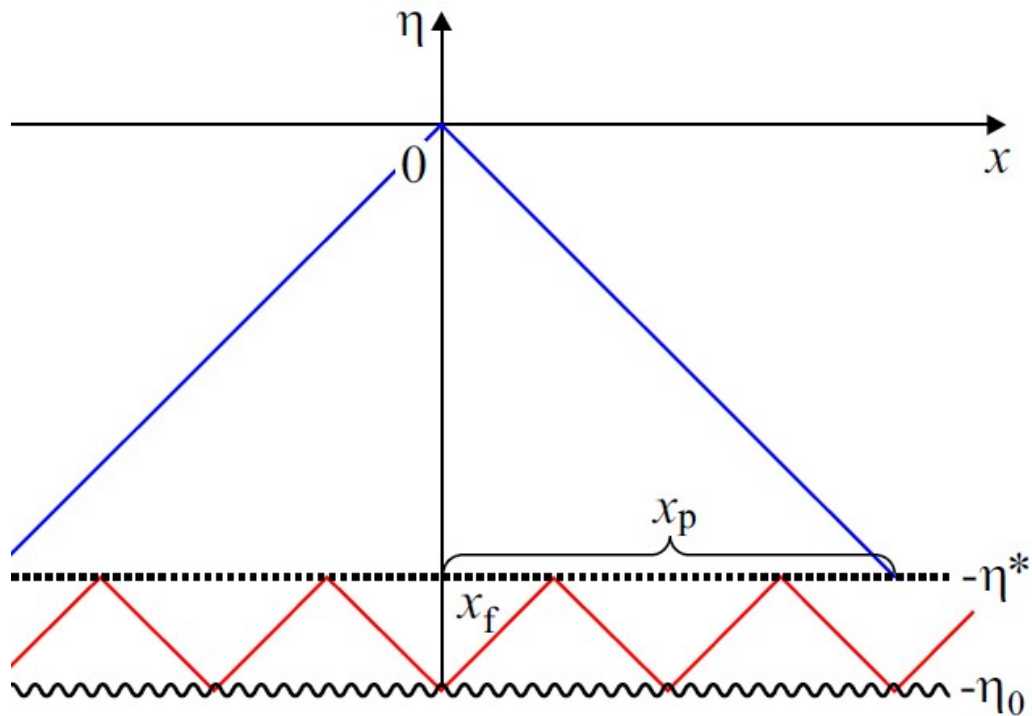
$$R(t) = R^* \exp \left[\sqrt{\frac{16\pi C_{UR}}{3}} \sqrt{\frac{t}{\tau}} \right]$$

- Moreover, this solution enables one to construct the “bounce model” of the Universe (*i.e.*, contraction followed by the expansion).

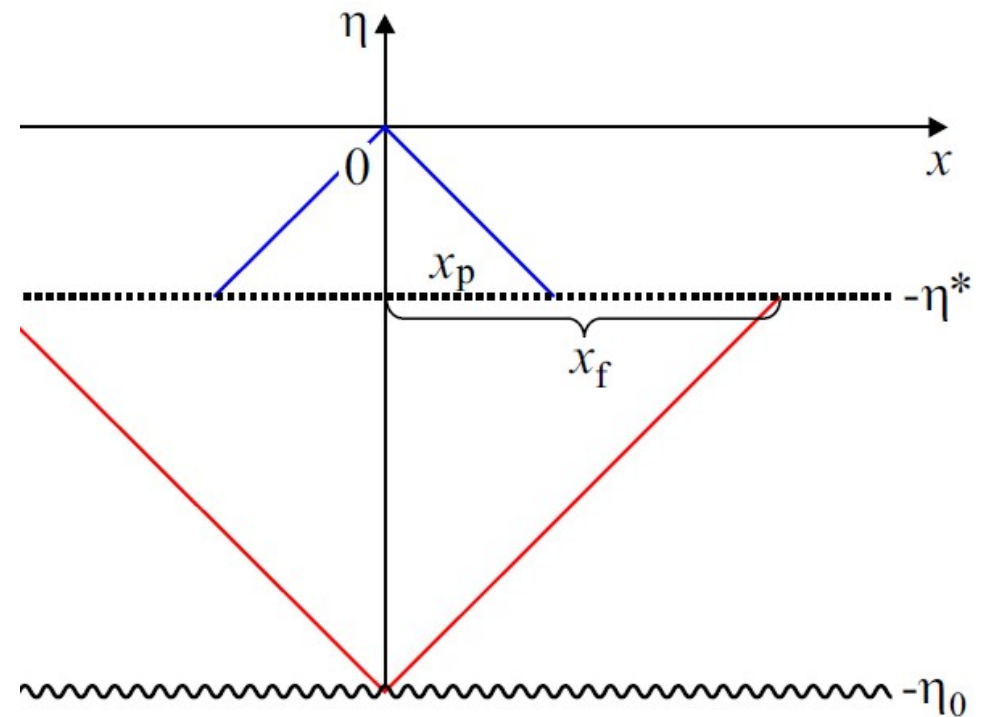
Properties of the Model - 3

II. The problem of causal connectivity between the remote spatial subregions can be conveniently analyzed in the conformal diagrams:

- the old “pre-inflationary” model



- inflationary models

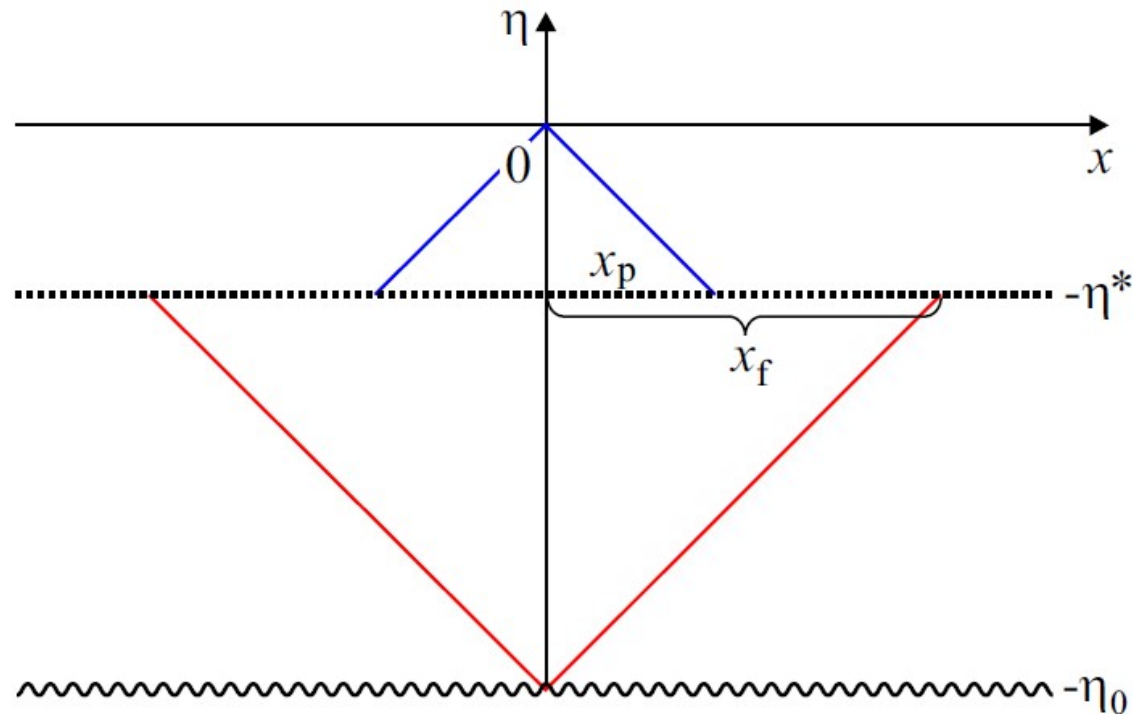


– The number of causally-disconnected domains observed by the instant η^* :

$$N_{\text{CD}} \approx \left\{ \left[1 - \left(\frac{T - t^*}{T} \right)^{\frac{1+3w}{3(1+w)}} \right]^{-1} - 1 \right\}^{-3} \gg 1 \quad N_{\text{CD}} \approx \exp \left[-\sqrt{3\Lambda} c (T - t^*) \right] \ll 1$$

Properties of the Model - 4

- Conformal diagram of the **uncertainty-mediated model** looks qualitatively the same as in the case of the “standard” inflation:



- and the number of causally-disconnected domains turns out to be:

$$N_{\text{CD}} \approx \left(\frac{16\pi C_{\text{UR}}}{3} \right)^{3/2} \left(\frac{T-t^*}{\tau} \right)^{3/2} \exp \left[-4\sqrt{3\pi C_{\text{UR}}} \left(\frac{T-t^*}{\tau} \right)^{1/2} \right] \ll 1$$

- Therefore, the entire observed region becomes causally connected, and the problems of homogeneity and isotropy of the space, the absence of topological defects, *etc.* should be naturally resolved.

Properties of the Model - 5

III. Possibility of a self-consistent (“dynamical”) formation of the approximately flat 3D space can be studied by analyzing a temporal behavior of the “curvature term” in the Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3c^2} \rho - \underbrace{kc^2 \frac{1}{R^2}}_{?} + \frac{c^2}{3} \Lambda$$

- In the **old “pre-inflationary” model**, the first term decays as $1/R^3$ or $1/R^4$ (for the dust-like matter and radiation, respectively), while the curvature term changes as $1/R^2$ and begins to dominate. Therefore, the observed flat 3D space cannot be formed dynamically, but rather should be postulated *a priori*.
- On the other hand, in the **standard inflationary models** with a slowly-varying Lambda-term, role of the curvature will quickly vanish in the course of time and the asymptotically-flat 3D space will be formed.
- At last, in the **uncertainty-mediated cosmological model** with Lambda-term inversely proportional to the time, ratio of the “vacuum” to “curvature” energies is:

$$\lim_{t \rightarrow \infty} \frac{1/t}{1/\tilde{R}^2(t)} = \lim_{t \rightarrow \infty} \frac{\tilde{R}^2(t)}{t} = \infty$$

– Therefore, a role of the curvature will also disappear, as in the standard inflation.

Conclusions:

- 1. The uncertainty-mediated cosmological model provides a unified description of the entire cosmological evolution and naturally explains existence of the decaying cosmological constant (Lambda-term).**
- 2. The well-known conceptual problems of the early Universe (such as the absence of singularity, a causal connectivity of the remote spatial subregions, dynamical formation of the approximately flat 3D space, *etc.*) can be resolved as efficiently as in the standard inflationary scenarios.**
- 3. A number of finer issues, such as the processes of leptogenesis and baryosynthesis, the spectrum of primordial fluctuations, *etc.* are still to be considered in more detail to derive a definitive conclusion on the viability of this model.**