Analysis of the Velocity Rotational Curves Via Weyl-Interaction Modified Gravity

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Plan of the Talk

1. Introduction
2. Dark Matter
3. Evidence for Dark Matter
4. Modified Theories of Gravity and Dark Matter
5. Modified Relativistic Dynamics

Outline

1. Introduction
2. Dark Matter
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4. Modified Theories of Gravity and Dark Matter
5. Modified Relativistic Dynamics

Theme

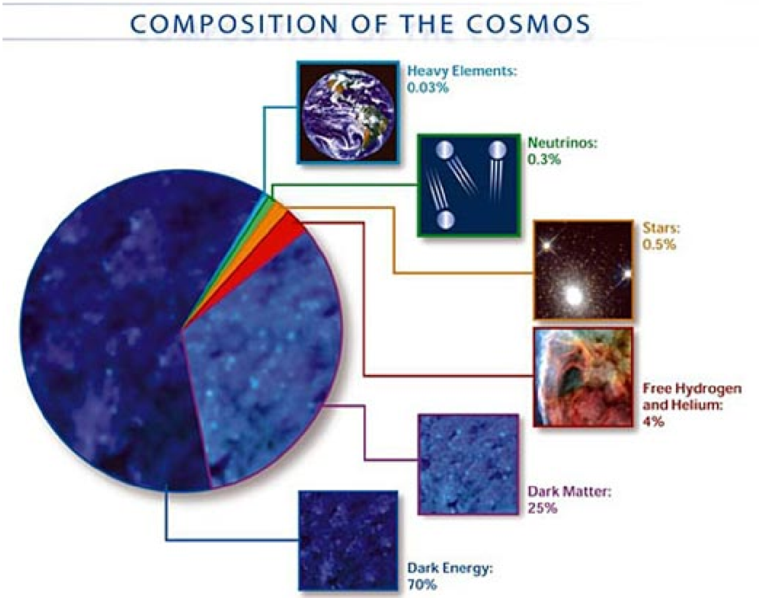
Various suggestions to modify the law of gravity had emerged since the observations of Vera Rubin, but all needed many parameters to solve a single problem, which is of no use. Qadir, Lee, and Kim suggested a modification of the Einstein-Hilbert Lagrangian including an explicit interaction term, RT, on the grounds that it might also solve the problem of dark matter and quantum gravity. However, the gravitational field is given by the Weyl tensor, $C^{\mu}_{\ \nu\rho\pi}$ and not the Ricci scalar, which gives the mass-energy.

Lee and Qadir suggested a modification of the Lagrangian by including the interaction term, $\lambda C_{\mu\nu\rho\pi} T^{\mu\rho} T^{\nu\pi}$. The idea had been that the same interaction coupling must explain the dark matter for all galaxies and systems, or clusters, of galaxies. In this work, we shall follow their calculations to get the coupling constant that fits both Andromeda and the Milky Way.

History and Background

Till the mid-twentieth century, it had been thought that most of the matter seen in the Universe is largely hydrogen, with some traces of other elements.

Why do we need to discuss the Composition of the Universe again?


COMPOSITION OF THE COSMOS
Heavy Elements:
0.03%
Neutrinos:
0.3%
Stars:
0.5%
Free Hydrogen
and Helium:
4%
Dark Matter:
25%
Dark Energy:
70%

Outline

Dark Matter

Dark Matter: It is well-known that 90% to 95% of the matter in the Universe is in some form that is yet to be discovered. An undetected form of mass that emits no photons, and cannot be detected by sensors, and detectors. This substance is a gravitational glue that ties galaxies and clusters of galaxies together, and it has a significant impact on the history and destiny of the Universe. This enigmatic substance neither produces nor absorbs significant amounts of electromagnetic radiation in any recognized waveband. It is known as dark matter (DM). One of science's largest unanswered mysteries is the nature of this substance.

Dark Energy: Dark energy is a hypothetical form of energy that exerts a negative, repulsive pressure, behaving like the opposite of gravity.

Dark Matter

There seems to be a lot of unseen matter in the Universe. An important question is whether this matter is baryonic (made of neutrons and protons like ordinary matter), or something more exotic. The ratio η of baryon number density to photon number density is

$$\eta \equiv \frac{n_{\mathrm{B}}}{n_\gamma} \simeq \frac{\Omega_{\mathrm{B}}\rho_{\mathrm{c}}/m_{\mathrm{B}}}{410\ \mathrm{cm}^{-3}} \simeq 2.76 \times 10^{-8}\Omega_{\mathrm{b}}h^2, \tag{1}$$

where Ω_{B} is the baryon density parameter, ρ_{c} is the closure density, m_{B} is the average mass of a baryon, and the density of photons has been approximated from the spectrum cosmic microwave background [1]. Observations of the cosmic microwave background and Big Bang nucleosynthesis studies have set constraints on the abundance of baryons in the Universe.

M. Guidry, *Modern General Relativity: Black Holes, Gravitational Waves, and Cosmology* (2019).

Density Parameters and Dark Matter

Independent analysis of the cosmic microwave background by the Planck satellite finds the value of η to be $\simeq 6.1 \times 10^{-10}$ Therefore, from the above equation, the value of Ω_{B} is

$$\Omega_{\mathrm{B}} \simeq \left(3.6 \times 10^{7} h^{-2}\right) \eta \simeq 0.04 \tag{2}$$

Most of these baryons are neutrons and protons, while most of the photons are in cosmic microwave background radiation. According to strong nucleosynthesis constraints, most of the baryons in the Universe are visible and are not dark.

Density Parameters and Dark Matter

The density parameter is the ratio of the average density of matter and energy in the Universe to the critical density (the density at which the Universe goes on expanding indefinitely).

The present estimate is, $\Omega_0 = 1$

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda \ , \tag{3}$$

where Ω_B is the normal baryonic matter density parameter, Ω_D is the DM density parameter, and Ω_Λ is the vacuum energy density parameter. Current observations suggest that we live in a dark energy dominated Universe with $\Omega_\Lambda = 0.73, \Omega_{\rm D} = 0.23$, and $\Omega_{\rm B} = 0.04$.

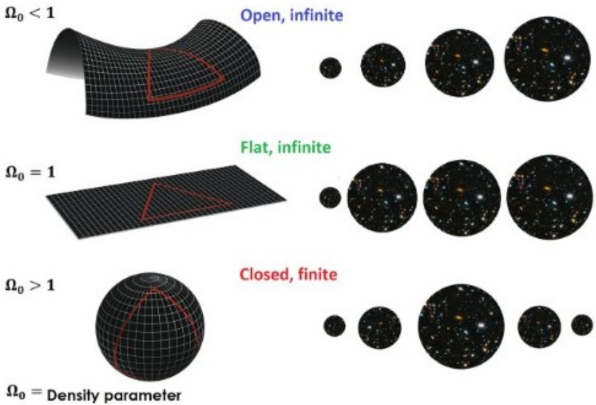


Figure: The density parameter and the predicted models of the Universe (Credit: Modified by Helen Klus, http://www.thestargarden.co.uk/Big-bang.html, original images by NASA and NASA/Hubble. Public domain).

Outline

Evidence for Dark Matter

There are several arguments mentioned below to believe that the Universe is filled with non-baryonic matter, which cannot be observed directly but impacts the evolutionary processes of the Universe gravitationally.

There is a substantial amount of indirect evidence for the presence of DM that has been gathered from the last century until now, such as the flat rotational velocity curves of the spiral galaxies, the galactic cluster mass, gravitational lensing, and hot gas-filled galactic clusters.

Rotational Velocity Curves of Spiral Galaxies

In 1970, an American astronomer, Vera Rubin, measured the red-shift of Andromeda as well as other galaxies and plotted a graph of the distance from the galactic center against the orbital velocities. Galaxies were investigated for the red-shift of separate parts. There were the parts moving away, due to their rotation, and parts moving towards the earth. Though the galaxies are generally red-shifted on average, one could look at the relative blue and red-shifts of the parts. The procedure adopted was to scan the galaxy through narrow slits and plot the red-shifts against the radial distance from the galactic center [2]. Vera Rubin measured the stars on the outskirts of the galactic center move around the galactic center at a similar speed as those closer to the center. She concluded that there had to be more mass. Otherwise, the galaxy would have fallen apart.

V. C. Rubin, W. K. Ford, Jr., "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions", *Astrophys. J.* **159**, (1970) 379.

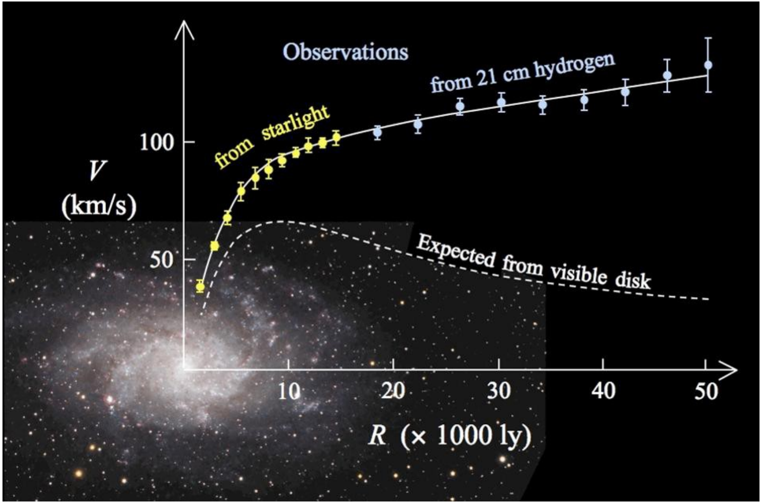


Figure: The Rotational velocity profile for galaxies. The dotted line gives the velocity expected for a sphere of Uniform density and 5 kpc radius. The dashed line gives the more realistic estimate with the tail is still going as $\frac{1}{\sqrt{r}}$. What is typically observed is shown by the blue line with a flat rotational velocity curve going out to about 80 kpc. (Credit: https://phys.org/news/2011-12-dark.html.

Rotational Velocity Curves of Spiral Galaxies

For the pressure-less dust sphere of total mass M, radius a and density $\rho(r)$, the mass required for stability, $M_s(r)$ is

$$M_s(r) \geq r^3\omega_0^2/G \ , \tag{4}$$

where the angular speed $\omega(r)$ is given by [3]

$$\omega(r) = \frac{v_{\text{rot}}}{r} = \sqrt{\frac{4\pi G \int_0^r \rho(\bar{r})\bar{r}^2 d\bar{r}}{r^3}}, \tag{5}$$

A. Qadir, "*Einstein's General Theory of Relativity*", *CSP*, (2020).

Rotational Velocity Curves of Spiral Galaxies

For a constant density, the expression of rotational velocity , v_{rot} , for a linear part inside the sphere is given by

$$v_{\text{rot}} = \sqrt{\frac{4\pi G\rho_0}{3}}r. \tag{6}$$

The expression of rotational velocity, v_{rot} outside the sphere is

$$v_{\text{rot}} = \sqrt{4\pi GM}/\sqrt{r}. \tag{7}$$

Thus outside the sphere, the rotational velocity varies with $\frac{1}{\sqrt{r}}$ [4]. So, the outer region does not follow that density profile.The ratio of the mass of the core, m_c, to the dark matter outside the core, m_d, for a galaxy of core radius a and total radius R, is

$$\frac{m_c}{m_d} = \frac{\int_0^a 4\pi\rho_0 r^2 dr}{\int_a^R 4\pi\rho_0 a^2 dr} = \frac{a}{3(R-a)} \tag{8}$$

A. Qadir, "*Einstein's General Theory of Relativity*", *CSP*, (2020).

Rotational Velocity Curves of Spiral Galaxies

This ratio comes out to be about 3% for the typical figures mentioned above [5]. Hence most of the matter in the galaxy must be dark and reside in large diffuse "haloes" around the galaxy. There must also be a significant dark component of the galactic disc.

A. Qadir, "*Einstein's General Theory of Relativity*", *CSP*, (2020).

Gravitational Lensing

The presence of massive objects curved the geometry of spacetime and the bending of spacetime alters the path of the light, which results in gravitational lensing, in which the intervening masses serve as lenses, distorting, delaying, and magnifying the images of distant objects. The gravitational lens magnifies or distorts distant objects and provides evidence of the existence of DM.

Gravitational lensing is used to determine how much-unseen matter is present in the lens as well as the distribution of mass in the lens. Moreover, DM accounts for more than 90% of the mass that contributes to the strength of the gravitational lensing.

The figure below shows two exceptional pieces of evidence of gravitational lensing in which an individual object appears as four images.

M. Guidry, *Modern General Relativity: Black Holes, Gravitational Waves, and Cosmology* (2019).

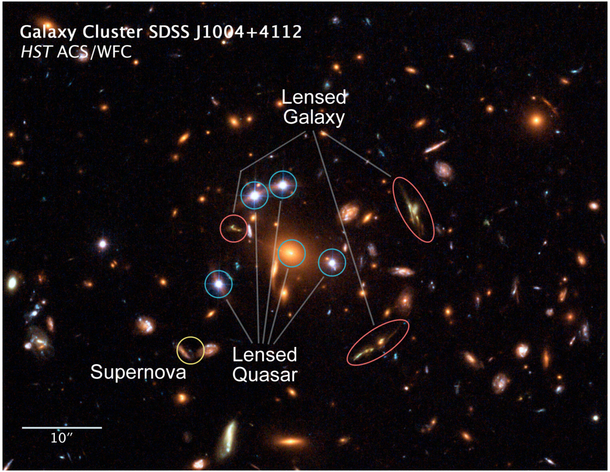


Figure: Because of gravitational lensing, three distorted images of the same background galaxy (red circles) and five images of the identical background quasar (blue circles) are seen in this Hubble view of the galaxy cluster SDSS J1004+4112. (Cite: https: // it.wikipedia.org/wiki/ File: Gravitational Lens SDSS J1004%2*B*4112 Annotated.jpg.

Cluster of Galaxies with Hot Gas

To produce X-rays, the gas particles must have very high velocities. The intensity of the X-rays measures the strength of the gravitational field. To trap a gas or to observe it in the clusters of galaxies, a large amount of matter is required in comparison to visible matter. The presence of hot and visible X-ray gas in the clusters of galaxies is an indication of the existence of DM.

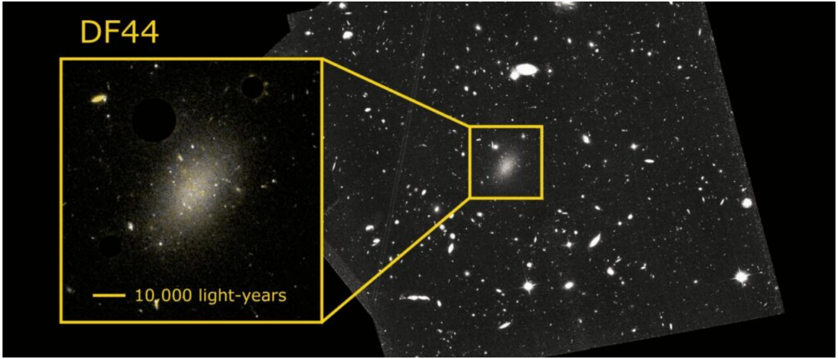


Figure: The images of the ultra-diffusive galaxies (UDG's) in the coma clusters taken by the dragonfly telescope indicate that they mostly consist of DM. It emits only 1% of light which indicates that it contains a large amount of DM. (Credits https: //scitechdaily.com/solved-the-puzzle-of-the-strange-galaxy-made-of-99-99-dark-matter).

Candidates for Dark Matter

The candidates for dark matter are divided into two groups.

- Baryonic candidates for Dark Matter.
- Strong nucleosynthesis constraints put a strong limit on the baryonic matter which is not sufficient to account for DM.
- Non-Baryonic candidates for Dark Matter.
- Cold dark matter (CDM) consists of particles that are not in a state of thermal equilibrium.
- Hot dark matter (HDM) consists of light particles that have relativistic velocities.

Results from Particle Physics

In decades of research, no one detected the DM particle. The high energy community believes that DM should be explained by yet-to-be-found new particles. The issue of whether this phenomenon necessitates a change to the standard model of particle physics or general relativity (GR) remains unanswered. Instead of appealing to the DM which can explain the flat rotational velocity curves, it had been proposed that the law of gravitation should be modified.

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Modified Newtonian Dynamics

In 1983, M. Milgrom suggested that the Newton's law of gravitation should be modified known as MoND as a substitute for non-baryonic DM.

According to Milgrom, maybe we merely need to change the theory of gravity so that it decreases by one over a radial distance squared at high acceleration, in Newton's equation [6]. The MoND gravitational acceleration is related to the Newtonian acceleration a_{N} by an equation

$$a_N = a\mu\left(a/a_0\right) \ , \tag{9}$$

where $\mathrm{a}_0 = 1.2\times10^{-8}\ \mathrm{cm\ s}^{-2}$. For $a >> a_0$, the interpolation function $\mu\left(a/a_0\right)$ shows the asymptotic behavior to agree with Newtonian mechanics.

M. Milgrom, "A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis", *Astrophys. J.* (1983) **270** 365–370.

Failure of MoND

Many expressions have been suggested for μ, but the mostly used function is

$$\mu\left(a/a_0\right) = \frac{a/a_0}{\sqrt{1+\left(a/a_0\right)^2}} \ .$$

Substitute the value of a, and we get

$$a = a_N \left(\frac{1}{2} + \frac{1}{2}\sqrt{1+\left(\frac{2a_0}{a_N}\right)^2}\right)^{1/2} \ .$$

However, MoND failed to deal with many problems especially those related to clusters of galaxies (e.g., the Bullet cluster), and large-scale structures [7].

P. Peebles, Dark matter. *Proc Natl Acad Sci USA* **112**, 12246–12248 (2015).

Modification of The Einstein-Hilbert Action

There are numerous ways to modify the theory of gravity. These theories are the result of a simple modification in the Einstein–Hilbert action. One of the modern methods is to pursue a variation of the Einstein-Hilbert Lagrangian by using an arbitrary function of the Ricci scalar instead of the Ricci scalar itself and that leads to a wide class of non-scientific theories es.

Let us take $f(R)$ gravity which is the simplest modified version

$$S_{\mathrm{EH}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} R \ , \tag{10}$$

where g is the determinant of the metric tensor and R is the Ricci scalar. For a general function of R, the action integral takes the form as

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) \ . \tag{11}$$

Modification of The Einstein-Hilbert Action

The model of the form $f(R) = R^n$ has been used extensively in the literature. By using spherically symmetric solutions to approximate galaxies for $\mathrm{n} = 1 - \alpha/2$, yields

$$\alpha = (3.07 \pm 0.18) \times 10^{-7} \left(\frac{M}{10^{10} M_{\odot}} \right)^{0.494}, \tag{12}$$

where M is the total mass of the galaxy. It is easy to see that α depends on the mass of the galaxy, which cannot be applied to all galaxies for the same function of $f(R)$.

S. Mendoza and Y. M. Rosas-Guevara, *AA*, **472(2)**, (2007) 367-371.

Asgari, Solmaz, and R. Saffari. arXiv preprint arXiv:1010.**1840** 2010.

Modification of The Einstein-Hilbert Action

Subsequently, attempts were made with $f(T)$-gravity [8], and $f(R, T)$-gravity [9], T is the torsion, rather than the stress-energy scalar. But, in these theories, many new parameters appear that do not provide any information regarding the flat rotational velocity curves.

The arbitrariness of the function in $f(R)$, $f(T)$, and $f(R, T)$ modified theories has no basis in explaining the flat rotational velocity curves.

R. Ferraro, *AIP Conf. Proc.* **1471** (2012) 103.

S. Capozziello and M. D. Laurentis, *Phys. Rep.* **509** (2011) 167.

Outline

Modified Relativistic Dynamics

Qadir, Lee, and Kim proposed a modification of the Einstein-Hilbert Lagrangian (EHL). There are two choices for an extra term in the Lagrangian to make sure that there is no quadratic term in the matter field or the curvature: (a) κRT and (b) $\kappa R_{\alpha\beta}T^{\alpha\beta}$. The modified Lagrangian for case (a):

$$\mathcal{L} = b_0 + b_1 R + b_2 T + b_3 RT, \tag{13}$$

where b_0, b_1, b_2, and b_3 are coupling constants. The corresponding modified field equations are

$$b_1\left(R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R\right) - \frac{b_0}{2}g_{\alpha\beta} = -b_2 T_{\alpha\beta} - b_3\left\{R_{\alpha\beta}T + RT_{\alpha\beta} + \left(g_{\alpha\beta}\Box - \nabla_\alpha\nabla_\beta\right)T\right\}. \tag{14}$$

If we take $b_0 = -2\Lambda$, $b_1 = 1$, $b_2 = -\frac{8\pi G}{c^4} := \kappa$ and $b_3 = 0$, we get the standard EFE's (with a cosmological constant).

Modified Relativistic Dynamics

For the second case, the modified Lagrangian is

$$\mathcal{L} = b_0 + b_1 R + b_2 T + b_3 R_{\alpha\beta} T^{\alpha\beta} \ , \tag{15}$$

The corresponding action is

$$S = \int \left(b_0 + b_1 R + b_2 T + b_4 R_{\alpha\beta} T^{\alpha\beta} \right) \sqrt{-g} d^4 x \ . \tag{16}$$

Modified Relativistic Dynamics

The corresponding modified field equations are

$$b_1\left(R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R\right)-\frac{b_0}{2}g_{\alpha\beta}=-b_2T_{\alpha\beta}-\frac{b_4}{2}\left\{\nabla^{\mu}\nabla_{\beta}T_{\alpha\mu}+\nabla^{\mu}\nabla_{\alpha}T_{\mu\beta}\right.$$
$$\left.-R_{\mu\beta}T^{\mu}_{\alpha}+R^{\nu}{}_{\alpha\mu\beta}T^{\mu}_{\nu}-R_{\mu\alpha}T^{\mu}_{\beta}+R^{\nu}{}_{\beta\mu\alpha}T^{\mu}_{\nu}+\Box T_{\alpha\beta}+g_{\alpha\beta}\nabla^{\mu}\nabla^{\nu}T_{\mu\nu}\right\}\ . \tag{17}$$

The above set of equations is still complicated and does not provide useful information.

Modified Relativistic Dynamics

Later, Lee and Qadir suggested that the Lagrangian should be taken as the most general linear combination of the modified Einstein-Hilbert Lagrangian with a term $C_{\mu\rho\nu\sigma}T^{\mu\nu}T^{\rho\sigma}$. To provide a better comparison with the quantum electrodynamics Lagrangian, a source (matter) term coming in and going out, and should interact with the gravitational field at the vertex.

Weyl Modified Gravity and Dark Matter

The model employs the Schwarzschild's interior metric

$$ds^2 = -e^{\eta(r)}c^2dt^2 + e^{\xi(r)}dr^2 + r^2d\Omega^2 \ , \tag{18}$$

The modified Einstein-Hilbert Lagrangian is

$$\mathcal{L} = \sqrt{-g}\left(R - 2\Lambda - \kappa T + \lambda C_{\mu\rho\nu\sigma}T^{\mu\nu}T^{\rho\sigma}\right) \ , \tag{19}$$

where Λ, κ and λ are coupling constants. The matter term is given by $T^{\mu\nu}$ and the Weyl tensor, $C_{\alpha\mu\beta\nu}$, gives the pure gravitational field. The first coupling constant is for the vacuum, the second for matter, and the third one is to be fitted for DM. The idea has been that the same interaction coupling must explain DM for all galaxies, or clusters of galaxies.

Weyl-Modified Gravity and Dark Matter

The modified field equations corresponding to the Lagrangian are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \kappa T_{\mu\nu} + \lambda I_{\mu\nu}, \tag{20}$$

where $I_{\mu\nu}$ is the modification term caused by the interaction term.

$$G_{tt} = \kappa e^{\eta}\rho c^2 + \frac{\lambda c^4 e^{\eta-\xi}}{2r}(r\eta'^2\rho^2 - 2r\eta'\rho\rho' - 4r\rho'^2 - 4r\rho\rho'' + 2r\xi'\rho\rho' - 8\eta'\rho\rho')\ , \tag{21}$$

$$G_{rr} = -\kappa e^{\xi}P + \frac{\lambda c^4}{2}\rho^2\eta'^2. \tag{22}$$

Using the field equations obtained from the modified Lagrangian, we get

$$\xi(r) = F(r) - \ln\left[\left\{\frac{r}{1-\lambda r c^4\rho\rho'}\left(\int\frac{e^{F(r)}}{r^2}dr - \kappa c^2\int\rho(r)e^{F(r)}dr\right)\right\}\right]\ , \tag{23}$$

where

$$F(r) = \frac{1}{2}\int \frac{4 + \lambda r c^4 \left(r\rho^2\eta'^2 - 2r\rho\rho'\eta' - 2r\rho'^2 - 2r\rho\rho'' - 8\eta'\rho\rho'\right)}{r\left(1 - \lambda r c^4 \rho\rho'\right)} dr \ , \tag{24}$$

and

$$\eta'(r) = \frac{1 + \sqrt{1 + 2\lambda\rho^2 c^4 \left(e^{\xi(r)} - 1\right)}}{2r\lambda\rho^2 c^4} \ . \tag{25}$$

Weyl-Modified Gravity and Dark Matter

The square of the rotational velocity is

$$v^2(r) = \frac{1 + \sqrt{1 + 2\lambda\rho^2 c^4 \left(e^{\xi(r)} - 1\right)}}{2\lambda\rho^2 c^2} \,, \tag{26}$$

where

$$\lambda = \frac{2(\frac{v}{c})^2 + e^{\xi(r)} - 1}{2\rho^2 v^4} \,. \tag{27}$$

Weyl-Modified Gravity and Dark Matter

For constant density,

$$e^{\xi(r)} = \frac{e^{F_{const}(r)}}{r\left(\int \frac{e^{F_{const}(r)}}{r^2}dr - \kappa\rho c^2 \int e^{F_{const}(r)}dr\right)} \,, \tag{28}$$

with

$$F_{const}(r) = 2\ln\left(\frac{rc^2}{2GM(r)}\right) + \frac{\lambda\rho^2 c^4}{2}\int r\eta'^2 dr \,, \tag{29}$$

where $M(r)$ denotes the mass within the radius r. The square of the rotational velocity is calculated numerically, assuming r to be the visible radius of the galaxy and consequently $M(r)$ to be the visible mass of the galaxy. Thus, the "gravitational force" is efficiently increased for positive λ.

M. Bilal, and A. Qadir " Testing Weyl-Modified Gravity on M31 and Milky Way" (World Scientific 2021).

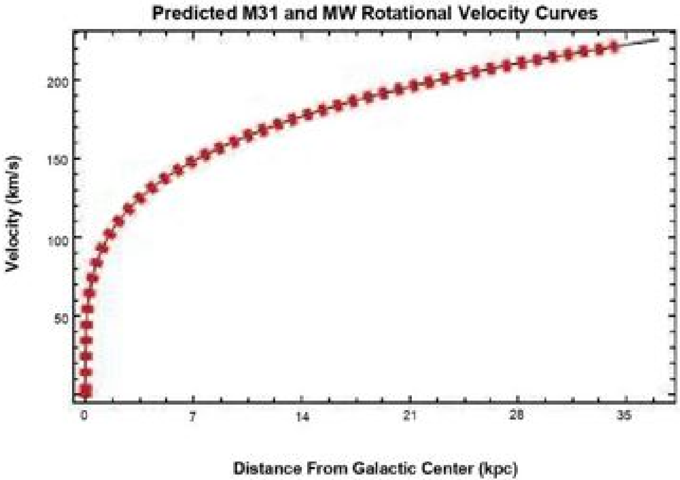

Figure: The black line represents M31 predicted rotation curve, while the red dotted line represents the Milky Way predicted rotation curve $\lambda = 1.14 \times 10^{16}\ km^2 s^4/kg^2$.

Conclusion

Due to the erroneous assumption of constant density, the rotational velocity curves for both galaxies are not flat. However, the curves of both galaxies coincide for $\lambda = 1.14 \times 10^{16}\ km^2 s^4/kg^2$, thus both need the same interaction coupling.

To check that the proposal of the Weyl-interaction modification of gravity is viable, it would be necessary to check the proposal for a more realistic model (variable density) for several different galaxies, taking into account the baryonic DM in that region and distinguishing between the density for the disc matter and the bulge matter.

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Thank You