Cosmic Strings in generalized hybrid metric-Palatini gravity

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Overview

1. Hybrid metric-Palatini
2. Cosmic Strings
3. Solutions for different scalar field potentials
4. Conclusions

Addressing the imbalance - Motivation for Modified Gravity

- Adding a previously unaccounted source. (e.g Λ,...)

- Changing the governing equations.(e.g Gauss-Bonnet gravity, $f(R)$ gravity,...)

Hybrid metric-Platini

Both metric (arXiv:astro-ph/0306438) and Palatini(arXiv:1101.3864) approach are successful in reproducing acceleration rate of the Universe, but are flawed, particularly changing the solar system dynamics. So, a new approach was proposed(arXiv:1508.04641), the hybrid metric-Palatini:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4 x \sqrt{-g} f(R, \mathcal{R}) + S_m \ , \tag{1}$$

$$\mathcal{R}_{\mu\nu} \equiv \hat{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \hat{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \hat{\Gamma}^{\alpha}_{\alpha\lambda}\hat{\Gamma}^{\lambda}_{\mu\nu} - \hat{\Gamma}^{\alpha}_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\alpha\nu} \, . \tag{2}$$

Dynamic equations(in scalar-tensor representation)

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left[(\varphi-\psi)R + \frac{3}{2\phi}\partial_\mu\psi\partial^\mu\psi - V(\varphi,\psi)\right] + S_m\,. \tag{3}$$

Performing the variation of the action with respect to the metric and the scalar fields, and redefining φ as $\xi^2 = \varphi - \psi$ yields the field equations

$$\xi^2 G_{\mu\nu} = \kappa^2 T_{\mu\nu} + \nabla_\mu\nabla_\nu\xi^2 + \frac{3}{2\psi}\partial_\mu\psi\partial_\nu\psi - \left(\Box\xi^2 + \frac{1}{2}\bar{V} + \frac{3}{4\psi}\partial^\alpha\psi\partial_\alpha\psi\right)g_{\nu\mu} \tag{4}$$

and

$$\Box\psi - \frac{1}{2\psi}\partial_\mu\psi\partial^\mu\psi - \frac{\psi}{3}\left[\frac{1}{2\xi}\bar{V}_{,\xi} + \bar{V}_{,\psi}\right] = 0 \tag{5}$$

$$\Box\xi^2 + \frac{1}{2\psi}\partial_\mu\psi\partial^\mu\psi + \frac{1}{6}\left[4\bar{V} - \xi\bar{V}_{,\xi}\right] = \frac{\kappa^2}{3}T\,, \tag{6}$$

Cosmic strings

- Topological defects formed at phase transitions after SSB.
- Featured on several Grand Unified scenarios.
- Possible impact on CMB anisotropies, small scale structure formation, Gamma Ray Bursts, stochastic background GW spectra.

Straight infinite Cosmic String

$$T_t^t = T_z^z = -\sigma(r)\,, \tag{7}$$

where σ is the string tension. We consider a general cylindrically symmetric static metric

$$ds^2 = -e^{2(K-U)}dt^2 + e^{2(K-U)}dr^2 + e^{-2U}W^2 d\theta^2 + e^{2U}dz^2, \tag{8}$$

where t, r, θ and z denote the time, radial, angular and axial cylindrical coordinates, respectively, and K, U and W are functions of r alone.

Full set of equations

In terms of two dynamical variables defined as $u = \varphi'$ and $v = \psi'$

$$\frac{dW}{dr} = \frac{1}{u - v}\left(\frac{3v^2}{4\psi} - \frac{V}{2}\right) W \tag{9}$$

$$\frac{du}{dr} = -\frac{v}{u - v}\left(\frac{3v^2}{4\psi} - \frac{V}{2}\right) - \frac{1}{4}\frac{v^2}{\psi} + \frac{\psi}{3}(V_\psi + V_\varphi) - \frac{V}{2} \tag{10}$$

$$\frac{dv}{dr} = -\frac{v}{u - v}\left(\frac{3v^2}{4\psi} - \frac{V}{2}\right) + \frac{1}{2}\frac{v^2}{\psi} + \frac{\psi}{3}(V_\psi + V_\varphi) \tag{11}$$

$$\kappa^2\sigma = \frac{1}{2}\left[V(\varphi, \psi) + V_\varphi(\varphi, \psi) - \frac{3v^2}{2\psi}\right] \tag{12}$$

Potential function and parameter space

- We need to specify a potential function $V(\varphi, \psi)$.
- We need also a set of boundary values $(\varphi(0), \psi(0), W(0), u(0), v_0)$
- In addition, more parameters can "appear" in the form of the potential.

Exact Solutions

- $V(\varphi, \psi) = 0$

$$W(r) = \frac{3c_1(r + c_2)}{4w_o} \qquad -\kappa^2\sigma = \frac{4{w_o}^2}{3(c_2+r)^2} \tag{13}$$

$$\tag{14}$$

- $V(\varphi, \psi) = \Lambda = cnst.$

$$W(r) = W_0(r + 2c_1) \qquad \kappa^2\sigma = \frac{\Lambda}{2} - \frac{3c_2}{4(2c_1+r^2)} \tag{15}$$

$$\tag{16}$$

Numerical solutions

We studied numerical solutions for different types of potentials:

- $V(\varphi, \psi) = a\varphi^2 + b\psi^2$
- $V(\varphi, \psi) = ae^{-\lambda_1\varphi} + be^{-\lambda_2\psi}$
- $V(\varphi, \psi) = a\varphi^2\psi^2$
- $V(\varphi, \psi) = ae^{-\lambda_1\varphi-\lambda_2\psi}$

What we learned?

- Unlike in Brans-Dicke gravity, string solutions are found in GHMP.
- String tension not necessarily positive.
- Number of parameters of the models allow a wide range of solutions.
- Solutions very sensitive on the initial conditions.
- Not asymptotically flat.

Behaviour of the string tension

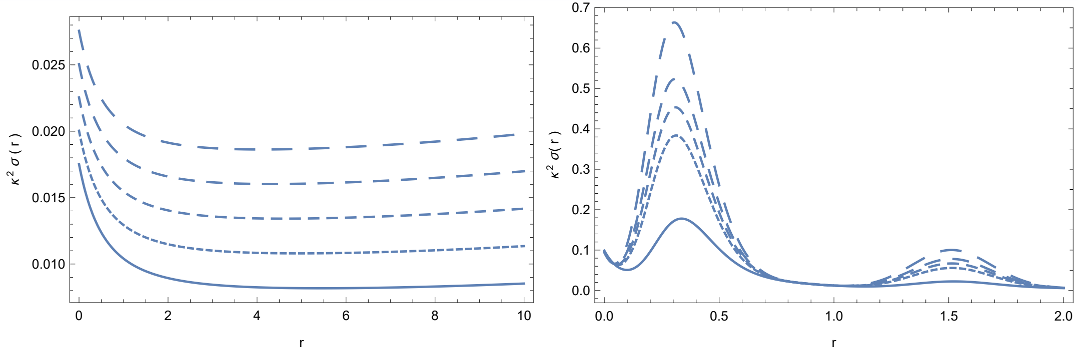

Figure: Variations of the string tension $\sigma(r)$ as a function of r for $V(\varphi, \psi) = a\varphi^2\psi^2$ (right panel) and for $V(\varphi, \psi) = ae^{-\lambda_1\varphi - \lambda_2\psi}$(left panel) for different values of a.

Thank you.