Precision Cosmology and Hubble tension in the era of LSS surveys

Giuseppe Fanizza
IA & FCUL - Lisbon

based on F, Fiorni, Marozzi arXiv:2102.12419
The best-fit of the CMB data (Planck 2018) allows us to infer the value of few cosmological parameters with the highest precision that we can achieve so far in cosmology.

Among all these parameters, the one we are interested in is $H_0$, namely the current expansion rate of the Universe, which can be determined to be $67.4 \pm 0.5$ km/s Mpc$^{-1}$.

The greatest concern that we can have about that is the dependency by the cosmology that we need to assume in order to fit data.
Local measurements

- At the background level, we can write luminosity distance as (datapoints from Union2)

\[ d_L = \frac{1 + z}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1 + z')^3 + \Omega_\Lambda}} \]

- For very low redshifts, these relations becomes independent on the chosen cosmology and leads to an estimator of the Hubble rate today as \( H_0 = \frac{z}{d_L} \)

- The analysis of local measurements from Supernovae (Riess et al. 2020) leads to an estimation of the Hubble rate today as \( 73.2 \pm 1.3 \text{ km/s Mpc}^{-1} \)
Figure: From Di Valentino, Mena, Pan, Visinelli, Yang, Melchiorri, Mota, Riess, Silk 2021
Cosmic variance in the ideal case

- Irreducible statistical error for a single measurement around the mean value which is given by the presence of stochastic inhomogeneities

The estimation of this dispersion\(^2\) is given under the hypothesis of infinite number of ideal sources for each redshift, stochastically distributed all across the observed sky and returns a cosmic variance at higher redshift which is of 10%.

\(^2\)Ben-Dayan, Marozzi, Nugier, Gasperini, Veneziano, 2013
Cosmic variance for realistic future surveys

For realistic surveys, sky-coverage is always partial and number of sources bin by bin is finite

Figure: Forecasted distribution of Superluminous Supernovae, from Inserra et al. 2020

▶ EDS: 135 sources up to $z = 3.5$ with two covered regions of 20 $\text{deg}^2$ each
▶ LSST: 929 sources up to $z = 3.85$ with sky coverage 9000 $\text{deg}^2$
▶ We deal with partial sky coverage surveys
In the case of finite number $N$ of sources, distributed observed within a partial sky coverage, we need a better suited estimator for the bias and dispersion of cosmic distances. We take

$$\langle \ldots \rangle \rightarrow \frac{1}{N^2} \sum_{i,j=1}^{N} (\ldots)$$

The impact of inhomogeneities on the distance-redshift relation can be obtained by inverting

$$d_L(z_i, n_i) = \overline{d_L} \left( 1 + \delta^{(1)} + \delta^{(2)} \right) (z_i, n_i) \Rightarrow$$

$$H_0(z_i, n_i) = \overline{H_0} \left[ 1 - \delta^{(1)} - \delta^{(2)} + (\delta^{(1)})^2 \right] (z_i, n_i)$$

Here $\delta^{(1)}$ and $\delta^{(2)}$ are respectively the linear and quadratic perturbations of the $d_L(z, n)$
In this way, it can be shown that the variance associated to the Hubble rate measurement, within a finite sample, around the bare value $\overline{H_0}$ is

$$\left( \frac{\Delta H_0}{H_0} \right)^2 = \frac{1}{N^2} \sum_{i,j=1}^{N} \delta^{(1)}(z_i, n_i) \delta^{(1)}(z_j, n_j)$$

Only linear inhomogeneities matter.

$\delta^{(1)}$ again is sourced at the leading order in the gradient expansion by lensing and Doppler.
Our estimator for the Hubble rate is then directly related to the 2-point correlation function of the luminosity distance redshift relations

\[ \xi(z_i, z_j, n_i \cdot n_j) = \bar{\delta^{(1)}}(z_i, n_i) \bar{\delta^{(1)}}(z_j, n_j) \]

Interestingly, a multipoles analysis of returns that the lensing 2-point correlation function has vanishing monopole and is extremely peaked when the 2 sources are aligned.

\[ z_1 = 0.75, \, z_2 = 0.15, \, \ell = 19 \]

\[ z_1 = 0.75, \, z_2 \text{ from } 0.15 \text{ to } 1.55 \]
Useful approximation for non-linear scales

- Lensing terms are the relevant ones when we aim to estimate the sky-limited cosmic variance for the $H_0$ measurement.
- Monopole for the lensing 2-point correlation function turns out to be exactly null.
- At higher redshift in the bin sampling, lensing 2-point correlation function is extremely peaked when the sources are aligned and rapidly becomes negligible beyond this angular scale $\nu_{th}^{ij}$.
- This allows us to approximate the lensing 2-point correlation function as follows:

$$\xi_{app}^{Lij}(\nu) = \Theta(\nu_{th}^{ij} - \nu) \left| \xi_+^{Lij} \right| - \Theta(\nu - \nu_{th}^{ij}) \left| \xi_-^{Lij} \right|,$$

where $\nu_{th}^{ij}$ is analytically determined by requiring that the monopole of $\xi_{app}^{Lij}(\nu)$ vanishes as well.
Forecasts for upcoming surveys

- According to the approximation scheme outlined above, it turns out that non-linear scales in the matter power spectrum are important only when the two sources are almost exactly aligned, in the range of redshift of our interest.

- This can be understood by looking at the geometrical structure of the line-of-sight integrals.

- By using the non-Linear Power spectrum, it turns out that Cosmic Variance is expected to affect by 0.03% for LSST and by 0.2% for EDS. These forecasts are indeed not too much sensitive to the non-linear scales.

- These forecasts are smaller than the ones obtained for closer sources, estimated to be of order 1% (Ben-Dayan, Durrer, Marozzi, Schwarz, 2014 and Macpherson, Heinesen 2021).
Conclusions

- Our first analytical estimations of the cosmic variance for limited-sky-coverage surveys indicate that forthcoming high-redshift surveys are well-suited to provide a precise determination of cosmological parameters, such as $H_0$.
- These forecasted errors for LSST and Euclid Deep Surveys are stable enough to be quite insensitive to the role of non-linearities in the matter power spectrum.
- Despite our analysis has been performed entirely within the $\Lambda$CDM model, from the geometrical structure of the light-cone general features emerge.
- Lensing 2-point correlation function has vanishing monopole, hence in the limit of large sky coverage and huge number of sources, the cosmic variance must be dominated by Doppler effect also on high redshift surveys.
- As a consequence of line-of-sight integration and stochastic isotropy, non-linear scales are important only when two sources are almost aligned.