# Finite Action Principle and black holes in Horava-Lifszyc gravity

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**Based on arXiv: 2102.13556 with J. Chojnacki**

# Finite action principle

- The action of a gravity / matter system should be finite for the quantum system to contribute to path integral $\int \mathscr{D} g_{\mu\nu} e^{iS[g_{\mu\nu}]}$.

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- Selection principle for actions 2012.08570.

# $R^2$ gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{\kappa^2}R - \Lambda - \frac{1}{2\sigma}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \frac{\omega}{3\sigma}R^2\right]$$

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- Possible resolutions: new degrees of freedom, infinitely many derivatives, non-perturbative unitarity, break Lorentz invariance.

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- $V \supset {}^{(3)}R^3$, which gives the same conditions as $R^2$ for black holes.

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- We require $S_s(r_{UV}, r_{IR})$ to be infinite as $r_{UV} \to 0$ for singular black-holes

# Singular black holes

- (Anti-) de Sitter Schwarzschild solution
  $$ds^2 = -dt^2 + e^{2\nu}(dr + e^{\mu-\nu}dt)^2 + r^2 d\Omega^2,$$
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- For $\lambda \neq 1$there are divergences for $r_{UV} = 0$ and $r_s = \left(\frac{3M}{|\Lambda|}\right)^{1/3}$.
- For Schwarzschild in orthogonal frame (not a solution in HL gravity) the $R_{ij}R^{ij}$ term is divergent.

# Regular black holes

- The Hayward metric - regular black hole solution in GR:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \quad f(r) = 1 - \frac{2Mr^2}{\left(r^3 + 2g^3\right)},$$

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- Check for HL regular blackholes

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- The action is again finite!

# Conclusions

- Quantum gravity shall resolve the singularities issue.
- Finite action principle: selects what can be in the exponent of $\int \mathscr{D} g_{\mu\nu} e^{iS_g[g_{\mu\nu}]}$.
- Horava gravity satisfies this principle.
- No-regular solutions found for HL: wormhole - black hole tradeoff
- Interesting consequences for cosmology, see 1912.12926,1909.01169, 2102.05550 and next talk