16th Marcel Grossman - 2021, AT6 Session

Horava-Lifshitz and Einstein-Aether Gravity in the light of Event Horizon Telescope Observations of M87

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Goal and Motivation

- Goal: To investigate Einstein-Aether gravity in light of the recent Event Horizon Telescope (EHT) observations of the M87*.

- M. Khodadi and E.N. Saridakis, Phys. Dark Univ. 32, 100835 (2021)

- The shape and size of the observed black hole shadow contains information of the geometry in its vicinity, and thus one can consider it as a potential probe to investigate different gravitational theories.

Einstein-Aether gravity

The action of the EA theory is [T. Jacobson D. Mattingly, Phys. Rev. D 64, 024028 (2001)]

$$S = S_{EH} + S_{AE} = \frac{1}{16\pi G_{AE}} \int d^4x \sqrt{-g}\Big(R + \mathcal{L}_{AE}\Big), \qquad (1)$$

which includes the standard Einstein-Hilbert action S_{EH} plus the Aether action.

The Lagrangian of the Aether field u^μ is defined as

$$\mathcal{L}_{AE} \equiv -\Big(c_1 g^{\alpha\beta} g_{\mu\nu} + c_2 \delta^\alpha_\mu \delta^\beta_\nu + c_3 \delta^\alpha_\nu \delta^\beta_\mu - c_4 u^\alpha u^\beta g_{\mu\nu}\Big) \cdot(\nabla_\alpha u^\mu)(\nabla_\beta u^\nu) + \lambda_0 (u^2 + 1)\ . \qquad (2)$$

λ_0 is a Lagrangian multiplier, ensuring that the Aether four-velocity u^α is always timelike (i.e. $u^2 = -1$).

All of four coupling constants (c_1, c_2, c_3, c_4) are dimensionless, and G_{AE} is linked to the Newtonian constant G_N as $G_{AE} = \frac{2G_N}{2-c_1-c_4}$.

Einstein-Aether gravity

Variations of the total action with respect to $g_{\mu\nu}$, u^α, λ_0 yield, respectively, the field equations

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G_{AE}T^{\mu\nu}_{AE}, \tag{3}$$

$$\nabla_\mu J^\mu{}_\alpha + c_4 a_\mu \nabla_\alpha u^\mu + \lambda_0 u_\alpha = 0, \tag{4}$$

$$g_{\mu\nu}u^\mu u^\nu = -1\ , \tag{5}$$

where

$$T^{Ae}_{\alpha\beta} \equiv \nabla_\mu\Big(J^\mu{}_{(\alpha}u_{\beta)} + J_{(\alpha\beta)}u^\mu - u_{(\beta}J_{\alpha)}{}^\mu\Big) + c_1\Big[(\nabla_\alpha u_\mu)(\nabla_\beta u^\mu) - (\nabla_\mu u_\alpha)(\nabla^\mu u_\beta)\Big] + c_4 a_\alpha a_\beta + \lambda u_\alpha u_\beta - \frac{1}{2}g_{\alpha\beta}J^\delta{}_\sigma\nabla_\delta u^\sigma, \tag{6}$$

$$J^\alpha{}_\mu \equiv \Big(c_1 g^{\alpha\beta}g_{\mu\nu} + c_2\delta^\alpha_\mu\delta^\beta_\nu + c_3\delta^\alpha_\nu\delta^\beta_\mu - c_4 u^\alpha u^\beta g_{\mu\nu}\Big)\nabla_\beta u^\nu, \tag{7}$$

$$a^\mu \equiv u^\alpha\nabla_\alpha u^\mu. \tag{8}$$

Spherically symmetric solutions

Then the metric for EA black holes can be written as

$$ds^2 = -e(r)dv^2 + 2f(r)dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2) \,, \tag{9}$$

with the Killing vector $\chi^a = (1, 0, 0, 0)$, where $e(r)$, $f(r)$ are r-dependent functions.
Additionally, we consider the Aether vector parametrization

$$u^a(r) = (\alpha(r), \beta(r), 0, 0) \,, \tag{10}$$

with $\alpha(r)$, and $\beta(r)$ the involved functions.
Concerning the metric components at infinity (boundary conditions) we require to correspond to the asymptotically flat solution, while those for the Aether components are set as $u^a = (1, 0, 0, 0)$.

Spherically symmetric solutions

Two types of exact solutions [E. Barausse et. al, Phys. Rev. D 83, 124043 (2011)]:

- The first solution corresponds to the special choice of coupling constants $c_{14} = 0$ (where $c_{14} \equiv c_1 + c_4$) and $c_{123} \neq 0$ (where $c_{123} \equiv c_1 + c_2 + c_3$):

$$e(r) = 1 - \frac{2M}{r} - \frac{27c_{13}}{256(1-c_{13})}\left(\frac{2M}{r}\right)^4, \tag{11}$$

$$f(r) = 1, \tag{12}$$

$$\alpha(r) = \left[\frac{3\sqrt{3}}{16\sqrt{1-c_{13}}}\left(\frac{2M}{r}\right)^2 + \sqrt{1 - \frac{2M}{r} + \frac{27}{256}\left(\frac{2M}{r}\right)^4}\right]^{-1}, \tag{13}$$

$$\beta(r) = -\frac{3\sqrt{3}}{16\sqrt{1-c_{13}}}\left(\frac{2M}{r}\right)^2, \tag{14}$$

Spherically symmetric solutions

- The second solution corresponds to $c_{123} = 0$ and reads as

$$e(r) = 1 - \frac{2M}{r} - \frac{2c_{13} - c_{14}}{8(1 - c_{13})}\left(\frac{2M}{r}\right)^2, \quad (15)$$

$$f(r) = 1, \quad (16)$$

$$\alpha(r) = \frac{1}{1 + \left(\sqrt{\frac{2-c_{14}}{2(1-c_{13})}} - 1\right)\frac{M}{r}}, \quad (17)$$

$$\beta(r) = -\sqrt{\frac{2 - c_{14}}{8(1 - c_{13})}\frac{2M}{r}}, \quad (18)$$

where $c_{13} \equiv c_1 + c_3$.

- It is clear that by fixing $c_{13} = 0$ in the first solution and $c_{13} = 0 = c_{14}$ in the second solution, then we recover the standard Schwarzschild BH, as expected.

Spherically symmetric solutions

Finally, since usually the metric is written in the form of the $(t,\ r,\ \theta,\ \phi)$ coordinates, using the coordinate transformation $dt = dv - \frac{dr}{e(r)},\ \ dr = dr$, the metric (9) in the Eddington-Finklestein coordinate system, can be re-expressed as

$$ds^2 = -e(r)dt^2 + \frac{dr^2}{e(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\ , \qquad (19)$$

and hence the Aether field becomes

$$u^a = \left(\alpha(r) - \frac{\beta(r)}{e(r)}, \beta(r), 0, 0\right). \qquad (20)$$

Spherically symmetric rotating solutions

- The difficulty in deriving BH solutions in Lorentz violating theories is related to the existence of casual boundaries, indicating an event horizon. Thus, in EA-gravity (as well as in Horava-Lifshitz one), due to complexities, we still do not have the fully rotating BH solution.
- Despite this, one can utilize spherically symmetric BH solutions in the Hartle-Thorne slow-rotation approximation (first order), in order to derive the rotating BH solutions in the slow limit.

Hence, applying the well-known Hartle-Thorne metric

$$
\begin{aligned}
ds^2 = -e(r)dt^2 + \frac{B(r)dr^2}{e(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \\
-\epsilon r^2 \sin^2\theta\Omega(r,\theta)dtd\phi + \mathcal{O}(\epsilon^2)\ ,
\end{aligned}
\tag{21}
$$

with ϵ a perturbative slow rotation parameter, we can derive the rotating BH solution in the slow rotation limit.

Spherically symmetric rotating solutions

In order to satisfy the asymptotically flat boundary conditions, we require θ-independence, namely $\Omega(r,\theta) = \Omega(r)$ and $\lambda(r,\theta) = \lambda(r)$.

For the first static solution there exists a corresponding slowly rotating black hole solution, with a spherically symmetric Aether field configuration ($\lambda(r) = 0$) and thus

$$\Omega(r) = \Omega_0 + \frac{4J}{r^3}\,, \tag{22}$$

with Ω_0 an integration constant that can be set to zero. Note that for convenience we replace the angular momentum J by introducing the rotation parameter a through

$$a \equiv \frac{J}{M}. \tag{23}$$

Black hole shadow in Einstein-Aether gravity

In order to study the geodesics structure of the photon trajectories, we begin with the Hamilton-Jacobi equation

$$\frac{\partial S}{\partial \lambda} = -\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^{\mu}} \frac{\partial S}{\partial x^{\nu}} \ , \tag{24}$$

where S and λ denote the Jacobi action of the particle (here photon) moving in the black hole spacetime, and the affine parameter of the null geodesic.
Concerning the massless photon propagating on the null geodesics, the Jacobi action S can be separated as

$$S = -Et + J\phi + S_r(r) + S_\theta(\theta) \ , \tag{25}$$

where E and J respectively address the energy and angular momentum of the photon in the direction of the rotation axis.

Black hole shadow in Einstein-Aether gravity

Thus, the photon propagation obeys the following four equations of motion, obtained from the variation of the Jacobi action with respect to the affine parameter λ:

$$\frac{dt}{d\lambda} = \frac{E}{e(r)} - \frac{2MJa}{e(r)r^3}, \tag{26}$$

$$\frac{dr}{d\lambda} = \frac{\sqrt{R(r)}}{r^2}, \tag{27}$$

$$\frac{d\theta}{d\lambda} = \frac{\sqrt{\Theta(\theta)}}{r^2}, \tag{28}$$

$$\frac{d\phi}{d\lambda} = \frac{J}{\sin^2\theta r^2} - \frac{2MEa}{r^3 e(r)}. \tag{29}$$

where $\mathcal{K}$ a separation constant and

$$R(r) = E^2 r^4 - (\mathcal{K} + J^2) r^2 e(r) - 4MaEJr\ , \tag{30}$$

$$\Theta(\theta) = \mathcal{K} - J^2 \cot^2\theta. \tag{31}$$

Black hole shadow in Einstein-Aether gravity

In order to investigate the photon trajectories one usually expresses the radial geodesics in terms of the effective potential $U_{eff}(r)$ as

$$\left(\frac{dr}{d\lambda}\right)^2 + U_{eff}(r) = 0 \tag{32}$$

with

$$U_{eff}(r) = -1 + \frac{e(r)}{r^2}(\xi^2 + \eta) + \frac{4M\eta a}{r^3}, \tag{33}$$

where $\xi = \frac{J}{E}, \;\; \eta = \frac{\mathcal{K}}{E^2}$. The above two impact parameters ξ and η are actually the principle quantities for determining the photon motion.

Black hole shadow in Einstein-Aether gravity

To obtain the geometric shape of the BH shadow, conventionally we have to find the photon critical circular orbit. This can be extracted from the following unstable conditions:

$$U_{eff}(r) = 0\ , \quad \frac{dU_{eff}(r)}{dr} = 0\ , \quad \frac{d^2 U_{eff}(r)}{dr^2} < 0\ . \tag{34}$$

We can extract the geometric shape of the shadow via the allowed values of ξ and η that satisfy the above conditions. Thus, with the implementation of (34) we arrive at

$$2(\eta + \xi^2) r e(r) - (\eta + \xi^2) r^2 e'(r) + 12 M \xi a = 0\ . \tag{35}$$

By solving this equation one acquires the radius r_{ps} of the photon sphere, which since we have taken the rotation effect into account is expected to be between the two values $r_{ps}^{\mp}$.

Black hole shadow in Einstein-Aether gravity

For slowly rotating BHs, solving conditions (34) we immediately find that for the spherical-orbit photon motion the two parameters ξ and η have the form

$$\xi(r) = \frac{r^3[re'(r) - 2\,e(r)]}{4Ma[e(r) + re'(r)]}\,,$$
$$\eta(r) = \frac{-r^6[-2e(r) + re'(r)]^2 + 48M^2a^2r^2\left[e(r) + re'(r)\right]}{16M^2a^2\left[e(r) + e'(r)\right]^2}\,.$$

Overall, the gravitational lensing effects result in deflection of the photon passing a BH. Some photons have the chance of reaching the distant observer after being deflected by the BH, while some others will fall into it.
The photons that cannot escape the black hole are the ones that create the black hole shadow.

Black hole shadow in Einstein-Aether gravity

To describe the shadow as seen by a distant observer, one introduces the following two celestial coordinates X and Y:

$$X = \lim_{r_* \to \infty} \left(-r_*^2 \sin\theta_0 \frac{d\phi}{dr} \right), \quad (36)$$

$$Y = \lim_{r_* \to \infty} r_*^2 \frac{d\theta}{dr}, \quad (37)$$

where r_* and θ_0 are respectively the distance between the observer and the black hole, and the inclination angle between the line of sight of the observer and the rotational axis of the black hole. By applying the geodesics equations we obtain

$$X = -\xi(r_{ps}) \csc\theta_0 \ , \quad (38)$$

$$Y = \sqrt{\eta(r_{ps}) - \xi^2(r_{ps}) \cot^2\theta_0} \ , \quad (39)$$

and therefore $X^2 + Y^2 = \xi^2(r_{ps}) + \eta_{\mathbf{ps}}(r_{ps})$, where r_{ps} is the radius of the unstable photon sphere. This is the expression of the EA BHs shadow in the slow rotation limit.

Einstein-Aether type I black hole solution

In order to draw the shadow of the BH solution we need to calculate two essential quantities, namely the event horizon radius r_e and the radius of the unstable photon sphere r_{ps}.

$$r_{e_{1,2}} = \frac{3M^2}{2} - s \pm \frac{1}{2}\sqrt{-4s^2 + 3M^2 - \frac{M^2}{s}}, \qquad (40)$$

$$r_{e_{3,4}} = \frac{3M^2}{2} + s \pm \frac{1}{2}\sqrt{-4s^2 + 3M^2 + \frac{M^2}{s}}, \qquad (41)$$

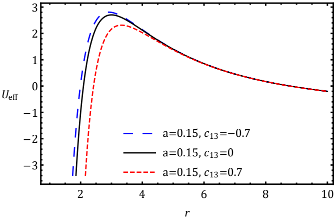
where

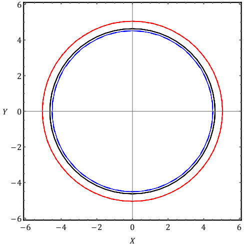
$$s = \sqrt{\frac{3M^2Q + Q^2 + \Delta_0}{12Q}}, \quad Q = \left(\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}\right)^{1/3} \qquad (42)$$

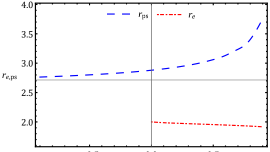
$$\Delta_0 = -\frac{81c_{13}}{4(1-c_{13})}M^4, \quad \Delta_1 = \frac{729c_{13}}{2(1-c_{13})}M^7. \qquad (43)$$

Einstein-Aether type I black hole solution

Solutions $r_{1,2}$ are imaginary and thus not physically interesting. Nevertheless, by setting c_{13} to zero the third solution becomes $r_3 = 2M$, as expected from Schwarzschild background. Thus, we deduce that r_3 addresses the event horizon radius r_e of the Einstein-Aether type I BH solution.

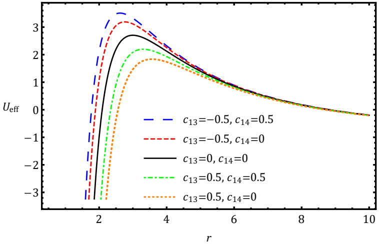


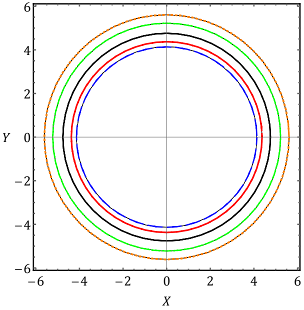




Einstein-Aether type II black hole solution

$$r_{e\mp} = M\left(1 \mp \sqrt{\frac{c_{14}-2}{2c_{13}-2}}\right) . \tag{44}$$





The Aether parameters c_{13} and c_{14} and M87* observations

In light of the report released by EHT collaboration for the shadow of M87* in [K. Akiyama et al.[Event Horizon Telescope Collaboration], Astrophys. J. Lett. 875, L1 (2019)] for the angular size of the shadow, the mass and the distance to M87* one respectively has the values

$$\delta = (42 \pm 3)\,\mu\text{arcsec}, \tag{45}$$

$$M = (6.5 \pm 0.9) \times 10^9\, M_\odot, \tag{46}$$

$$D = 16.8^{+0.8}_{-0.7}\ \text{Mpc}, \tag{47}$$

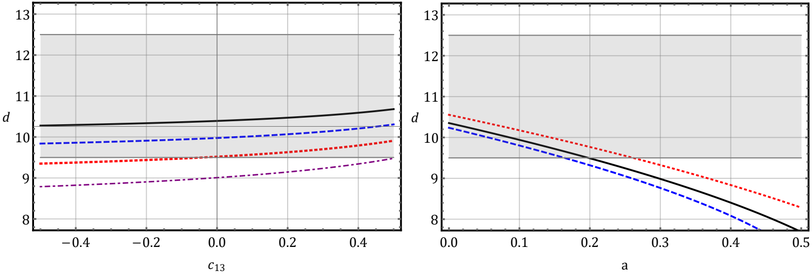
where $M_\odot$ is the Sun mass.

One can merge this information by introducing the single number d_{M87*}, which quantifies the size of M87*'s shadow in unit mass:

$$d_{M87*} \equiv \frac{D\delta}{M} \approx 11.0 \pm 1.5\,. \tag{48}$$

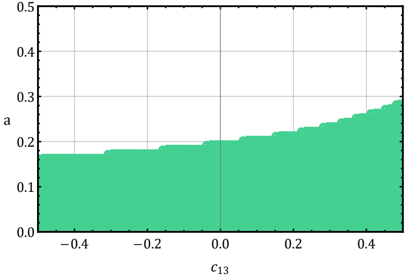
This combination can be used in order to confront with the theoretically predicted shadows.

Einstein-Aether type I black hole solution



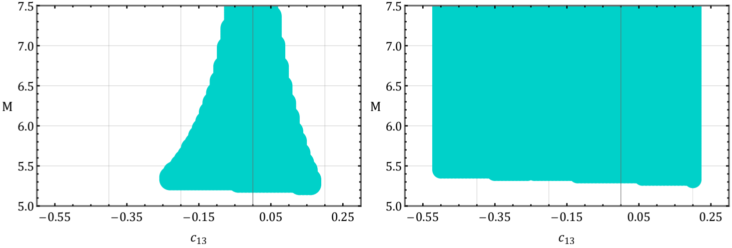
Left graph: The predicted diameter per unit mass d_I for the Einstein-Ae ther type I black hole solution, as a function of the Aether parameter c_{13}, for several values of the rotational parameter: $a = 0$ (black - solid), $a = 0.1$ (blue - dashed), $a = 0.2$ (red - dotted), $a = 0.3$ (purple - dashed-dotted). **Right graph:** d_I as a function of the rotational parameter a, for $c_{13} = -0.5$ (blue - dashed), $c_{13} = 0$ (black- solid), $c_{13} = 0.5$ (red- dotted). In both graphs the shaded area mark the observationally determined diameter per unit mass of M87*'s shadow, namely d_{M87*}, within 1σ-error.

Einstein-Aether type I black hole solution



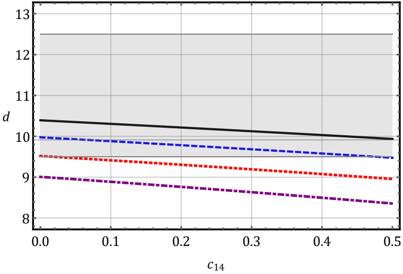
The allowed parameter region (green) in the $c_{13} - a$ plane, for fixed BH mass ($M = 6 \times 10^9 M_\odot$), for the Einstein-AEther type I black hole solution, that leads to diameter per unit mass d_I in agreement with the observationally determined one d_{M87*} within 1σ-error

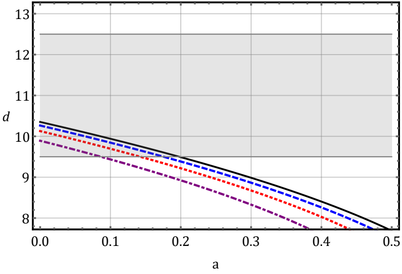
Einstein-Aether type I black hole solution



The allowed parameter region (green) in the $c_{13} - M(10^9 M_\odot)$ plane, for fixed rotation parameter values $a = 0$ (left graph) and $a = 0.2$ (right graph), for the Einstein-Ae ther type I black hole solution, that leads to diameter d_I in agreement with the observationally determined one d_{M87*} within 1σ-error.

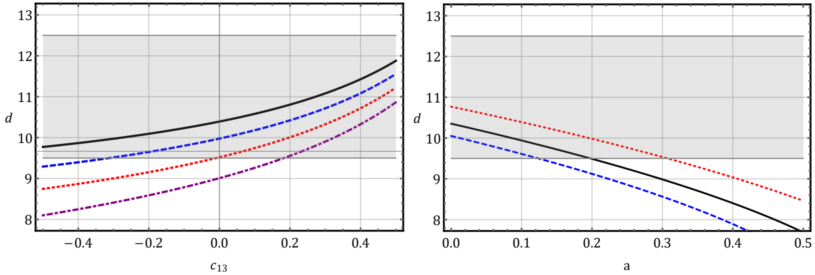
Einstein-Aether type Î black hole solution





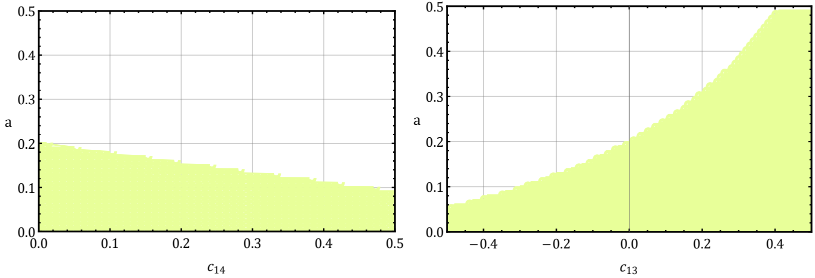
Left graph: The predicted diameter per unit mass d_{II} for the Einstein-Ae ther type II black hole solution, as a function of the Aether parameter c_{14}, for fixed $c_{13} = 0$ and for several values of the rotational parameter: $a = 0$ (black-solid), $a = 0.1$ (blue - dashed), $a = 0.2$ (red - dotted), $a = 0.3$ (purple - dashed-dotted). **Right graph:** d_{II} as a function of the rotational parameter a, for fixed $c_{13} = 0$ and for $c_{14} = 0$ (black-solid), $c_{14} = 0.1$ (blue - solid), $c_{14} = 0.25$ (red - dotted), $c_{14} = 0.5$ (purple - dashed-dotted). In both graphs the shaded area mark the observationally determined diameter per unit mass of M87*'s shadow, namely d_{M87*}, within 1σ-error.

Einstein-Aether type ÎI black hole solution



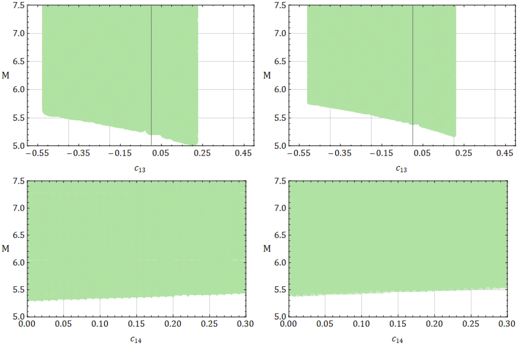
Left graph: The predicted diameter per unit mass d_{II} for the Einstein-Ae ther type II black hole solution, as a function of the Aether parameter c_{13}, for fixed $c_{14} = 0$ and for several values of the rotational parameter: $a = 0$ (black-solid), $a = 0.1$ (blue - dashed), $a = 0.2$ (red - dotted), $a = 0.3$ (purple - dashed-dotted). **Right graph:** d_{II} as a function of the rotational parameter a, for fixed $c_{14} = 0$ and for $c_{13} = 0$ (black-solid), $c_{13} = -0.2$ (blue - solid), $c_{13} = 0.2$ (red - dotted). In both graphs the shaded area mark the observationally determined diameter per unit mass of M87*'s shadow, namely d_{M87*}, within 1σ-error.

Einstein-Aether type I black hole solution



The allowed parameter regions (light green) in the $c_{14} - a$ and $c_{13} - a$ planes, for fixed BH mass ($M = 6 \times 10^9 M_\odot$), for the Einstein-Ae ther type II black hole solution, that leads to diameter per unit mass d_{II} in agreement with the observationally determined one d_{M87*} within 1σ-error.

Einstein-Aether type I black hole solution



The allowed parameter regions (green) in the $c_{13} - M(10^9 M_{\odot})$ plane (upper graphs) and $c_{14} - M(10^9 M_{\odot})$ planes (lower graphs), for fixed rotation parameter values $a = 0$ (left graphs) and $a = 0.2$ (right graphs), for the Einstein-Ae ther type II black hole solution, that leads to diameter d_{II} in agreement with the observationally determined one d_{M87*} within 1σ-error.

Conclusions

- We extracted the black hole solutions for EA gravity, and we calculated the corresponding effective potential $U_{eff}(r)$ for the photons, the resulting event horizon radius r_e and the radius of the unstable photon sphere r_{ps}.
- We calculated the induced angular size δ, which combined with the mass and the distance can lead to a single prediction that quantifies the black hole shadow size, namely the diameter per unit mass d.
- Since d_{M87*} is observationally known from the EHT Probe, we extracted the corresponding parameter regions of instein-Aether theory in order to obtain consistency.
- In summary, Einstein-Aether black hole solutions are in agreement with EHT M87* observation, and this may act as an advantage for Einstein-Aether gravity.

THANK YOU!

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