Static, equipotential photon surfaces have no hair

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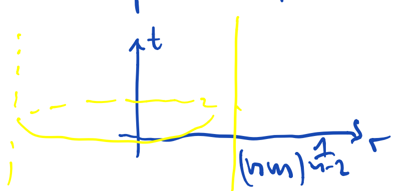
Schwarzschild – Tangherlini spacetime in $1+n$ dimensions:

$$-\left(1-\frac{2m}{r^{n-2}}\right)dt^2+\frac{1}{1-\frac{2m}{r^{n-2}}}dr^2+r^2\,\underbrace{d\Omega^2}_{\text{canonical metric on } \mathbb{S}^{n-1}}$$

$m>0$: photon sphere at $r=(nm)^{\frac{1}{n-2}}$ $\quad [n=3:\ r=3m]$



$\underline{n=2}$: Foertsch, Hasse, Perlich '03

↳ ex. plethora of spherically symmetric "photon surfaces" ← umbilic timelike surfaces

umbilic ← extrinsic curvature ~ induced metric

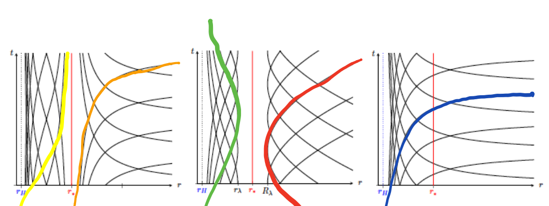


Figure 2: Examples of photon surfaces for umbilicity factors $\lambda=\lambda_*$, $0<\lambda<\lambda_*$, and $\lambda>\lambda_*$, respectively, including time-translates and time-reflections thereof. The vertical red line represents the photon sphere at $r=r_*$ which has $\lambda=\lambda_*$.

Taken from Cederbaum – Jahns – Vičánek Martínez

$n \geq 3$ C. – Galloway '21 (JMP)

1. The exact "same" plethora of rotationally symmetric photon surfaces exists in $n \geq 3$.
   ↖ umbilic timelike hypersurfaces

2. More generally, in spacetimes of the form

$$-f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + r^2\,d\Omega^2 \,, \quad n \geq 3 \qquad \text{"class } \mathcal{S}\text{"}$$

   have a photon sphere at $r = r_*$ if and only if

$$f'(r_*)\, r_* = 2 f(r_*) \,,$$

   and a rotationally symm. photon surface with radial profile $r = r(t)$ if and only if

$$\dot{r}^2 = \frac{f(r)^2\,(\lambda^2 r^2 - f(r))}{\lambda^2 r^2} \qquad \text{for some } \lambda > 0 \,.$$

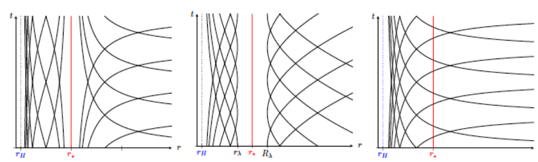


Figure 2: Examples of photon surfaces for umbilicity factors $\lambda = \lambda_*$, $0 < \lambda < \lambda_*$, and $\lambda > \lambda_*$, respectively, including time-translates and time-reflections thereof. The vertical red line represents the photon sphere at $r = r_*$ which has $\lambda = \lambda_*$.

3. Every photon surface in a spacetime in class $\mathcal{S}$ is either rotationally symmetric or a "partial hyperplane"
   ↳ provided the spacetime is nowhere conformal to Minkowski

4. In static spacetimes

$$-N^2\,dt^2 + g \,, \quad 1+n \geq 1+3 \,,$$

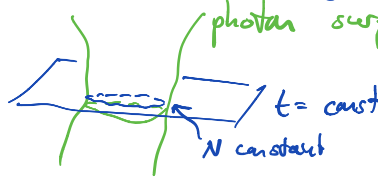
   VACUUM (no cosmol. constant), asympt. Schwarzschild:
   If inner boundary has black hole horizon and

"equipotential" photon surface boundary

then $\cong$ Schwarzschild–Tangherlini.

No hair theorem

equipotential:



$N = N(t)$ along the photon surface

5. Rot. symm. photon surfaces in spacetimes of class $\mathcal{S}$

= timelike hypersurfaces generated by null geodesics via rotation

$$\lambda = \frac{E}{\ell}$$

6. Null section of phase space of spacetime of of class $\mathcal{S}$ is partitioned by rot. symm. photon surfaces and rot. symm. principal null hypersurfaces. $\ell = 0$

work in progress C.– Wolff

1. Above analysis generalizes to

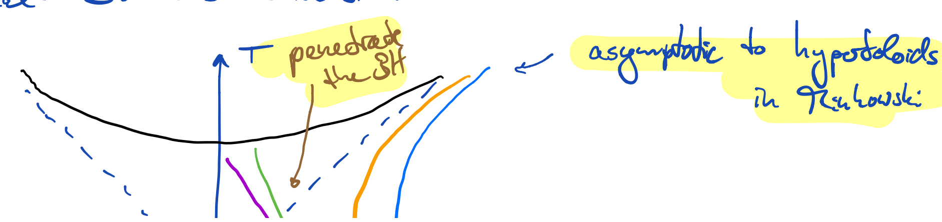
$$-f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + r^2\, g^W$$

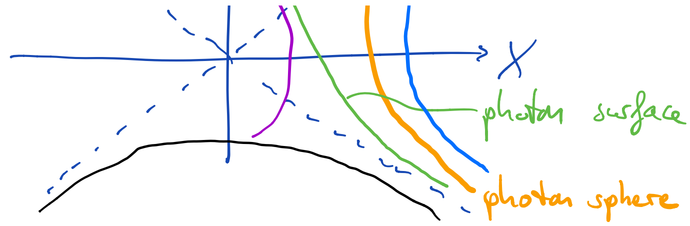
class $\mathcal{K}$

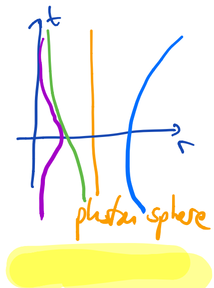
(N, g^W) $(n-1)$-dim. Riem. mfd.

2. Can construct Kruskal–Szekeres extensions for such spacetimes across non-degenerate Killing horizons

3. Photon surface analysis (rot. symm.) extends to Kruskal–Szekeres extension







work in progress C. – Jahns – Vičanek Martínez

1. careful ODE-solvability analysis for photon surface ODE

$$\boxed{\dot{r}^2 = \frac{f(r)^2\,(\lambda^2 r^2 - f(r))}{\lambda^2 r^2}}$$

$\lambda > 0$
$\shortparallel$
$\frac{E}{\ell}$

using effective potential
$V_{eff}(r) = \frac{f(r)}{r^2}$

$-f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + r^2\,d\Omega^2$

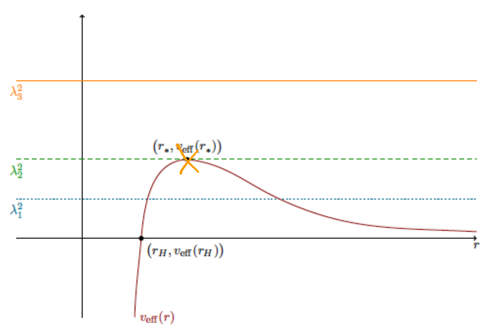


Figure 3: Sketch of the function $v^f_{\text{eff}}(r) = \frac{f(r)}{r^2}$ with three different cases for λ

Taken from Cederbaum – Jahns – Vičanek Martínez

in a wide variety of spacetimes of class $\mathcal{S}/\mathcal{H}$
including Schwarzschild-Tangherlini;
Reissner-Nordström $1+n \geq 1+3$,
AdS (-Schwarzschild /- Reissner-Nordström)
$\vdots$

2. Generalize no hair theorem to electrovacuum, AF