

# **Review of Tokamak Physics and GW conditions in relic conditions before $10^{-26}$ reduction in frequency with predictions as to what may be obtained in eLISA GW measurements from $10^{-4}$ Hz down to $10^{-16}$ Hz for eLISA**

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*Abstract:* - We consider an inverse procedure as to predict what may be obtained in eLISA, near Earth Orbit, in GW frequency. Among other issues would be the duration of the GW pulse so observed, in eLISA measurements, the relative degree of noise in the signal, as observed by eLISA, and this by the device of a step down in frequency of GW from about  $10^{19}$  Hz, in the early universe, or at a minimum  $10^{10}$  Hz down to  $10^{-4}$  Hz to a low of  $10^{-16}$  Hz, as could be ascertained by eLISA. We use the Tokamak in order to obtain GW signals an average of  $10^{25}$  to  $10^{26}$  times larger than what eLISA would observe as a way to make guesses as to the turbulence of the eLISA signal, how to consider and prepare for inevitable isotropic stochastic noise in the signal as well as guesses as to sources as to the noise and the duration of the signal. Which may be observed by eLISA. We do this as was mentioned before using Grischuk and Sachin (1975) amplitude for the GW generation due to plasma in a toroid, we generalize this result for Tokamak physics. We obtain evidence for strain values up to  $h_{2nd-term} \sim 10^{-25} - 10^{-26}$  in a Tokamak centre. The GW frequency created by a Tokamak are due to Plasma physics interactions within the Tokamak Toroid, but can with an application of common sense allow us to know what to look for in eLISA in its commissioning and GW runs.

**Key-Words:** - Tokamak physics, confinement time (of Plasma), GW amplitude, Drift current,

## **I. Introduction**

The author has in prior work given the idea that a decay of millions of Planck sized BHs as within the exceedingly early universe as in [1] could generate GW and gravitons, due to a breakup of black holes as predicted in [1] but with the present GW spectrum of today very conservatively following [2]. The breakup of black holes may commence due to what is stated in [1] and be complimented by what is addressed in [3] which would be if Gravitons acting as similar to a Bose-Einstein condensate contribute to a resulting DE [1]. Either the strict breakup of black holes as in [4] or some conflation with [3] would lead to, likely GW (and Graviton frequencies) initially of the order of  $10^{10}$  Hz to maybe  $10^{19}$  Hz. In doing so we can consider the duration of an observed signal, its relative noisiness and stochastic noise contributions of a sort which are covered in [5]. In addition, the generation of GW in a Tokamak if commensurate with eLISA data after a step down of  $10^{-25}$  to  $10^{-26}$  due to 60 or more e folds [6] may allow for a review of adequate polarization states for GW which may or may not need higher dimensions to be in fidelity to the data sets obtained [7]. Having said that, what are the justifications as to using Tokamaks?

First of all, there is the question of what sort of polarization would be produced in initial processes. Secondly, if we were able to ascertain  $10^{10}$  Hertz gravitational waves, via our laboratory arrangements and if we were later able to confirm, say the existence of  $10^{-16}$  Hz GW frequencies via LISA in the present era, this would be a proof of the big bang hypothesis, and so we summarize what this inquiry may answer .

**So, what is our inquiry good for?**

- A. Determination of the fidelity of the e fold value of 60 in the big bang**
- B. Issues of initial GW polarization which may be configured at the start of the big bang.**

**C. Determination of the relative stability of the production of the GW signal (i.e., if we had a Tokamak running and the resulting GW amplitude and the characteristics of the signal are stable, over a “long time interval” does this imply stability of the eLISA signal? Over time and space?**

**D. Likelihood of noise and Stochastic fluctuations in a produced GW signal.**

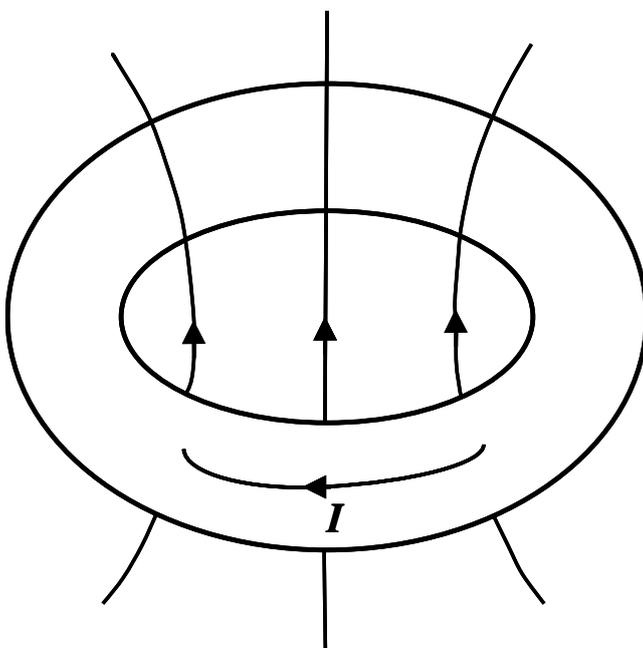
If A,B,C and D were determined as to the Tokamak, and GW, we may be able to infer what to look for and to model when examined directly, what a LISA GW signal set of characteristics may be inferring as to early universe conditions. Having said that, let us go to the Tokamak information.

## II. Comparison with Grishchuk and Sachin results.

Russian physicists Grishchuk and Sachin [8] obtained the amplitude of a Gravitational wave (GW) in a plasma as

$$A(\text{amplitude-GW}) = h \sim \frac{G}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 .$$

This is compared with [9], and we diagram the situation out as follows [10]  
[3]



**Fig. 1 We outline the direction of Gravitational wave “flux”. If the arrow in the middle of the Tokamak ring perpendicular to the direction of the current represents the z axis, we represent where to put the GW detection device as 5 meters above the Tokamak ring along the z axis. This diagram was initially from Wesson [10]**

Note that a simple model of how to provide a current in the Toroid is provided by a transformer core. This diagram is an example of how to induce the current I, used in the simple Ohms law derivation referred to in the first part of the text. Here, E is the electric field whereas  $\lambda_{Gw}$  is the gravitational wavelength for GW generated by the Tokamak in our model. In the original Griskchuk model, we would have very small strain values, which will comment upon but which require the following relationship between GW wavelength and resultant frequency. Note, if

$\omega_{GW} \sim 10^6 \text{ Hz} \Rightarrow \lambda_{Gw} \sim 300 \text{ meters}$ , so we will be assuming a baseline of the order of setting  $\omega_{GW} \sim 10^9 \text{ Hz} \Rightarrow \lambda_{Gw} \sim .3 \text{ meters}$ , as a baseline measurement for GW detection above the Tokamak. Furthermore,

$$A(GW.amplitude) \sim h \sim \frac{G \cdot W_E \cdot V_{volume}}{c^4 \cdot \tilde{a}} \quad (1)$$

Where

$W_E = \text{Average} - \text{energy} - \text{density}, V_{volume} = \text{Volume} - \text{Toroid}, \tilde{a} = \text{inner} - \text{radii}(\text{Toroid})$   
(1a)

This Eq. (1) above is due to the 1<sup>st</sup> term of a two-part composition of the strain, with the 2<sup>nd</sup> term of the strain value significantly larger than the first term and due to ignition of the Plasma in the Tokamak. The first term of strain is largely due to what was calculated by Grishkuk [8]et. al. The

second term is due to Plasma fusion burning. This plasma fusion burning contribution is due to non-equilibrium contributions to Plasma ignition, which will be elaborated on in this document. Note that the first term in the strain derivation is due to the electric field within a Toroid, not Plasma fusion burning, and we will first discuss how to obtain the requisite strain, for the electric field contribution to the current, inside a Tokamak. making use of Ohms law. See [9] for additional details.

### III. Derivation of strain generated by an electric field, and small strain values.

We will examine the would-be electric field, contributing to a small strain values similar in part to Ohms law. A generalized Ohm's law ties in well with Figure 1 above

$$J = \sigma \cdot E \quad (2)$$

In order to obtain a suitable electric field, to be detected via 3DSR technology [11,12], we will use a generalized Ohm's law as given by Wesson [3] (page 146), where E and B are electric and magnetic fields, and v is velocity.

$$E = \sigma^{-1} J - v \times B \quad (3)$$

Note that the term in Eq. (4) given as  $v \times B$  deserves special commentary. If  $v$  is perpendicular to  $B$  as occurs in a simple equilibrium case, then of course, Eq. (4) would be, simply put, Ohms law, and spatial equilibrium averaging would then lead to

$$E = \sigma^{-1} J - v \times B \xrightarrow{v \text{ perpendicular to } B} E = \sigma^{-1} J \quad (4)$$

What saves the contribution of Plasma burning as a contributing factor to the Tokamak generation of GW, with far larger strain values commencing is that one does not have the velocity of ions in Plasma perpendicular to B fields in the beginning of Tokamak generation. It is, fortunately for us, a non-equilibrium initial process, with thermal irregularities leading to both terms

in Eq. (5) contributing to the electric field values. We will be looking for an application for radial free electric fields being applied e.g., Wesson [10] ( page 120)

$$n_j e_j \cdot (E_r + v_{\perp j} B) = -\frac{dP_j}{dr} \quad (5)$$

Here,  $n_j$  = ion density, jth species,  $e_j$  = ion charge, jth species,  $E_r$  = radial electric field,  $v_{\perp j}$  = perpendicular velocity, of jth species,  $B$  = magnetic field, and  $P_j$  = pressure, jth species. The results of Eq. (3) and Eq. (4) are

$$\begin{aligned} \frac{G}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 &\sim \frac{G}{c^4} \cdot \left[ \frac{Const}{R} \right]^2 \cdot \lambda_{GW}^2 + \frac{G}{c^4} \cdot \left[ \frac{J_b}{n \cdot e} + v_R \right]^2 \cdot \lambda_{GW}^2 \\ &= (1^{st}) + (2^{nd}) \end{aligned} \quad (6)$$

Here, the 1<sup>st</sup> term is due to  $\nabla \times E = 0$ , and the 2<sup>nd</sup> term is due to

$$E_n = \frac{dP_j}{dx_n} \cdot \frac{1}{n_j \cdot e_j} - (v \times B)_n \text{ with the 1}^{st} \text{ term generating}$$

$h \sim 10^{-38} - 10^{-30}$  in terms of GW amplitude strain 5 meters above the

Tokamak, whereas the 2<sup>nd</sup> term has an  $h \sim 10^{-26}$  in terms of GW amplitude above the Tokamak. The article has contributions from amplitude from the 1<sup>st</sup> and 2<sup>nd</sup> terms separately. The second part will be tabulated separately from the first contribution assuming a minimum

temperature of  $T = Temp \sim 10KeV$  as from Wesson [10]. We should also consider the issues in [9],[11], and [12]

#### IV. GW h strain values when the first term of Eq. (4) is used.

We now look at what we can expect with the simple Ohm's law calculation for strain values. As it is, the effort led to non-usable GW amplitude values of up to  $h \sim 10^{-38} - 10^{-30}$  for GW wave amplitudes 5 meters above a Tokamak, and  $h \sim 10^{-36} - 10^{-28}$  in the centre of a Tokamak. I.e. this would be using Ohm's law and these are sample values of the Tokamak generated GW amplitude, using the first term of Eq. (4) and obtaining the following value [8] with

$$h_{First-term} \sim \frac{G}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{G}{c^4} \cdot \left[ \frac{J}{\sigma} \right]^2 \cdot \lambda_{GW}^2 \quad (7)$$

We summarize the results of such in our first table as given for when  $\omega_{GW} \sim 10^9 Hz \Rightarrow \lambda_{Gw} \sim .3 meters$  and with conductivity

$\sigma(tokamak - plasma) \sim 10 \cdot m^2/sec$  and with the following provisions as to initial values. What we observe are a range of Tokamak values which are, even in the case of ITER (not yet built) beyond the reach of any technological detection devices which are conceivable in the coming decade. This table and its results, assuming fixed conductivity values  $\sigma(tokamak - plasma) \sim 10 \cdot m^2/sec$  as well as

$\lambda_{Gw} \sim .3 meters$  is why the author, results as to the 2<sup>nd</sup> term of Eq. (4) which lead to even for when considering the results for the Chinese Tokamak in Hefei to have [13]

$$h_{Second-term} \sim \frac{G}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{G}{c^4} \cdot \left[ \frac{J_b}{n \cdot e} + v_R \right]^2 \cdot \lambda_{GW}^2 \quad (8)$$

or values 10,000 larger than the results in ITER due to Eq. (6).'

Note that we are setting  $\lambda_{Gw} \sim .3 \text{ meters}$ ,

$\sigma(\text{tokamak} - \text{plasma}) \sim 10 \cdot m^2 / \text{sec}$ , using Eq.6 above for Amplitude of GW. What makes it mandatory to go the 2<sup>nd</sup> term of Eq. (4) is that even in the case of ITER, 5 meters above the Tokamak ring, the GW amplitude is 1/10,000 the size of any reasonable GW detection device, and this including the new 3DSR technology (Li et al, 2009) [11,12]. Hence, we need to come up with a better estimate, which is what the 2<sup>nd</sup> term of Eq. (5) is

### **V .Enhancing GW strain Amplitude via utilizing a burning Plasma drift current: Eq.(4)**

The way forward is to go to Wesson, [10][3] (2011, page 120) and to look at the normal to surface induced electric field contribution

$$E_n = \frac{dP_j}{dx_n} \cdot \frac{1}{n_j \cdot e_j} - (v \times B)_n \quad (9)$$

If one has for  $v_R$  as the radial velocity of ions in the Tokamak from Tokamak centre to its radial distance,  $R$ , from centre, and  $B_\theta$  as the direction of a magnetic field in the ‘face’ of a Toroid containing the Plasma, in the angular  $\theta$  direction from a minimal toroid radius of  $R = a$ , with  $\theta = 0$ , to  $R = a + r$  with  $\theta = \pi$ , one has  $v_R$  for radial drift velocity of ions in the Tokamak, and  $B_\theta$  having a net approximate value of: with  $B_\theta$  not perpendicular to the ion velocity, so then[10]

$$\left( \mathbf{v} \times \mathbf{B} \right)_n \sim v_R \cdot B_\theta \quad (10)$$

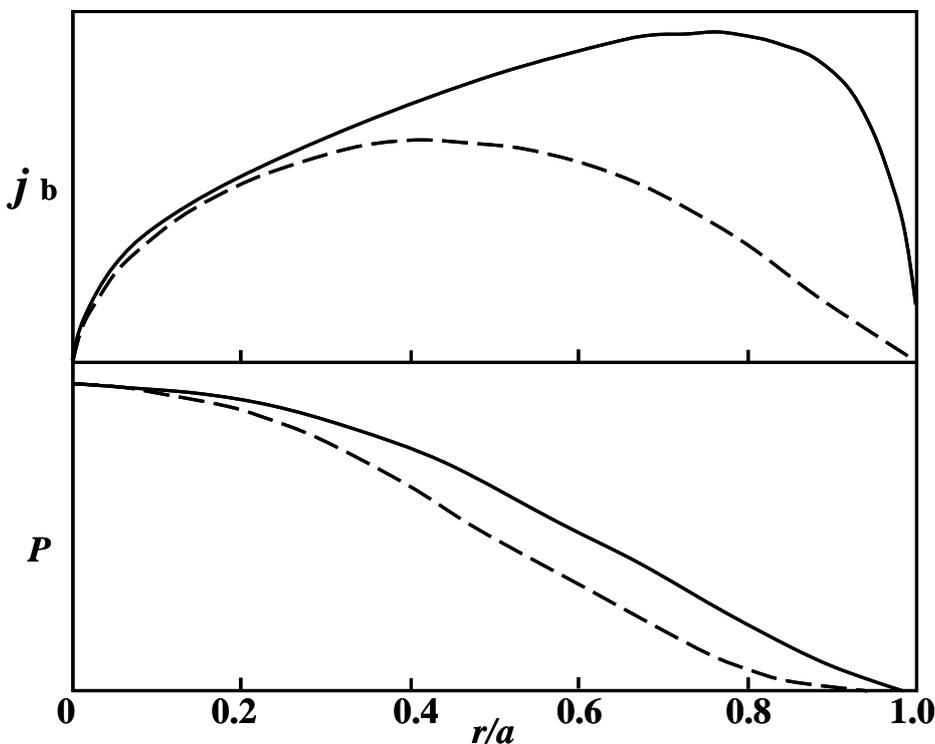
Also, From Wesson [3] (page 167) the spatial change in pressure denoted.

$$\frac{dP_j}{dx_n} = -B_\theta \cdot j_b \quad (11)$$

Here the drift current, using  $\xi = a/R$ , and drift current  $j_b$  for Plasma charges, i.e.

$$j_b \sim -\frac{\xi^{1/2}}{B_\theta} \cdot T_{Temp} \cdot \frac{dn_{drift}}{dr} \quad (12)$$

Figure 2 below introduces the role of the drift current, in terms of Tokamaks [10]



**Fig. 2 Typical bootstrap currents with a shift due to  $r/a$  where  $r$  is the radial direction of the Tokamak, and  $a$  is the inner radius of the Toroid. This figure is reproduced from Wesson [10] Then one has**

$$B_{\theta}^2 \cdot (j_b/n_j \cdot e_j)^2 \sim \frac{B_{\theta}^2}{e_j^2} \cdot \frac{\xi^{1/4}}{B_{\theta}^2} \cdot \left[ \frac{1}{n_{drift}} \cdot \frac{dn_{drift}}{dr} \right]^2 \sim \frac{\xi^{1/4}}{e_j^2} \cdot \left[ \frac{1}{n_{drift}} \cdot \frac{dn_{drift}}{dr} \right]^2 \quad (13)$$

Now, the behaviour of the numerical density of ions, can be given as follows, namely growing in the radial direction, then [3]

$$n_{drift} = n_{drift} \Big|_{initial} \cdot \exp[\tilde{\alpha} \cdot r] \quad (14)$$

This exponential behaviour then will lead to the 2<sup>nd</sup> term in Eq. (4) having in the centre of the Tokamak, for an ignition temperature of  $T_{Temp} \geq 10KeV$  a value of

$$h_{2nd-term} \sim \frac{G}{c^4} \cdot B_{\theta}^2 \cdot (j_b/n_j \cdot e_j)^2 \cdot \lambda_{GW}^2 \sim \frac{G}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp}^2}{e_j^2} \cdot \lambda_{GW}^2 \sim 10^{-25} \quad (15)$$

As shown in [10] there is a critical ignition temperature at its lowest point of the curve in the having  $T_{Temp} \geq 30KeV$  as an optimum value of the

Tokamak ignition temperature for  $n_{ion} \sim 10^{20} m^{-3}$ , with a still permissible temperature value of  $T_{Temp} \Big|_{safe-upper-bound} \approx 100KeV$  with a

value of  $n_{ion} \sim 10^{20} m^{-3}$ , due to from page 11, [10] the relationship

of Eq.(16), where  $\tau_E$  is a Tokamak confinement of plasma time of about 1-3 seconds, at least due to [10]. Then

$$n_{ion} \cdot \tau_E > .5 \times 10^{20} \cdot m^{-3} \cdot sec . \quad \text{Also, if,}$$

$T_{Temp} \Big|_{safe-upper-bound} \approx 100KeV$ , then one could have at the Tokamak centre, i.e. even the Hefei based Tokamak [10,13]

$$h_{2nd-term} \Big|_{T_{Temp} \geq 100KeV} \sim \frac{G}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp}^2}{e_j^2} \cdot \lambda_{GW}^2 \sim 10^{-25} - 10^{-26} \quad (16)$$

This would lead to, for a GW reading 5 meters above the Tokamak, then lead to for then the Tokamak [10,13]

$$\left[ h_{2nd-term} \Big|_{T_{Temp} \geq 100KeV} \right]_{5-meters-above-Tokamak} \sim \frac{G}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp}^2}{e_j^2} \cdot \lambda_{GW}^2 \sim 10^{-27} \quad (17)$$

Note that the support for up to 100 KeV for temperature can yield more stability in terms of thermal Plasma confinement.

**V. Restating the energy density and power using the formalism of Eq.(1) directly**

$$W_E \cdot V_{volume} \sim \tilde{\alpha} \cdot \lambda_{GW}^2 \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp-plasma-fusion-burning}^2}{e_j^2} \quad (18)$$

The temperature for Plasma fusion burning, is then about between 30 to 100 KeV, as given by Wesson [10] The corresponding power as given by Wesson is then for the Tokamak [10]

$$P_{\Omega} = E \cdot J \leq \frac{E}{\mu_0} \cdot \frac{B_{\phi}}{R} \quad (19)$$

The tie in with Eq.(18) by Eq. (20) can be seen by first of all setting the E field as related to the B field, via E (electrostatic)  $\sim 10^{12} Vm^{-1}$  as equivalent to a magnetic field  $B \sim 10^4 T(Torr)$  as given by[9]. In a one second interval, if we use the input power as an experimentally supplied quantity, then the effective E field is

$$E_{applied} \sim \frac{\xi^{1/8} \cdot \tilde{\alpha}}{e_j} \times T_{Tokamak-temperature} \quad (20)$$

What is found is, that if Eq. (19) and Eq.(20) hold. Then by Wesson[10] , pp. 242-243, if  $Z_{eff} \sim 1.5, q_a q_0 \sim 1.5, (R/\tilde{a}) \approx 3$  Then the temperature of a Tokamak, to good approximation would be between 30 to 100 KeV, and then one has[10]

$$B_{\phi}^{4/5} \sim .87 \cdot \left( \tilde{T} = T_{Tokamak-temperature} \right) \quad (21)$$

Then the power for the Tokamak is

$$P_{\Omega} \Big|_{Tokamak-toroid} \leq \frac{\xi^{1/8} \cdot \tilde{\alpha}}{\mu_0 \cdot e_j \cdot R} \times \frac{\left( T_{Tokamak-temperature} \right)^{9/4}}{(.87)^{5/4}} \quad (22)$$

Then, per second, the author derived the following rate of production per second of a  $10^{-34} eV$  graviton, as, if  $\tilde{a} = R/3$

$$n|_{\text{massive-gravitons/second}} \propto \frac{3 \cdot \hbar \cdot e_j}{\mu_0 \cdot R^2 \cdot \xi^{1/8} \cdot \tilde{\alpha}} \times \frac{(T_{\text{Tokamak-temperature}})^{1/4}}{\lambda_{\text{Graviton}}^2 \cdot m_{\text{graviton}} \cdot c^2 \cdot (.87)^{5/4}} \sim 1/\lambda_{\text{Graviton}}^2 \text{ scaling} \quad (23)$$

If there is a fixed mass for a massive graviton, the above means that as the wavelength decreases, that the number of gravitons produced between plasma burning temperatures of 30 to 100 KeV changes. See [10,11,12,13,14,15]

**vi. GW generation due to the Thermal output of Plasma burning , and linkage to the initial GW strain and frequency problem versus values of strain and frequency of GW today, from the initial pre big bang.**

Further elaboration of this matter in the experimental detection of experimental data sets for massive gravity lies in the viability of the expression derived , namely Eq. (19)  $h \sim 10^{-27}$  for a GW detected 5 meters above a Tokamak represents the decrease in strain, by a factor of about 100, from details which are further elaborated upon in [15] , whereas in the center of the Tokamak, we would have, say,  $h_{\text{2nd-term}} \sim 10^{-26} - 10^{-27}$  . I.e a difference of 2 orders of magnitude. We state that our rough estimate is that we would see about the same strain values, in the initial starting point of the universe we

would have, say  $h \sim 10^{-25}$  decreasing to  $h \sim 10^{-26} - 10^{-27}$  today. I.e. a comparatively small change in strain amplitude. Contrast this with the e folding issues, of [16] whereas we would have a difference of  $10^{26}$  in frequency magnitude, with  $10^{10}$  Hz initially, for GW at start of big bang, decreasing to  $10^{-16}$  Hz, due to inflation, and [9]. If we confirm that last statement observationally, we have confirmed the [16] e folding prediction and taken a huge step forward in observational cosmology. Eventually we could investigate, also, early universe polarization of GW.

**VII. What can we say about the stability of a Plasma generated signal creating GW? And its relevance to eLISA?**

Among other things to consider, if we do Tokamak generation of GW simulations right as to GW generation, we will be able to enhance the likelihood [17][18] of having a stable signal (if that is what the Tokamak predicts), or an unstable signal (if that is what the Tokamak predicts) of the

LISA data sets. Not necessarily in a fool proof way, but it would be a baseline to review and to refer to. Another item to consider, not just as to the type of GW polarizations, and stability of the signal, but also a way to infer through trial and error the duration of the phenomenon creating very early universe GW generation. References [1],[2], and [3] are verifiable portals as far as model building exercises which may commence once we have data sets. This is in outlook like the opportunities which may be given to us by [19]. The author also refers readers to [20] by Moniz, as well as [21] for the old wavefunction of the universe problem. We hope that a fully developed research and development program may enable full investigation of all these issues via the medium of GW astronomy. And we also ascertain that these same techniques may be useful in evaluating Brane world physics cosmology [22],[23]. This all involves using the following frequency relationship [24]

$$\begin{aligned} (1 + z_{initial-era}) &\equiv \frac{a_{today}}{a_{initial-era}} \approx \left( \frac{\omega_{Earth-orbit}}{\omega_{initial-era}} \right)^{-1} \\ \Rightarrow (1 + z_{initial-era}) \omega_{Earth-orbit} &\approx 10^{25} \omega_{Earth-orbit} \approx \omega_{initial-era} \end{aligned} \quad (24)$$

And a goal eventually of determining if the following wave functions are applicable to GW astronomy, i.e. [21] where on page 239 for a quantum cosmology similar to a “dust universe” we are given by Kieffer that

$$\langle E \rangle_{\kappa=n, \lambda} = \frac{(\kappa = n) + 1/2}{\lambda} \xrightarrow{\lambda \approx 1/\hbar\omega} \hbar\omega \cdot ((\kappa = n) + 1/2) \quad (25)$$

What we can do, is to ascertain the last step would be to make a cosmology wavefunction in a sense partly related to the simple harmonic oscillator. But we should take into consideration the normalization using that if  $\hbar = \ell_P = G = t_P = k_B = 1$  is done via Plank unit normalization [25][26]. If so, then we have that frequency is proportional to 1/t, where t is time. I.e., hence if there is a value of n=0 and making use of the frequency, we then would be able to write

$$\Psi_{1,\kappa=n=0} \approx \sqrt{\frac{\omega}{\pi}} \cdot \left[ \frac{1}{\omega + i \cdot (t+r)} - \frac{1}{\omega + i \cdot (t-r)} \right] \quad (26)$$

Or,

$$\Psi_{2,\kappa=n=0} \approx \frac{1}{\sqrt{\pi}} \sqrt{\frac{\sqrt{8\pi}}{t}} \cdot \left[ \frac{1}{\frac{\sqrt{8\pi}}{t} + i \cdot (t+r)} - \frac{1}{\frac{\sqrt{8\pi}}{t} + i \cdot (t-r)} \right] \quad (27)$$

With, say.

$$\omega \approx \frac{\sqrt{8\pi}}{t} \quad (28)$$

## VIII. Acknowledgements

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