



MGM16

16th Marcel Grossmann Meeting



July 5 – 10, 2021

Relativistic measurements with the Galileo constellation: the Galileo for Science (G4S_2.0) Project

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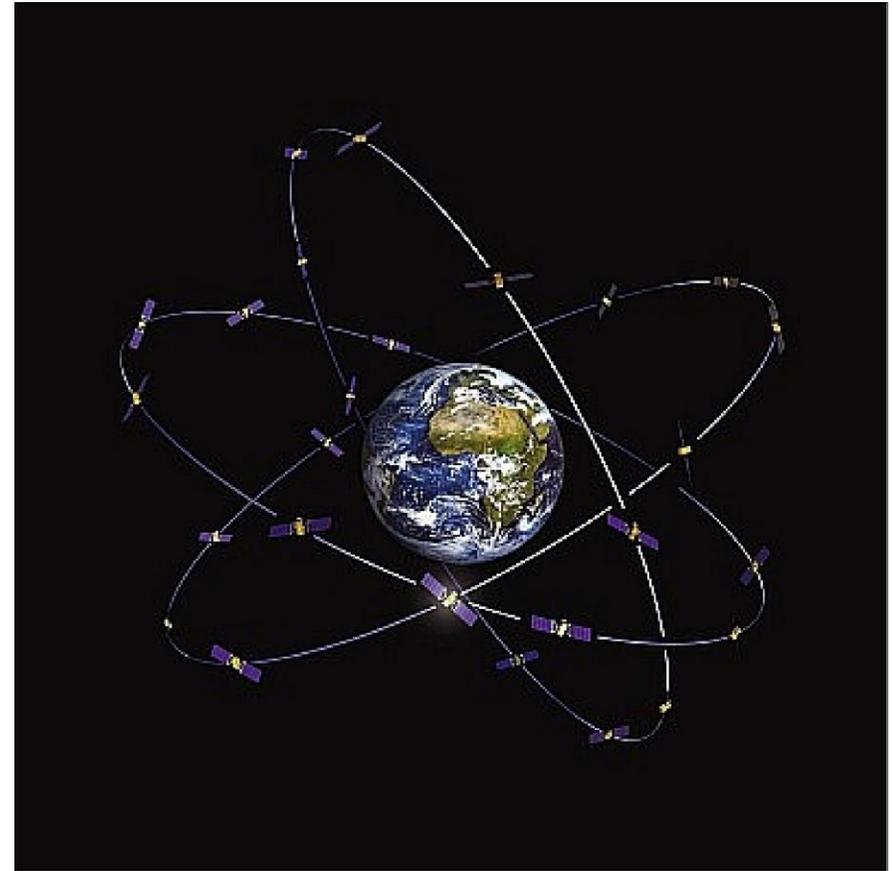
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Agenzia Spaziale Italiana

Summary

- High-level goals of **G4S_2.0**
- Main activities at **IAPS/INAF**
- Conclusions



The Galileo constellation of 30 spacecraft
(image credit: ESA)

High-level goals of G4S_2.0

- The **G4S_2.0** project, financed by the **Italian Space Agency (ASI)**, aims to perform a set of measurements in the field of **gravitation** with the two **Galileo** satellites **GSAT0201** and **GSAT0202** exploiting their relatively high eccentricity ($\cong 0.16$) with respect to that ($\cong 0$) of the other satellites of the **Full Operational Capability (FOC)** constellation and taking advantage of the accuracy of their on-board atomic clocks
- Three research centers in Italy are involved in this project:
 - **ASI-CGS** (Center for Space Geodesy) in Matera
 - Istituto di Astrofisica e Planetologia Spaziali (**IAPS/INAF**) in Roma and **OATO/INAF** in Torino
 - Politecnico (**POLITO**) in Torino

High-level goals of G4S_2.0



1. A new measurement of the **Gravitational Redshift** exploiting the orbits of the **Galileo** satellites **GSAT0201** and **GSAT0202**
2. A measurement of the **General Relativity (GR) precessions** on the orbits of **GSAT0201** and **GSAT0202**
3. Constraints on **Dark Matter (DM)** in the Milky Way
4. Realize (in a reverse use) a pure **Relativistic Positioning System (RPS)**

SVN N.	PRN N.	
GSAT0201	E18	Doresa
GSAT0202	E14	Milena

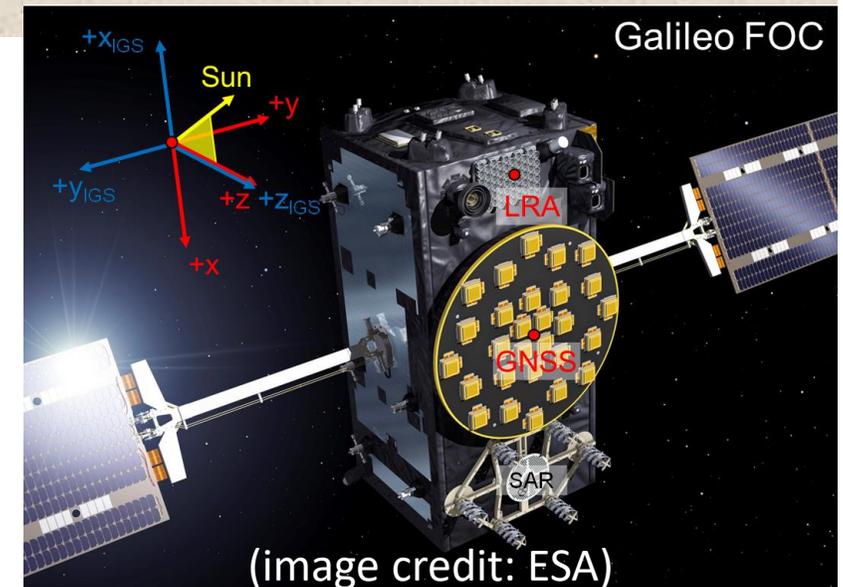
	<i>Nominal</i>	<i>Injection</i>	<i>Resonant (19/10) (orbit 1)</i>	<i>Max. Perigee (orbit 2)</i>	<i>Resonant (37/20) (orbit 3)</i>
Semi-Major Axis [km]	29600	26180.99	27484.8	27932.86	27.977,9
eccentricity	0.00027	0.2326	0.156	0.1563	0.1544
Inclination	55.12°	49.80°	49.8°	49.8°	49.8°
RAAN	100.66°	83.82°	83.82°	83.82°	83.82°
Arg-Per	241.98°	28.00°	28.00°	28.00°	28.00°

High-level goals of G4S_2.0

Other fundamental goals of the project

5. To develop new and more accurate models for the **Non-Gravitational Perturbations**
6. To develop a new **Accelerometer concept** for next generation of **Galileo** satellites to support the **Precise Orbit Determination** and **Science Measurements**

SVN N.	PRN N.	
GSAT0201	E18	Doresa
GSAT0202	E14	Milena



High-level goals of G4S_2.0

Gravitational Redshift

- **Special Relativity (SR)** and **GR** are responsible of frequency shifts of the onboard clocks that can be carefully measured by exploiting both the precise onboard **H-maser clocks** and the **high eccentricity** of the orbits
 - ✓ among these effects the **Gravitational Redshift** plays a special role, since it represents a test of the **Local Position Invariance (LPI)**
- Indeed, **LPI** is one of the ingredients of **Einstein Equivalence Principle (EEP)** which is at the basis of Einstein's **GR** and of all metric theories of gravitation:
 1. **Universality of Free Fall (UFF)**
 2. **Local Lorentz Invariance (LLI)**
 3. **Local Position Invariance (LPI)**

G4S_2.0 approach (orbit models + systematics):

- FEM for the satellite
- SLR data: NP and FR
- Gravity Field Free (GFF) technique
- Galileo constellation

$$y = \frac{\Delta v}{v} = \frac{GM_{\oplus}}{c^2} \left(\frac{1}{R_{\oplus}} - \frac{1}{r} \right) \quad \longrightarrow \quad y_{\alpha} = -(1 + \alpha) \frac{GM_{\oplus}}{rc^2} \quad \alpha = 0 \text{ in GR} \quad \text{Goal } |\alpha| \leq 2 \times 10^{-5}$$

High-level goals of G4S_2.0

$$y_\alpha = -(1 + \alpha) \frac{GM_\oplus}{rc^2}$$

$$\alpha = 0 \text{ in GR}$$

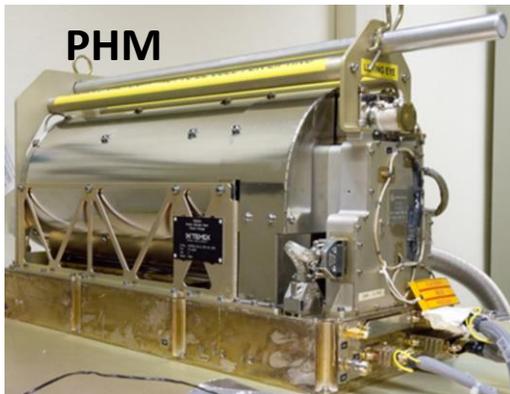


Gravitational Redshift: past and recent measurements

- Gravity Probe A (GPA), 1976-1980: $|\alpha| = (1 \pm 2) \times 10^{-4}$ $|\alpha| \leq 7 \times 10^{-5}$
R.F.C. Vessot, M.W. Levine., *Gen. Rel. Grav*, 10, 3, 181-204 (1979)
R.F.C. Vessot, et al., *Phys. Rev. Letter*, 45, 2081 (1980)
- Galileo gravitational Redshift Experiment with eccentric sATellites (GREAT), 2018
 - SYRTE: $\alpha = (0.19 \pm 2.48) \times 10^{-5}$ P. Delva, et al., *Phys. Rev. Letter*, 121, 231101 (2018)
 - ZARM: $\alpha = (4.5 \pm 3.1) \times 10^{-5}$ S. Herrmann, et al., *Phys. Rev Lett.*, 121, 231102 (2018)

Systematics:

- Temperature effects
- Earth's magnetic field
- Orbit/clock solution
- Ongoing clocks



PHM (Passive Hydrogen Maser): $<4.5 \times 10^{-14}$ at 30000 s

RAFS (Rubidium Atomic Frequency Standard): $<5.1 \times 10^{-14}$ at 10000 s



High-level goals of G4S_2.0

Relativistic precessions:

- **Schwarzschild** or Einstein precession (gravitoelectric field)
- **Lense-Thirrig (LT)** precession (gravitomagnetic field)
- **Geodetic precession** (Coriolis-like acceleration)

$$\dot{\omega}^{Ein} = \frac{3 (GM_{\oplus})^{3/2}}{c^2 a^{5/2} (1-e^2)}$$

$$\dot{\omega}^{LT} = \frac{-6 GJ_{\oplus}}{c^2 a^3 (1-e^2)^{3/2}} \cos i$$

$$\dot{\Omega}^{LT} = \frac{2 GJ_{\oplus}}{c^2 a^3 (1-e^2)^{3/2}}$$

$$\dot{\Omega}^{ds} = \frac{3}{2} \frac{GM_{\oplus}}{c^2 R_{\oplus\odot}^3} |(V_{\oplus} - V_{\odot}) \times R_{\oplus\odot}| \cos \epsilon_{\odot}$$

Rate (mas/yr)	GSAT-201/202	GSAT-203
	1 mas = 1 milli arc sec	
$\Delta \dot{\omega}_E$	+ 428.88	+ 362.74
$\Delta \dot{\omega}_{LT}$	- 5.21	- 3.67
$\Delta \dot{\Omega}_{LT}$	+ 2.69	+ 2.18
$\Delta \dot{\Omega}_{ds}$	+ 17.60	+ 17.60

Rate (mas/yr)	LAGEOS II	LAGEOS
	1 mas = 1 milli arc sec	
$\Delta \dot{\omega}_E$	+ 3351.95	+ 3278.77
$\Delta \dot{\omega}_{LT}$	- 57.00	+ 32.00
$\Delta \dot{\Omega}_{LT}$	+ 31.50	+ 30.67
$\Delta \dot{\Omega}_{ds}$	+ 17.60	+ 17.60

The measurements of the relativistic precessions are much more challenging in the case of the Galileo satellites

- In particular in the case of the **LT** effect (the **GR** precession is more than a factor of 10 smaller than the **LAGEOS** one)
- However, the measurement of **Schwarzschild** precession is favorable, particularly if **LAGEOS II** is also considered
- Consequently, also the constraints in **Yukawa-like** interactions are promising

High-level goals of G4S_2.0



Constraints on dark matter (DM):

- **DM** could arise from very light quantum fields that form macroscopic objects or clumps
- These may produce **topological defects (TD)**, as **domain walls**, that could be responsible of glitches and transients in **GNSS** clocks time measurements related to transient variations of fundamental constants
- Roberts et al. (2017) have found no evidence for the existence of **domain walls** after an analysis of about 16 years of **GPS** data at the sensitivity level of the onboard atomic clocks
- These limits can be further improved exploiting the higher performance of the atomic clocks of the **Galileo** constellation

High-level goals of G4S_2.0

Constraints on dark matter (DM):

- Topological defects may be formed during the cooling of the early universe through a spontaneous symmetry breaking phase transition
- From a practical point of view, this requires the existence of hypothesized self-interacting **DM** fields: φ
- The exact nature of **TDs** is model-dependent
- the spatial scale d of the **TD** is the Compton wavelength λ of the particles that make up the **DM** field

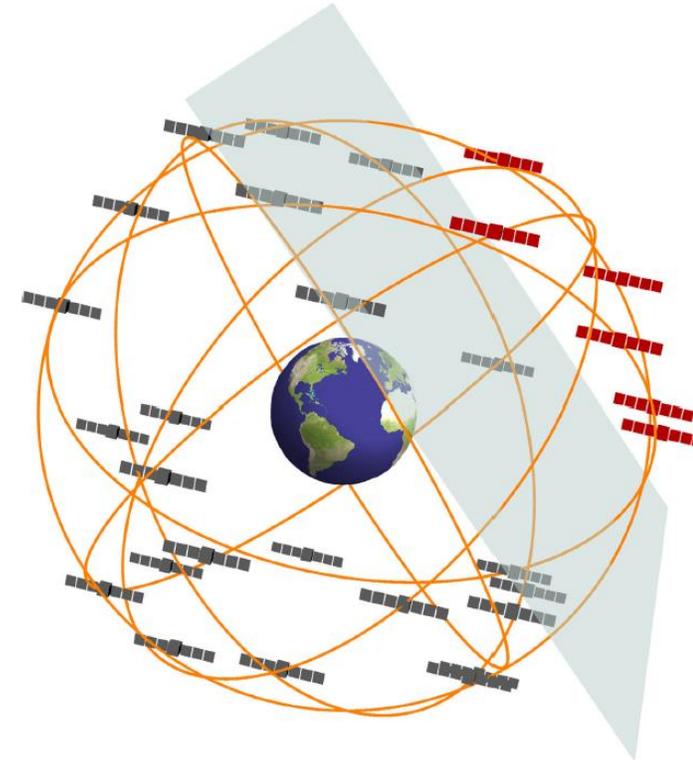


Fig. 1 Domain wall crossing. As a domain wall sweeps through the Global Positioning System constellation at galactic velocities, $v_g \sim 300 \text{ km s}^{-1}$, it perturbs the atomic clocks on board the satellites causing a correlated propagation of glitches through the network. The red satellites have interacted with the domain wall, and exhibit a timing bias compared with the grey satellites. Image generated using Mathematica software⁴⁸

$$d \cong \lambda = \frac{h}{2\pi m_\varphi c}$$

$$m_\varphi \cong 10^{-14} \text{ eV}/c^2$$

Earth-sized object

High-level goals of G4S_2.0

Constraints on dark matter (DM): quadratic scalar interaction between DM and clock atoms

$$\alpha = 2\pi \frac{e^2}{hc} \cong \frac{1}{137} \quad \text{Dimensionless electromagnetic fine-structure constant}$$

$$q = \frac{m_q}{\Lambda_{QCD}} \quad \text{Ratio of the light quark mass to the quantum chromodynamics (QCD) energy scale}$$

m_e, m_p Electron and proton masses

$$\frac{\delta X(r, t)}{X} = \Gamma_X \varphi(r, t)^2$$

$$\frac{\delta \omega(r, t)}{\omega_c} = \sum_X K_X \frac{\delta X(r, t)}{X} = \Gamma_{eff}^c \varphi(r, t)^2$$

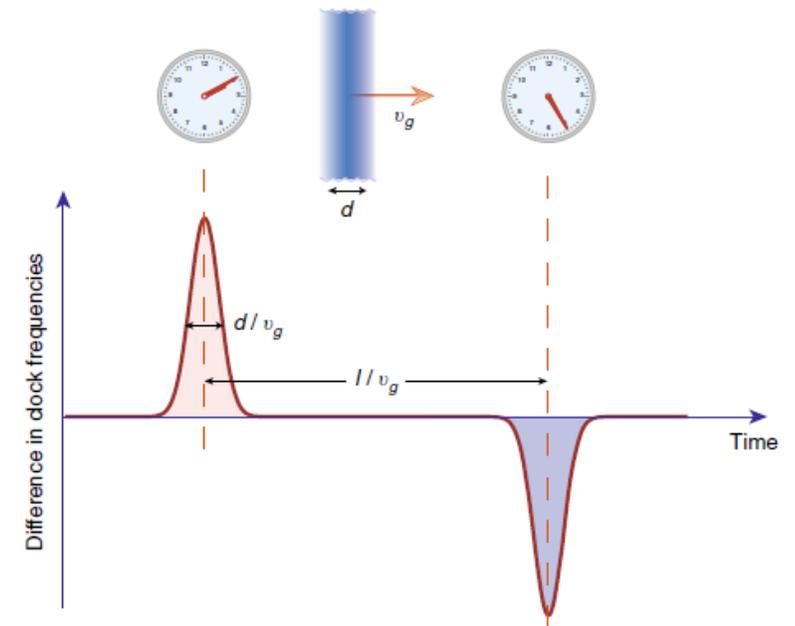


Fig. 2 Time dependence of the dark matter-induced signal. The frequency difference between two identical ideal clocks separated by distance l . The time delay in the signals encodes the kinematics of the dark matter object

High-level goals of G4S_2.0



- **Relativistic Positioning System (RPS):**

- The idea is to use a **RPS** based on emitters fixed to the surface of the Earth to provide an accurate orbit determination of the two S/C
- This represents a fully **RPS** based on the complete exploit of **SR** and **GR** and on the concept of space-time
- In the case of **GNSS** satellites the positioning is “traditional” and the relativistic effects are introduced as perturbations, i.e., as corrections to a Newtonian formulation
- The counting of the pulses for a set of different emitters whose positions on the Earth and periods are assumed to be known, is used to provide **null emission (or light) coordinates** for the receiver on a satellite
- The measurement of the **proper time intervals** between successive arrivals of the signals from the various emitters is used to give the final localization of the receiver, within an accuracy controlled by the precision of the onboard clock

Main activities at IAPS/INAF



1. Improve the dynamical model of the satellites orbits, in particular the modelling of the **Non-Gravitational Perturbations (NGPs)** and, among these, that of the direct solar radiation pressure, the largest **NGP** on **GNSS** satellites
2. Improve the **precise orbit determination (POD)** of the **Galileo FOC** satellites on eccentric orbit in relationship to the measurements to be performed in the field of **gravitation** and **fundamental physics**
3. Perform a set of measurements of the general relativistic effects on the orbits of the satellites with possible constraint to **alternative theories** of gravitation
4. Develop a **new concept** of **accelerometer** for the next generation of **Galileo** satellites
5. Support the activities carried out by **ASI-CGS**, **OATO** and **POLITO**

Main activities at IAPS/INAF



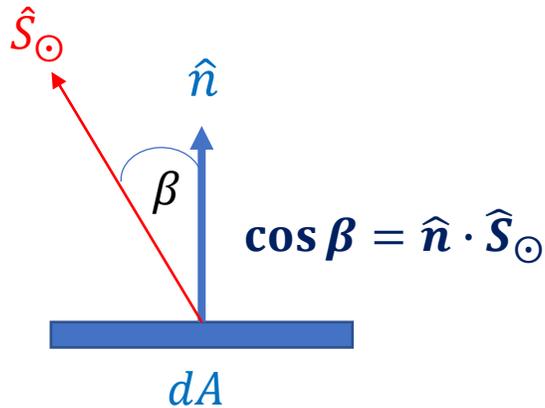
- Direct solar radiation pressure (SRP)
 - It represents the largest **NGP** on the **Galileo FOC** satellites, about 1×10^{-7} m/s² in magnitude, and about two orders-of-magnitude larger than the albedo and infrared radiation from the Earth
 - Our ultimate goal is to develop a **Finite Element Model (FEM)** of the spacecraft and apply a ray-tracing technique to compute, first and foremost, the acceleration due to the direct **SRP** on the satellite
 - Preliminary to the final **FEM**, we have developed a **Box-Wing (BW)** model of the satellite based on current **Galileo Metadata** provided by **ESA**

Main activities at IAPS/INAF

Towards a **FEM** of the **Galileo FOC S/C**

- The direct **SRP** depends on:
 1. the solar flux Φ_{\odot}
 2. the distance of the Earth-satellite system from the Sun
 3. the satellite attitude and its shape/geometry
 4. the physical properties (optical, thermal,...) of the various surfaces (constituents) and their time degradation

Main activities at IAPS/INAF



The resulting elementary acceleration on a surface element

$$\alpha + \rho + \delta = 1$$

$$d\vec{a} = -\frac{\Phi_{\odot}}{mc} \left[(1 - \rho)\hat{S}_{\odot} + 2 \left(\frac{\delta}{3} + \rho \cos \beta \right) \hat{n} \right] dA |\cos \beta|$$

$$d\vec{a} = d\vec{a}_{\alpha} + d\vec{a}_{\rho} + d\vec{a}_{\delta}$$

Simplifying assumptions:

A. Milani, A.M. Nobili, P. Farinella. Adam Hilger, Bristol (1987)

- the absorbed light is not-re-emitted
- for a given direction, the intensity of diffused light is proportional to the cosine of the angle with the unit vector \hat{n} (**Lambert's law**, i.e. spherical diffusion lobe)
- the reflection is perfectly specular
- dA behaves like a linear combination of a black body, a perfect mirror and a Lambert diffuser

$$d\vec{a}_{\alpha} = -\frac{\Phi_{\odot}}{mc} \alpha dA |\cos \beta| \hat{S}_{\odot}$$

$$d\vec{a}_{\rho} = -\frac{\Phi_{\odot}}{mc} 2\rho \cos \beta dA |\cos \beta| \hat{n}$$

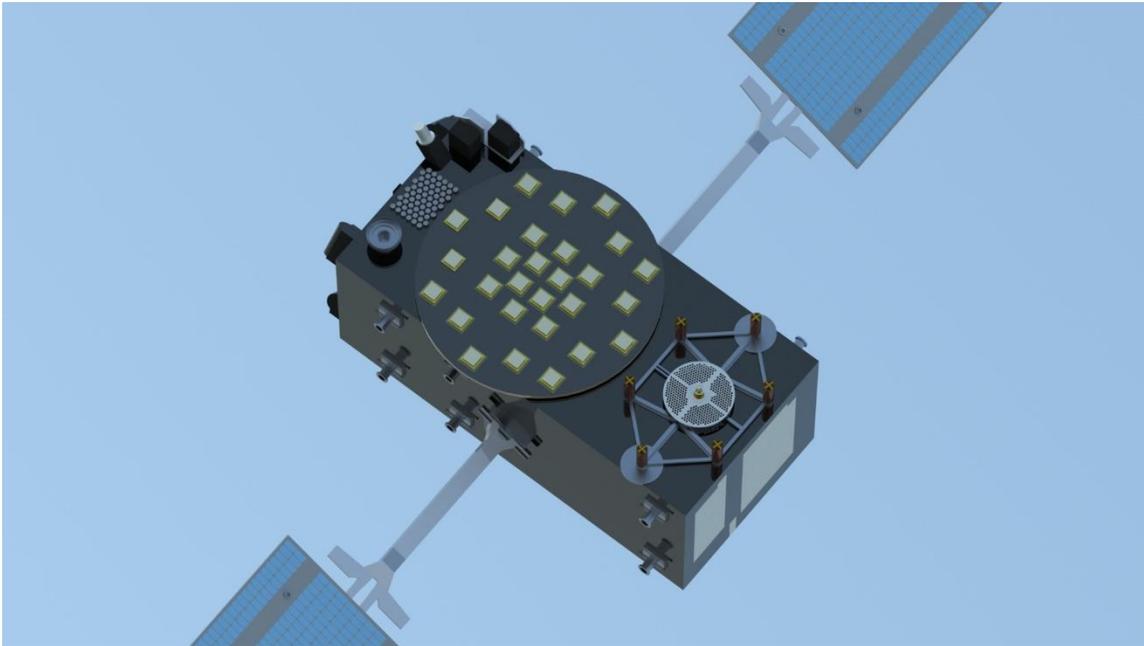
$$d\vec{a}_{\delta} = -\frac{\Phi_{\odot}}{mc} \delta dA |\cos \beta| \hat{S}_{\odot} - \frac{\Phi_{\odot}}{mc} \frac{2}{3} \delta dA |\cos \beta| \hat{n}$$

Main activities at IAPS/INAF

- We need to improve the modeling of the **NGPs** down to an acceleration level $\leq 10^{-10} \text{ m/s}^2$
- On the basis of our experience with **LAGEOS** satellites, the main challenge is represented by the knowledge of the temperature distribution of the S/C and the development of a reliable model for the thermal perturbations
- We need almost all the physical information characterizing the S/C materials:
 - The characteristics provided in the **FOC metadata** are not enough for a **FEM**
 - Currently, we are using a **BW** model

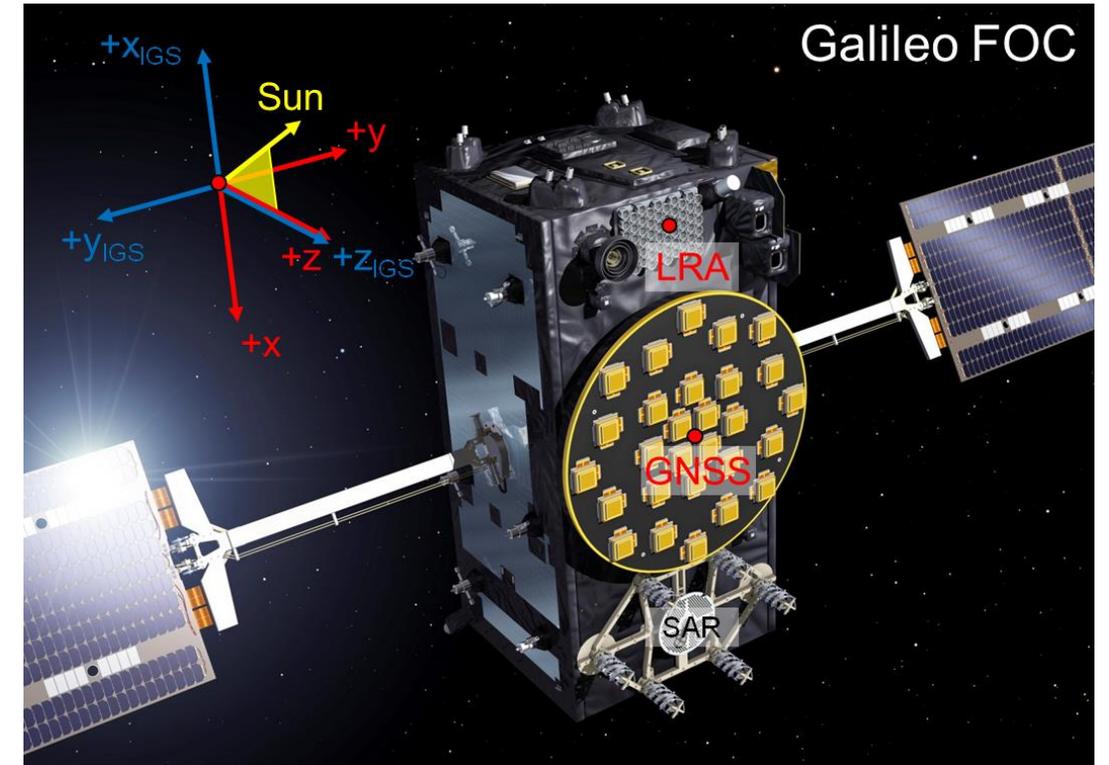
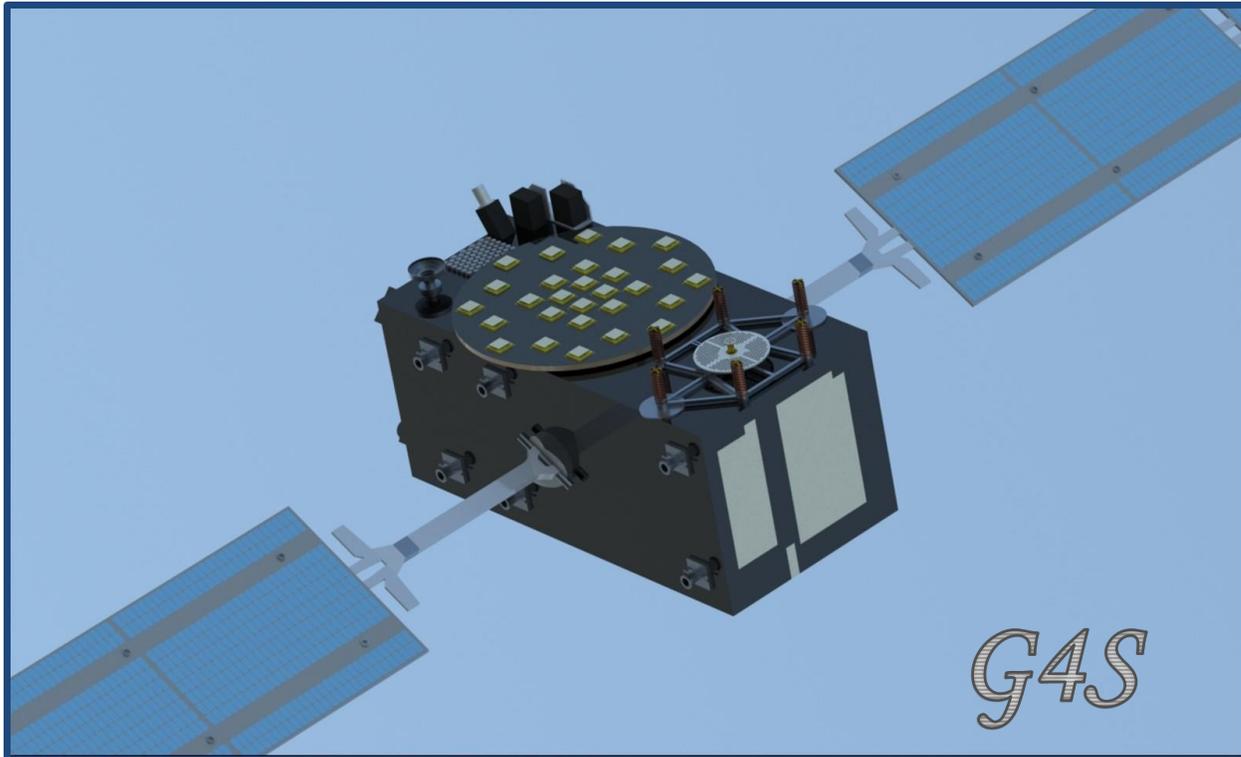
Main activities at IAPS/INAF

We have developed a realistic **3D model** of the **Galileo FOC S/C**



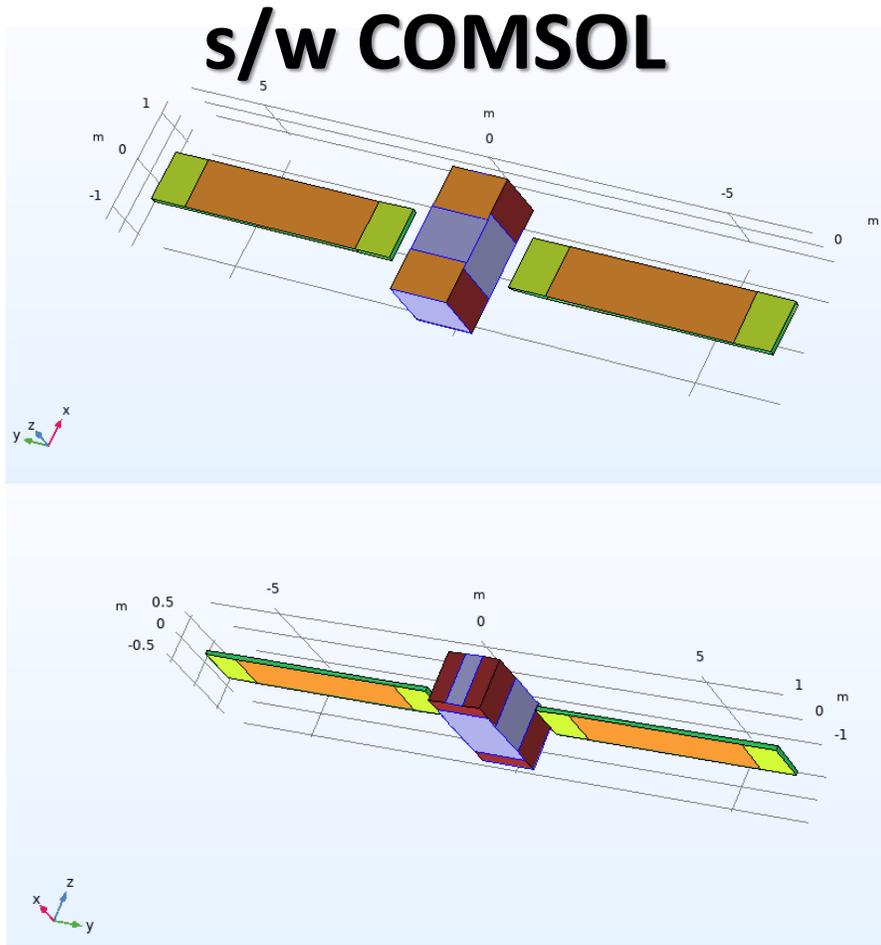
Main activities at IAPS/INAF

3D Model for FEM analysis



Main activities at IAPS/INAF

Simplified Box-Wing model



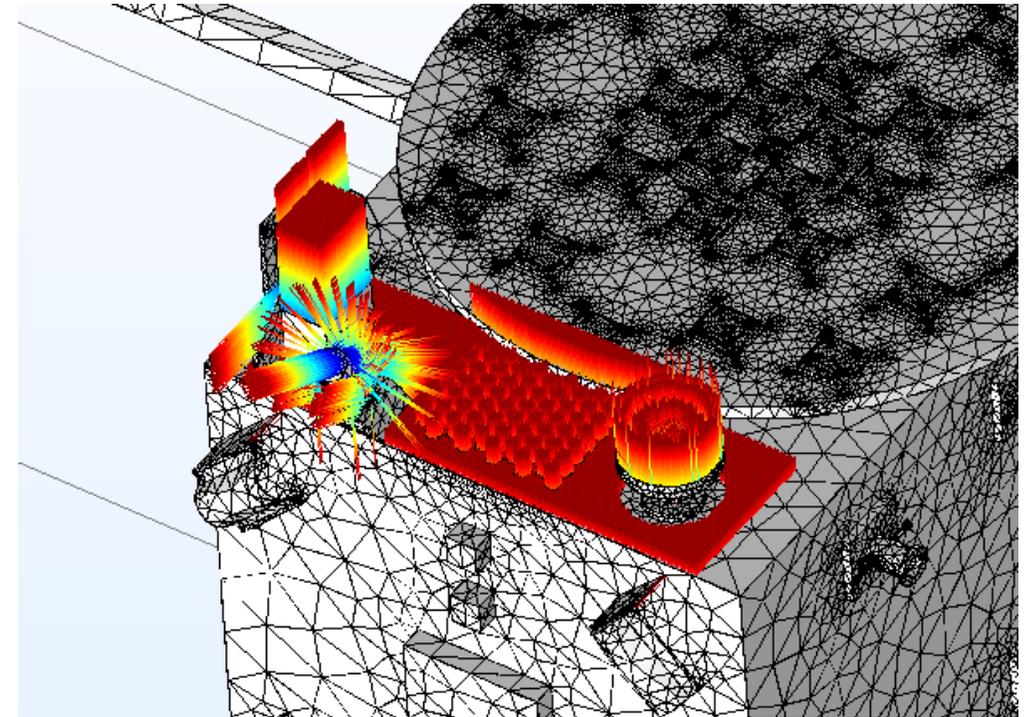
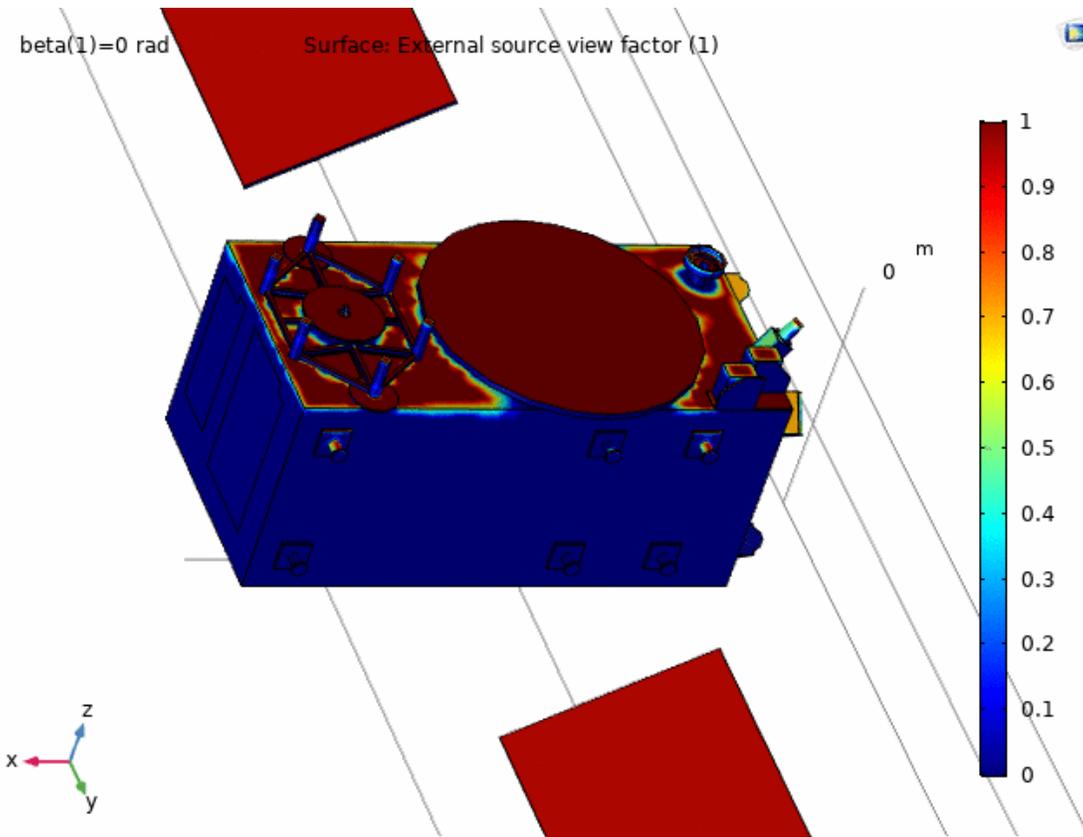
Optical properties from ESA Galileo Metadata

Surface	Material	Area [m ²]	α [-]	ρ [-]	δ [-]	
Box	+X	A	0.440	0.93	0.00	0.07
		C	0.880	0.08	0.73	0.19
	-X	A	1.320	0.93	0.00	0.07
		C	1.654	0.08	0.73	0.19
	+Y	A	1.129	0.93	0.00	0.07
		C	1.654	0.08	0.73	0.19
	-Y	A	1.244	0.93	0.00	0.07
		C	1.539	0.08	0.73	0.19
	+Z	A	1.053	0.93	0.00	0.07
		B	1.969	0.57	0.22	0.21
-Z	A	2.077	0.93	0.00	0.07	
	C	0.959	0.08	0.73	0.19	
Wing	+SA	E	3.880	0.92	0.08	0.00
		D	1.530	0.90	0.10	0.00
	-SA	E	3.880	0.92	0.08	0.00
		D	1.530	0.90	0.10	0.00

Main activities at IAPS/INAF

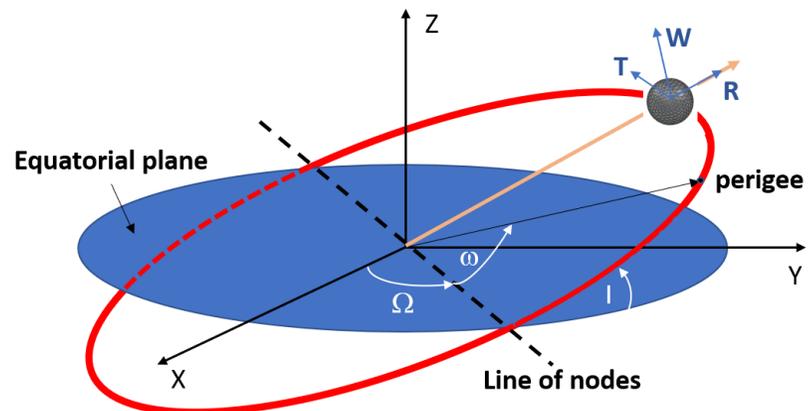
Preliminary activity: towards Ray-Tracing

s/w COMSOL



Main activities at IAPS/INAF

- In the following, we show the results for the perturbing accelerations due to direct **SRP**, Earth's Albedo radiation pressure (**ARP**) and Earth's Infrared radiation pressure (**IRP**) in the case of the **Galileo FOC E08** satellite approximated by means of the **S-BW** model
- The accelerations are given in the **Gauss co-moving** frame and plotted
 - in the time domain
 - as colour map as a function of the Sun height β above the orbital plane and of the difference Δu between the argument of latitude of the satellite with that of the Sun



$$\vec{a} = R\hat{r} + T\hat{t} + W\hat{w}$$

$$R = R(\beta, \Delta u)$$

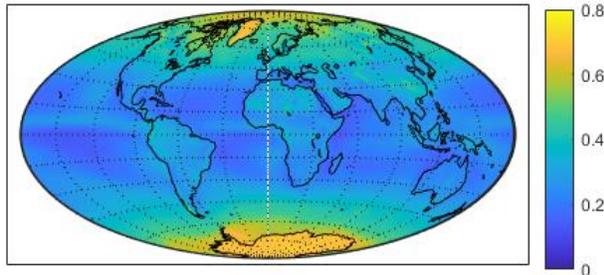
$$T = T(\beta, \Delta u)$$

$$W = W(\beta, \Delta u)$$

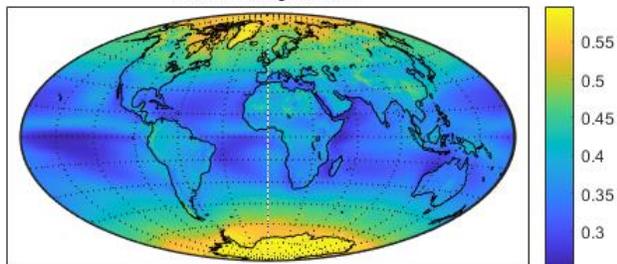
Main activities at IAPS/INAF

- We used **CERES** data to model the Earth's radiation sources

Average on March
from hourly values



Variance on March
from hourly values



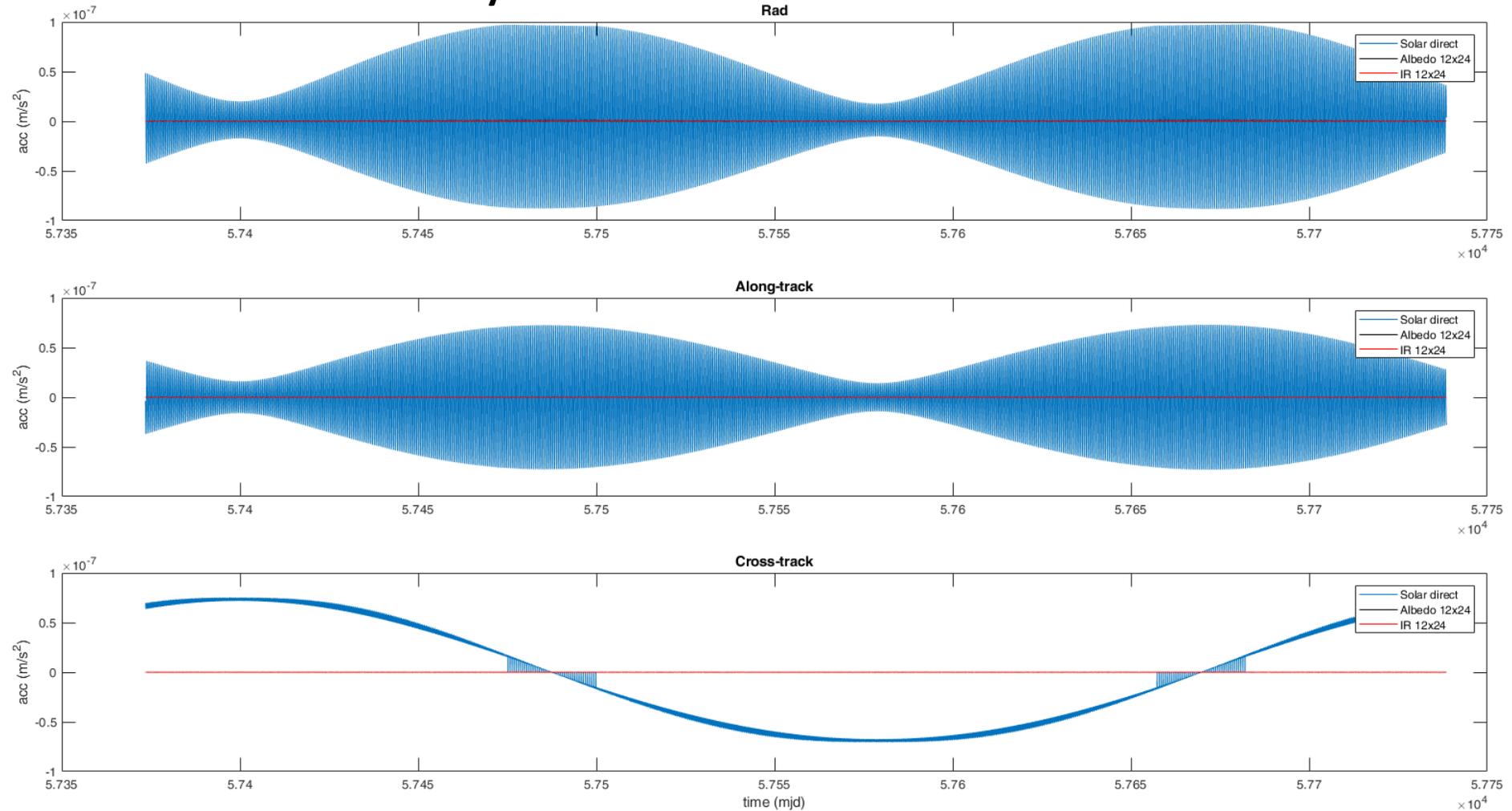
- **CERES (Clouds and Earth's Radiant Energy System)** measures in three bands solar-reflected and Earth-emitted radiation from the top of the atmosphere to the Earth's surface
- Using data from **CERES** we can calculate different averages (hourly, monthly, etc.) of Earth's Albedo and Infrared radiation
- <https://ceres.larc.nasa.gov/data/>

Main activities at IAPS/INAF

Galileo FOC E08



A 1-year simulation with the S-BW model

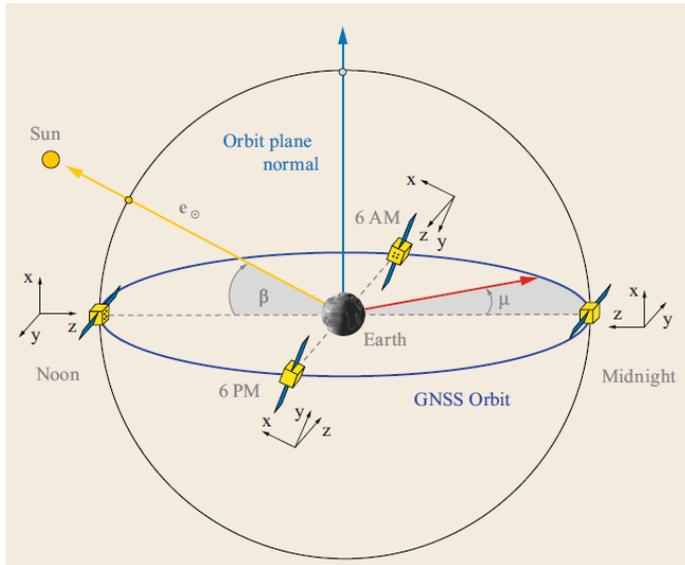
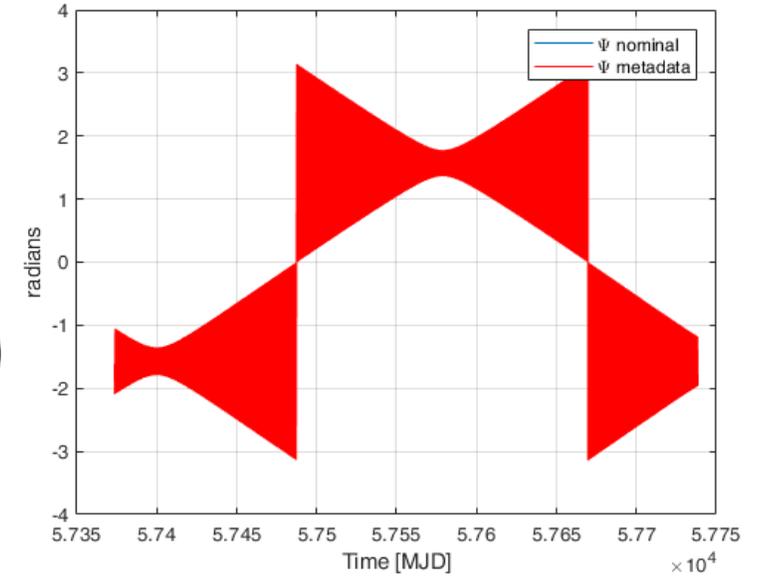


Main activities at IAPS/INAF

The attitude law:
Yaw Steering law of Galileo FOC satellites in agreement with the metadata

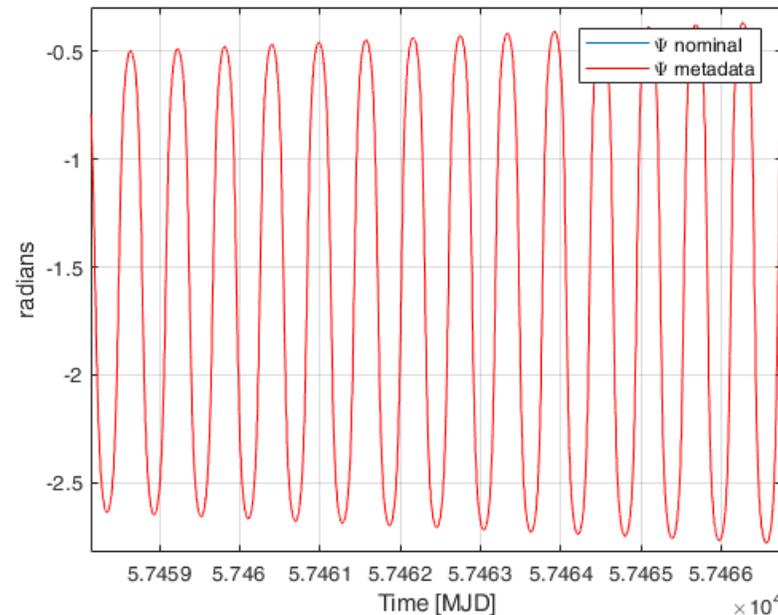
$$\psi(t) = \text{atan2}[\hat{s} \cdot \hat{n}, \hat{s} \cdot (\hat{r} \times \hat{n})]$$

$$\psi_{mod}(t_{mod}) = \frac{\pi}{2} \text{sign}\psi + \left(\psi - \frac{\pi}{2} \text{sign}\psi \right) \cos\left(\frac{2\pi}{5656} t_{mod} \right)$$

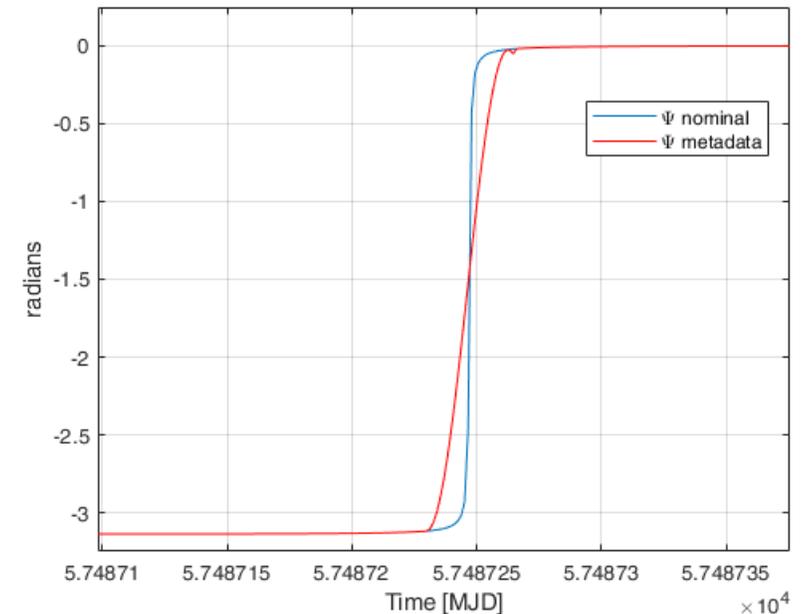


O. Montenbruck, et al., Adv. Space Res., 56, 1015, 2015

Particular 1



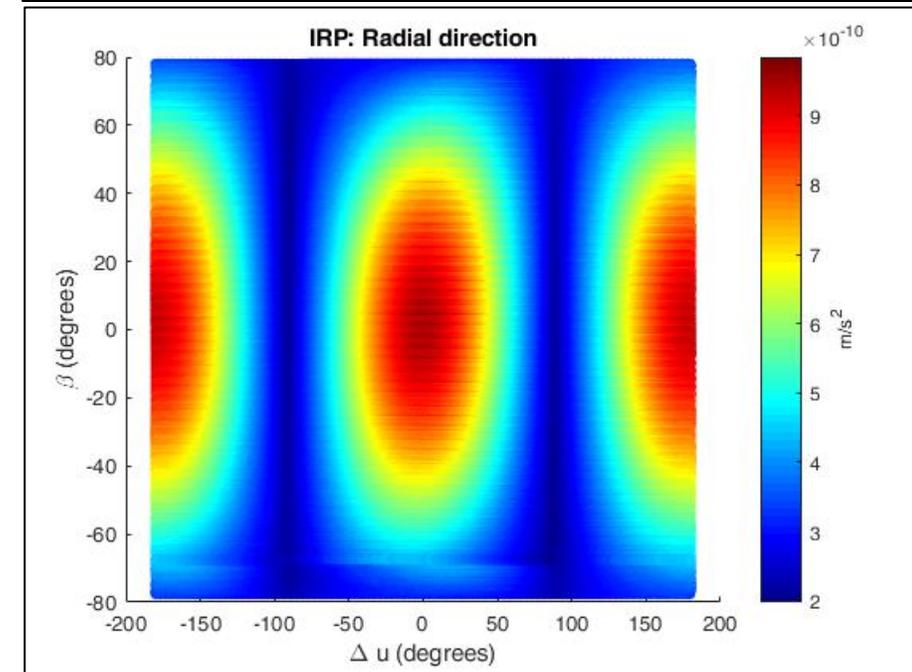
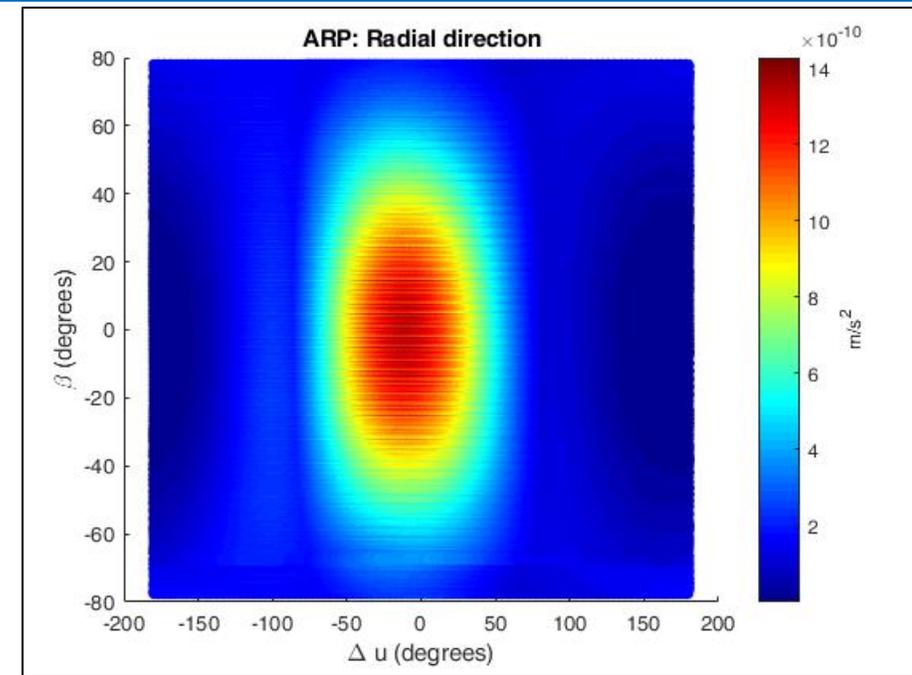
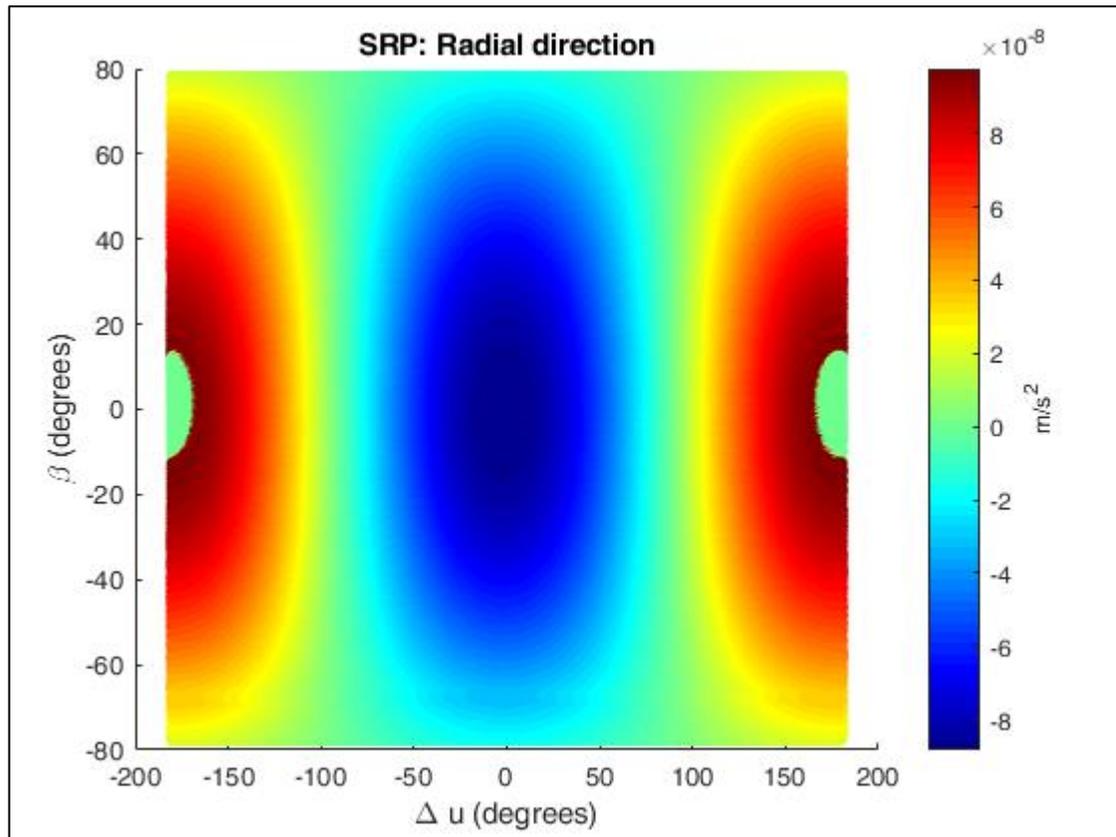
Particular 2



Main activities at IAPS/INAF

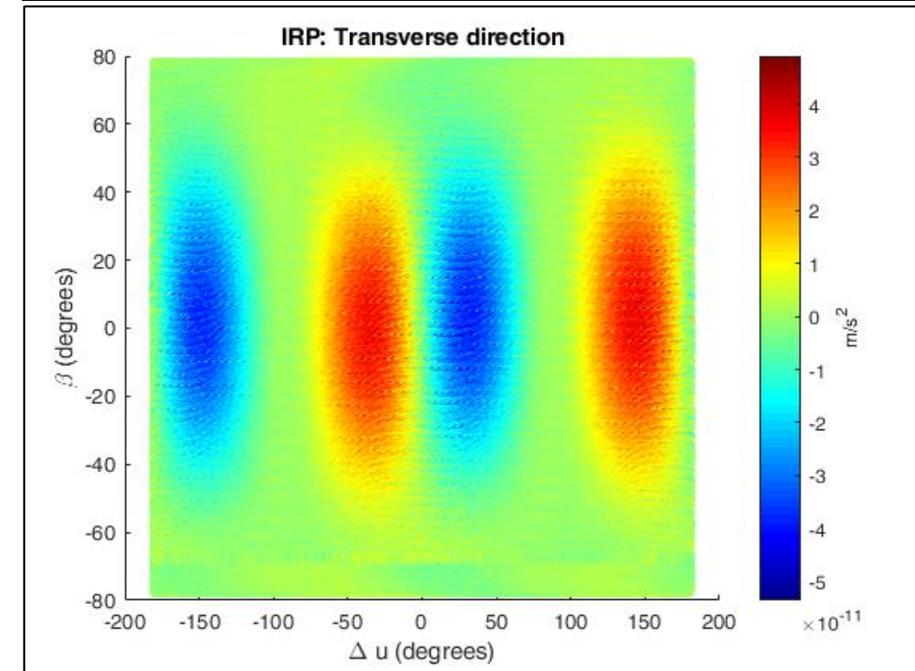
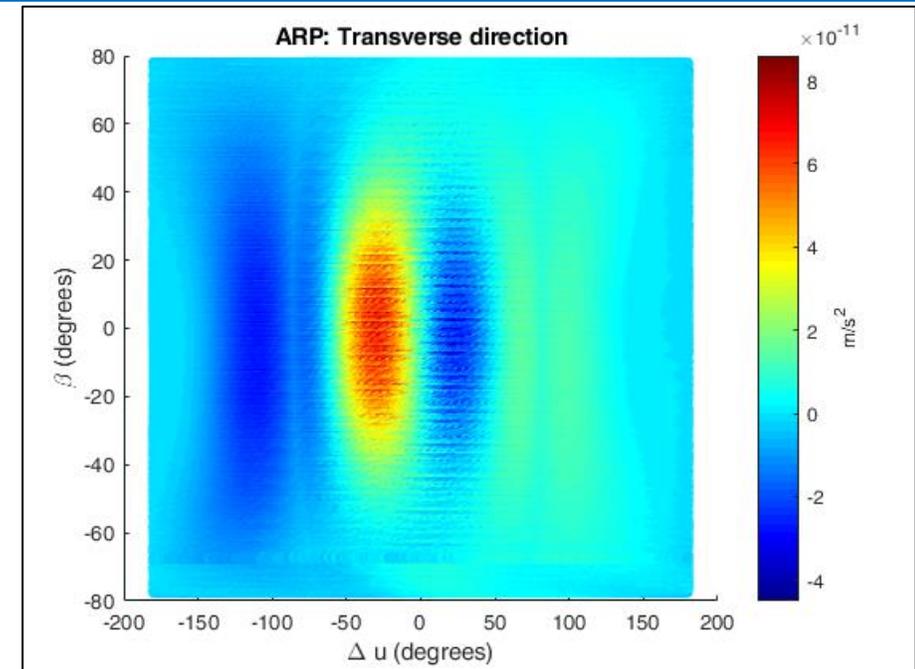
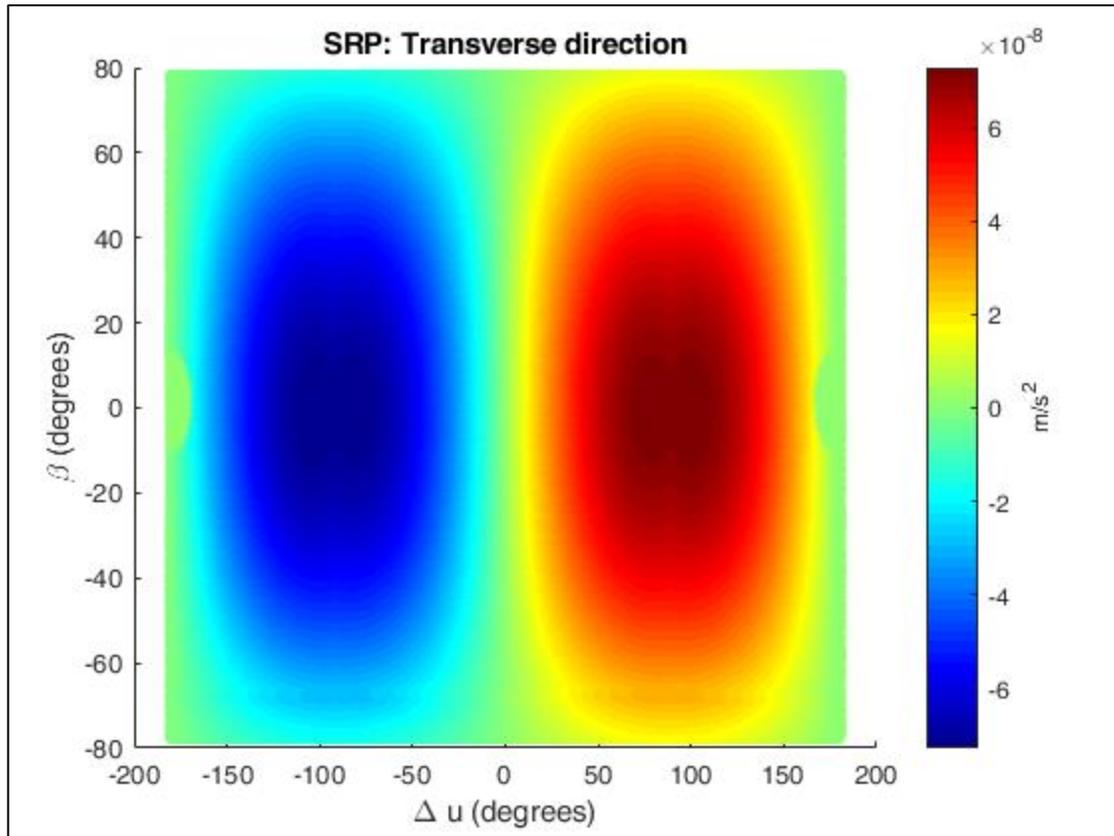
Radial acceleration

Galileo FOC E08



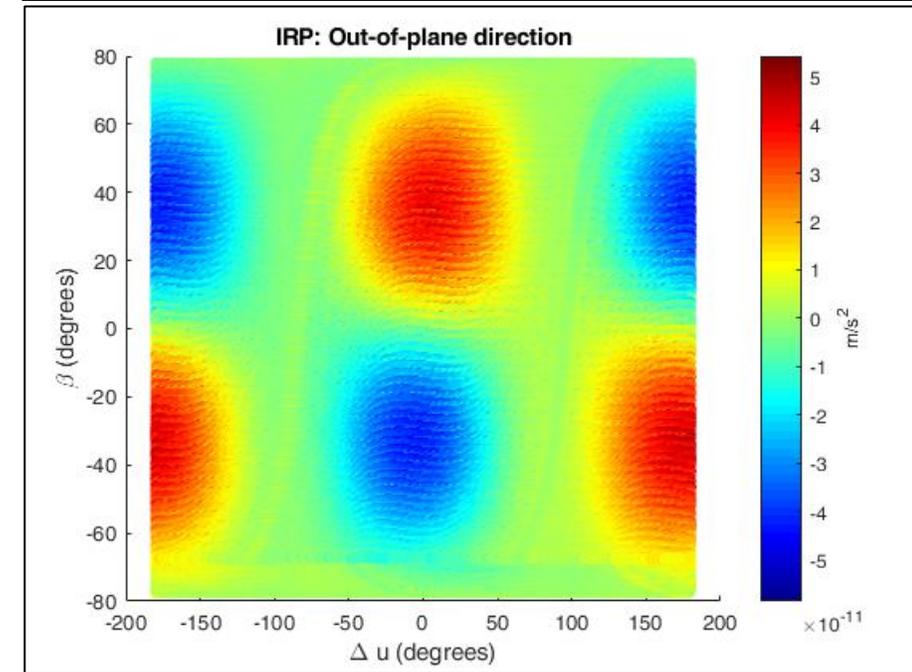
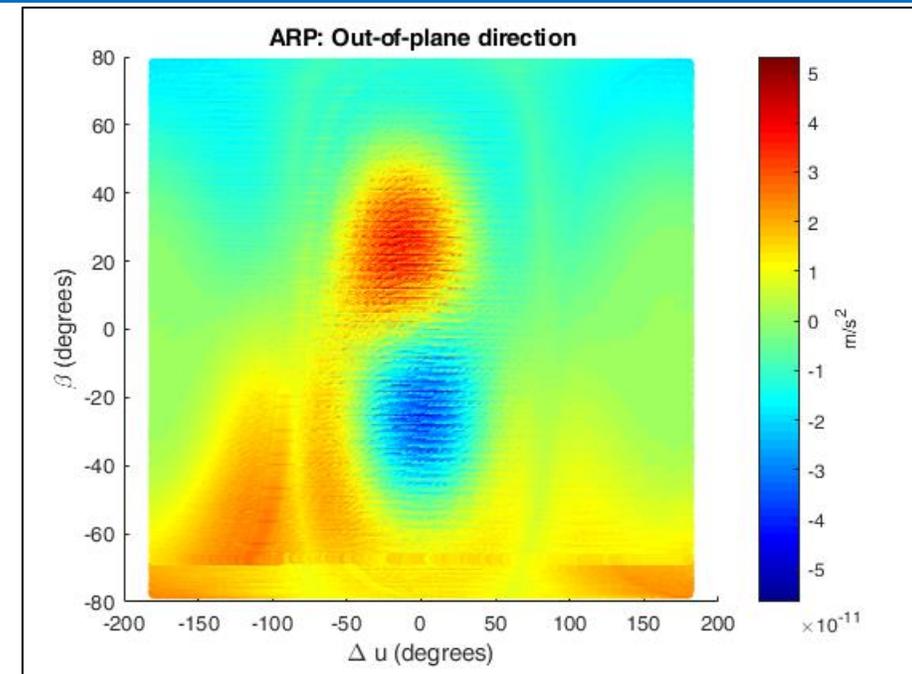
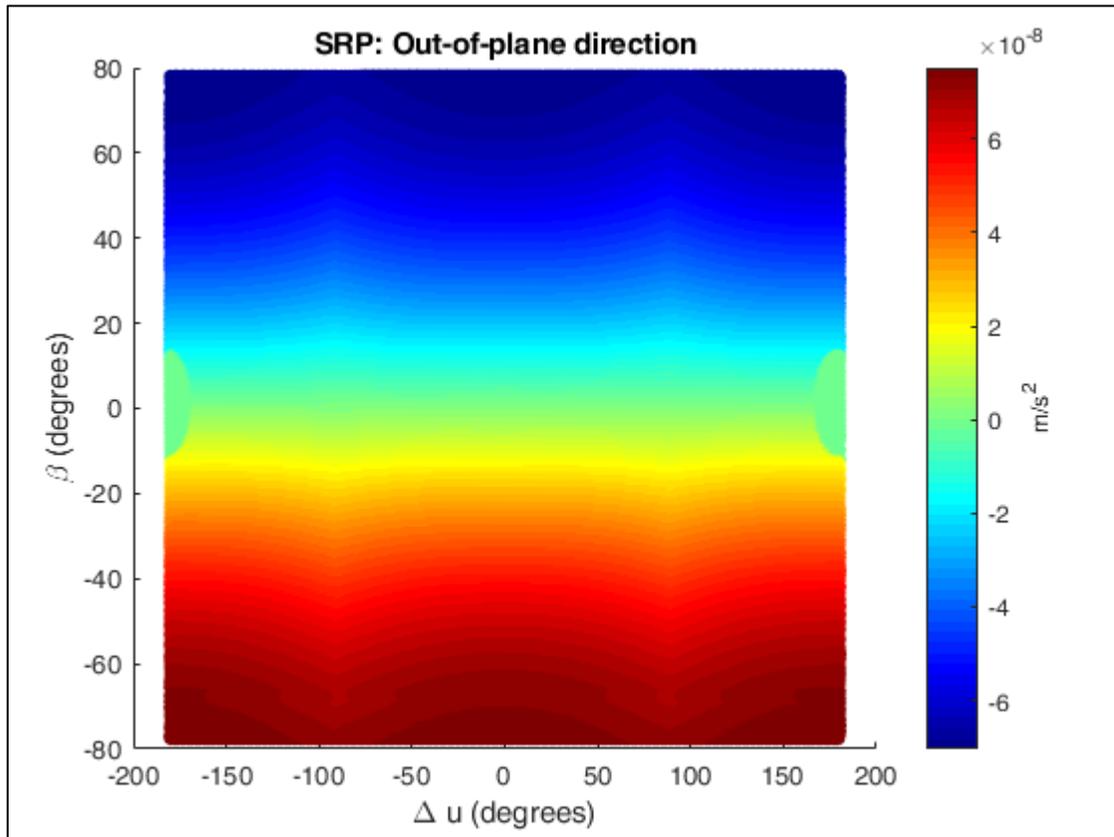
Main activities at IAPS/INAF

Transverse acceleration **Galileo FOC E08**



Main activities at IAPS/INAF

Out-of-plane acceleration Galileo FOC E08



Main activities at IAPS/INAF



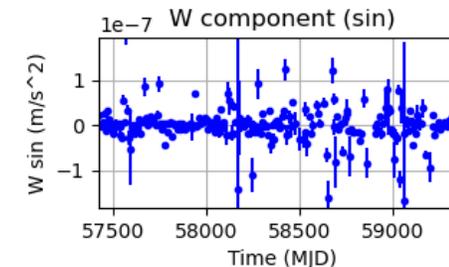
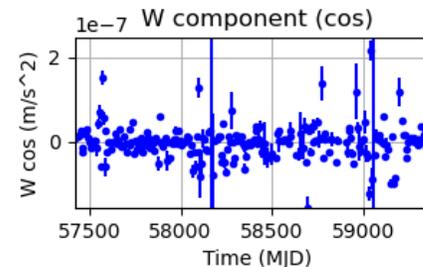
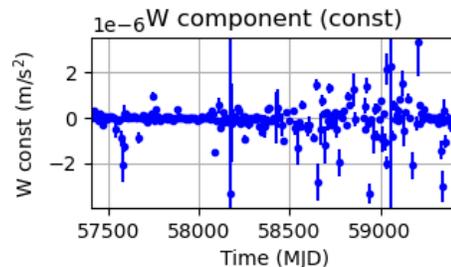
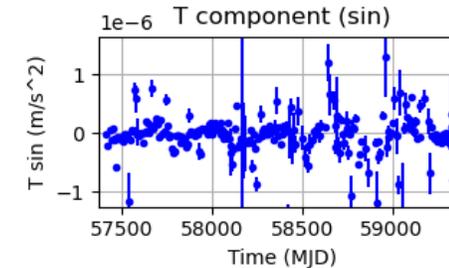
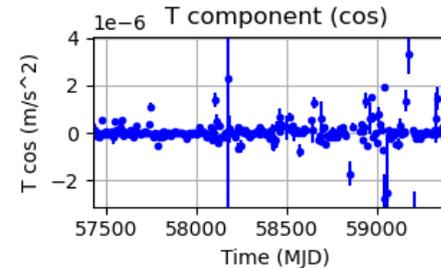
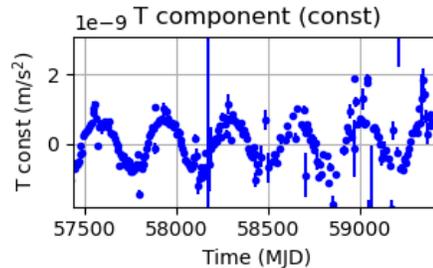
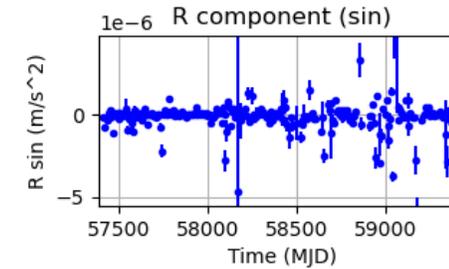
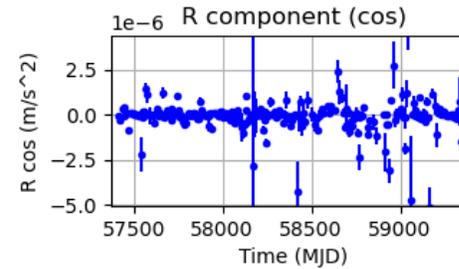
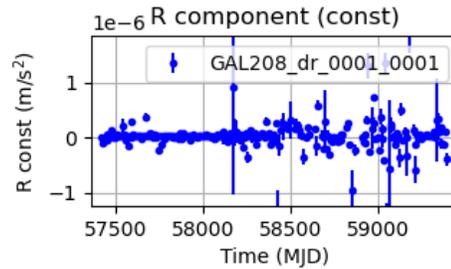
Precise orbit determination (**POD**)

- The analyses will be mainly performed with **GEODYN II (NASA/GSFC)** and **Bernese** s/w
- The best setup with the current models implemented in the code needs to be established and improved with the models we will develop for the **NGPs**
- We are evaluating the **maximum degree ℓ** and **order m** of the gravity field development in spherical harmonics that should be considered in the analysis
- Both **Satellite Laser Ranging (SLR)** normal points and full rate tracking data will be analyzed
- The most favorable **arc length** must be established on the basis of the measurements to be made

Main activities at IAPS/INAF

A “rough” preliminary orbit determination (**POD**): empirical accelerations

E08 satellite
as a cannon ball s/c

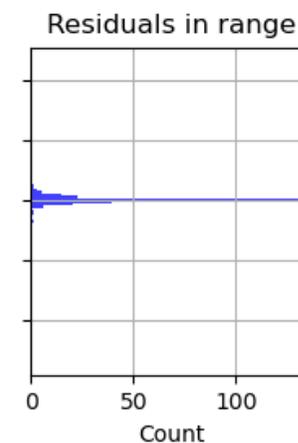
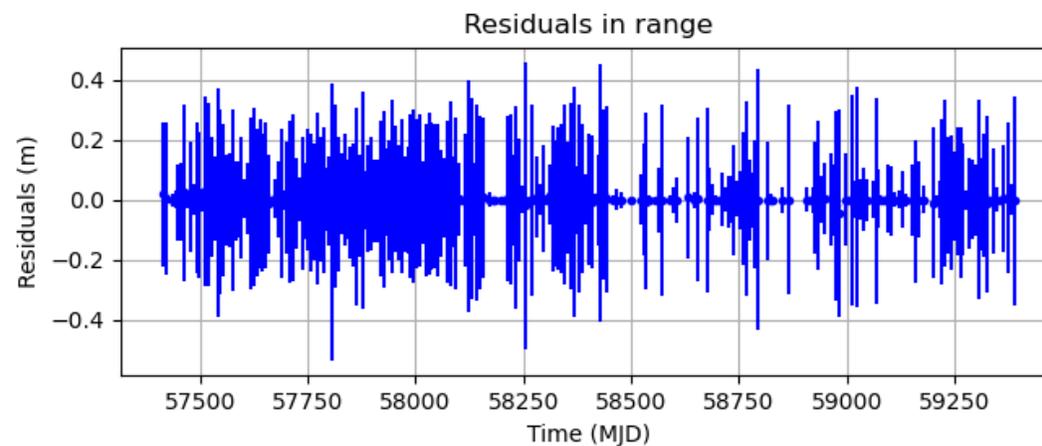
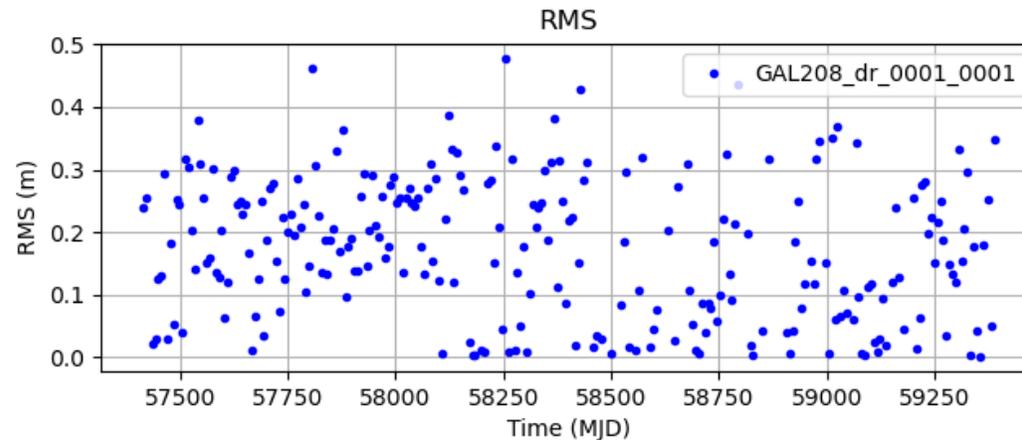


Main activities at IAPS/INAF

A “rough” preliminary orbit determination (**POD**): range residuals

E08 satellite

as a cannon ball s/c



Main activities at IAPS/INAF

Raw Clock bias analysis

$$y_\alpha = -(1 + \alpha) \frac{GM_\oplus}{rc^2}$$

$\alpha = 0$ in GR

- We started a pre-analysis of the clock data of the **E14** and **E18** satellites
- These are the same data analyzed by the **GREAT** experiment
- This analysis is very important to obtain, in the end, corrected clock bias to compare them with **GR** and extract the constraint in the **parameter α**

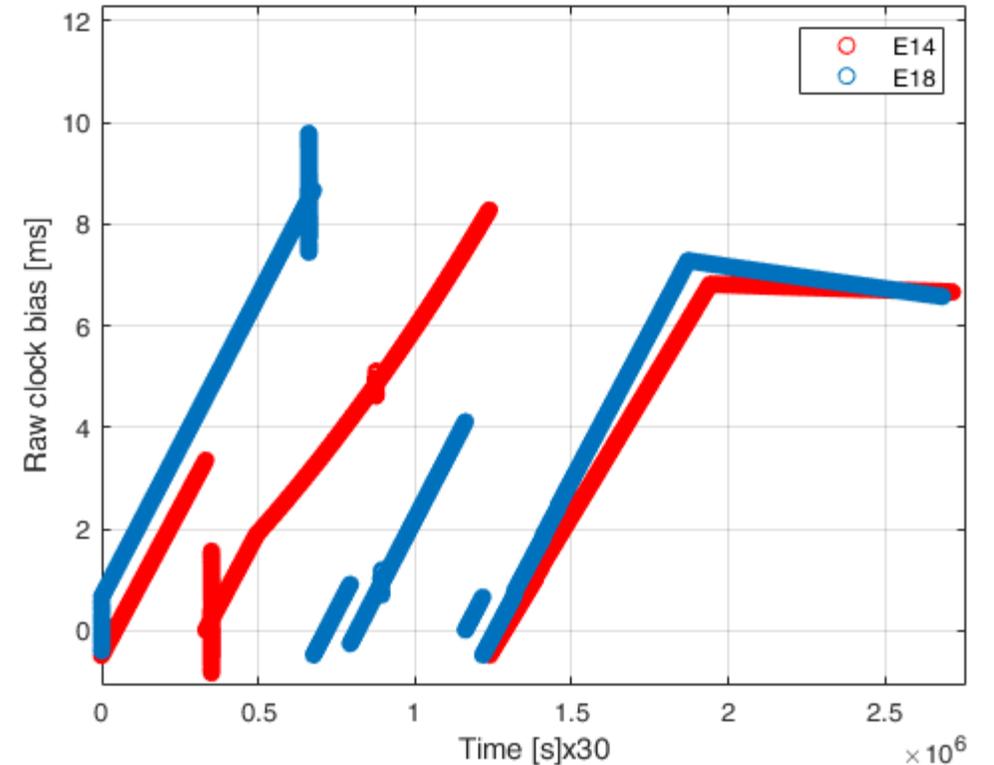
$$\tau_{GR} = \int \frac{d\tau}{dt} dt = \int \left[1 - \frac{1}{2} \frac{v^2}{c^2} - \frac{U_\oplus + U_{tides}}{c^2} \right] dt$$

$$U_\oplus = \frac{GM_\oplus}{r} + \frac{1}{2} \frac{GM_\oplus}{R_\oplus} \left(\frac{R_\oplus}{r} \right)^3 J_2 (1 - 3 \cos^2 \vartheta) \quad \cos \vartheta = \sin i \sin(\omega + f)$$

$$U_{tides} = \sum_i GM_i \left[\frac{1}{|\mathbf{r} - \mathbf{r}_i|} - \frac{1}{r_i} - \frac{\mathbf{r} \cdot \mathbf{r}_i}{r_i^3} \right] \quad \tau_\alpha = -\alpha \int \left[\frac{U_\oplus + U_{tides}}{c^2} \right] dt$$

$$\tau_{Kepler} = -2 \frac{\mathbf{r} \cdot \mathbf{v}}{c^2}$$

From 05/12/2014 to 16/12/2017

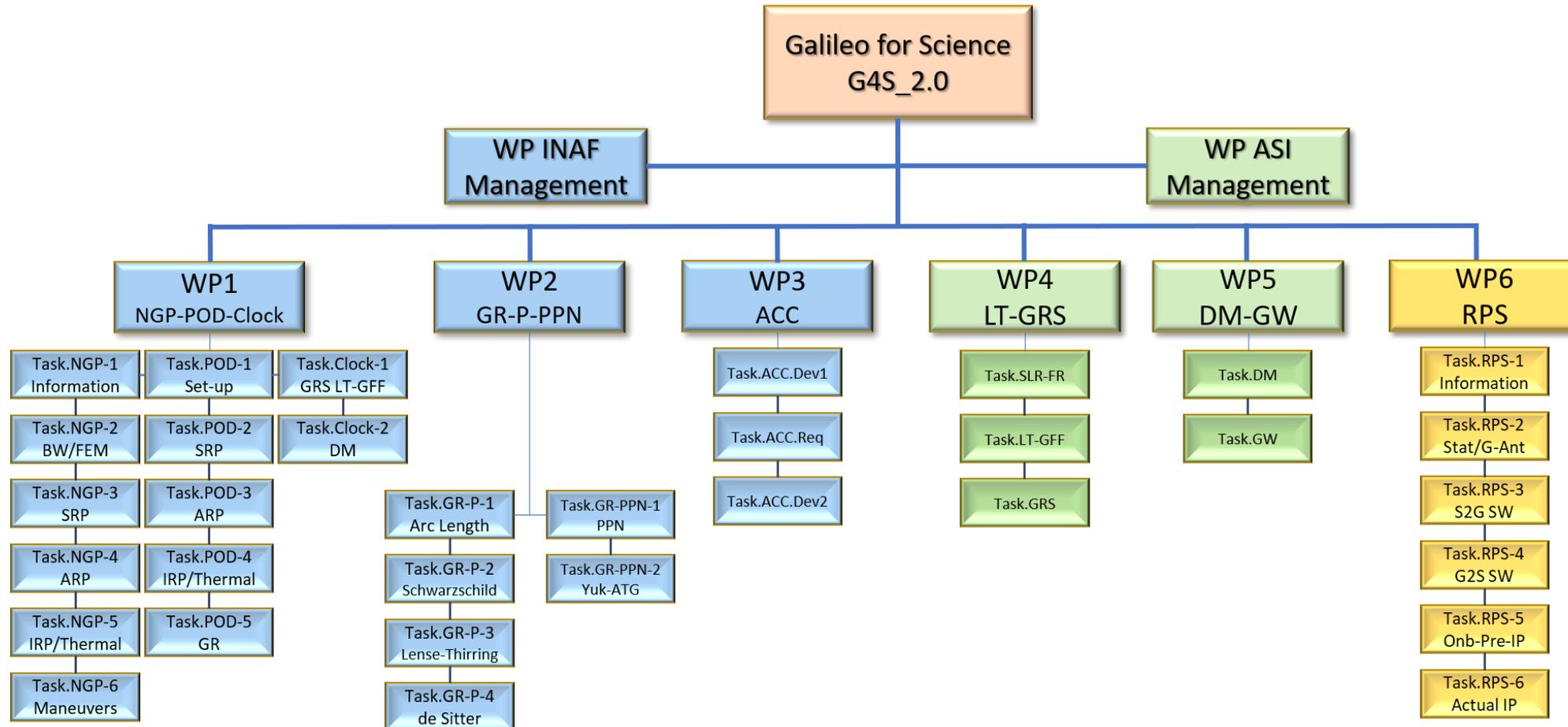


$$\tau_{ESOC} = \tau_{Raw} - \tau_{Kepler}$$

$$\tau_{Corr} = \tau_{ESOC} + \tau_{Kepler} - \tau_{GR}$$

$$\tau_{Corr} = \tau_{Raw} - \tau_{GR}$$

Main activities at IAPS/INAF



Conclusions



Agenzia Spaziale Italiana



- We have introduced the general objectives of the joint proposal **G4S_2.0** and the main activities that will be carried out at the **IAPS/INAF** institute in Rome
- Special care is devoted, as shown, to the modelling of the **NGPs** with the aim to improve the current state of the art
- Next step is to apply the **S-BW** model to the satellites **GSAT0201** and **GSAT0202** and start their orbit determination
- Furthermore, an improved **BW** model is expected based both on deeper information on the S/C characteristics from an update (from **ESA**) of **Galileo FOC** metadata, and on gathering a deeper information on the values for the **optical properties** of the **antennae** and **panels** of these satellites
- The experimental activity for the development of a **new accelerometer** will start shortly

Many thanks for your kind attention