Non-unitarity problem in quantum gravity corrections to quantum field theory with Born-Oppenheimer approximation

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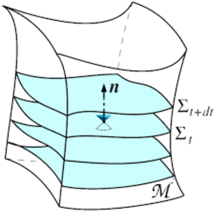
The problem of time in quantum gravity

Canonical quantization approach to gravity with ADM formalism:

Universe wave function: $\Psi = \Psi(\{h_{ij}\}, \phi_a)$
h_{ij} = equivalence class of 3-geometries on Σ
ϕ_a = matter fields



The dynamics of the system is encoded in the Wheeler-deWitt equation:

$$H\Psi = \left(-\frac{2\hbar^2\kappa}{\sqrt{h}}\nabla_g^2 - \frac{\sqrt{h}\,R^{(3)}}{2\kappa}\right)\Psi + \left(-\frac{\hbar^2}{2\sqrt{h}}\nabla_m^2 + u(h_{ij}, \phi_a)\right)\Psi = 0$$

Applying the canonical quantization scheme: $\hat{H}\Psi = \mathrm{i}\hbar\frac{\partial}{\partial t}\Psi = 0$
$\rightarrow$ the Universe wave function does not evolve in time
$\rightarrow$ **Definition of a relational time**

The Wentzel-Kramer-Brillouin expansion

WKB method

$$\Psi(c,q) = e^{\mathrm{i}S/\hbar}$$

Perturbative expansion in parameter K: $S = \sum_{n=0}^{\infty} K^n S_n$
Decomposition $S_n = \sigma_n(c) + \eta_n(c,q)$ into a semiclassical part for the background, and a part for the quantum subsystem.

Applied to the WDW equation:

$$H\Psi = \left(-K\nabla_c^2 + U_c + H_q\right)\Psi = 0$$

$\rightarrow$ identification of a semiclassical background, which can be used to define the time evolution of the quantum subsystem.
What is the dynamics that emerges from expansion?

WKB-expanded gravity-matter systems

- *A. Vilenkin (1989)* 10.1103/PhysRevD.39.1116
  expansion in $\hbar$: only up to $\mathcal{O}(\hbar)$, where a Schrödinger eq. for matter fields is found with suitable time.
  Expansion to $\mathcal{O}(\hbar^2)$: $\mathrm{i}\hbar\frac{\partial\chi_2}{\partial\tau} = H_q\chi_2 - \left(2\mathrm{i}\hbar^2\nabla_c\sigma_1\cdot\nabla_c + \hbar^2\nabla_c^2\right)\chi_2$
  $\rightarrow$ non-unitary dynamics for matter fields
- *C. Kiefer, T.P. Singh (1991)* 10.1103/PhysRevD.44.1067
  expansion in $M \propto m_{\mathrm{Pl}}^2$ up to $\mathcal{O}(1/M)$: non-unitary matter dynamics is found due to quantum gravity corrections.
- *C. Kiefer, D. Wichmann (2018)* 10.1007/s10714-018-2390-4
  redefinition of the quantum state χ to make H_{tot} Hermitian
  Procedure not valid in general due to $[H_{tot}, H_m] \neq 0$

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New proposal for the non-unitarity problem

WKB expansion: $M \equiv \dfrac{1}{4c^2\kappa} = \dfrac{cm_{\mathrm{Pl}}^2}{4\hbar}$

$\rightarrow$ expansion valid for particles with small $m/\lambda_{Compton}$ ratio

Born-Oppenheimer separation: $\Psi(h_a, \phi, y^\mu) = \psi(h_a)\chi(\phi, y^\mu; h_a)$

where

- ★ $\frac{\hat{H}\chi(\phi, y^\mu; h_a)}{\hat{H}\psi(h_a)} = \mathcal{O}\left(\frac{1}{M}\right) \rightarrow \psi$ satisfies $\hat{\mathcal{H}}^g\psi = 0$, $\hat{\mathcal{H}}_i^g\psi = 0$
- ★ $\frac{\delta}{\delta h_a}\chi(\phi, y^\mu; h_a) \simeq \mathcal{O}\left(\frac{1}{M}\right)$

B-O applied in *Bertoni et al (1996)* 10.1088/1361-6382/aa8fb3 and *Venturi et al (2017)* 10.1088/1361-6382/aa8fb3 :
when WKB expanded, matter has non-unitary dynamics

Insertion of the kinematical action

First defined in *K.V. Kuchař (1981), "Canonical Methods of quantization"* as a way to naturally insert the system constraints into the action.
First used in gravity in 10.1016/S0550-3213(02)00301-2

$$S^{kin} = \int d^4x(p_\mu \partial_t y^\mu - N^\mu p_\mu)$$

is inserted as a fast component.
The kinematical action reinstates covariance under ADM foliation of the gravitational background (choice of N and N^i).
Additional contributions to the total constraints of the system (for Ψ):

$$\mathcal{H}^{kin} = n^\mu p_\mu$$

$$\mathcal{H}_i^{kin} = b_i^\mu p_\mu$$

Constraints of the model

Gravitational wave function:

$$\left[-\frac{\hbar^2}{2M}\left(\nabla_g^2 + g\cdot\nabla_g\right) + MV\right]\psi = 0$$

$$2\mathrm{i}\hbar\, h_i \mathrm{D}\cdot\nabla_g\psi = 0$$

Total wave function:

$$\left[-\frac{\hbar^2}{2M}\left(\nabla_g^2 + g\cdot\nabla_g\right) + MV - \hbar^2\nabla_m^2 + U_m\right]\Psi = \mathrm{i}\hbar\, n^\mu \frac{\delta}{\delta y^\mu}\Psi$$

$$\left(2h_i\,\mathrm{D}\cdot\nabla_g - \partial_i\phi\cdot\nabla_m\right)\Psi = \mathrm{i}\hbar\, b_i^\mu \frac{\delta}{\delta y^\mu}\Psi$$

Ansatz:

$$\Psi(h_a,\phi,y^\mu) = e^{\frac{\mathrm{i}}{\hbar}\left(M\sigma_0+\sigma_1+\frac{1}{M}\sigma_2\right)}\cdot e^{\frac{\mathrm{i}}{\hbar}\left(\eta_1+\frac{1}{M}\eta_2\right)}$$

Unitary emerging dynamics for matter field

$\mathcal{O}\left(M^1\right)$: H-J equation for gravity sector:

$$\frac{1}{2}\nabla_g\sigma_0 \cdot \nabla_g\sigma_0 + V = 0$$

$\rightarrow$ correct gravitational limit (Einstein equations).

- $\mathcal{O}\left(M^0\right)$: Schrödinger equation for matter sector

$$\mathrm{i}\hbar\frac{\delta}{\delta\tau}\chi_1 = \hat{H}\chi_1 = \int_\Sigma d^3x\left(N\mathcal{H}^m + N^i\mathcal{H}_i^m\right)\chi_1$$

with relational time

$$\mathrm{i}\hbar\frac{\delta}{\delta\tau} \equiv \mathrm{i}\hbar\int_\Sigma d^3x\left(Nn^\mu + N^i b_i^\mu\right)\frac{\delta}{\delta y^\mu}$$

Unitary emerging dynamics for matter field

$\mathcal{O}\left(M^{-1}\right)$: matter dynamics with quantum gravity corrections

$$\mathrm{i}\hbar\frac{\delta}{\delta\tau}\chi = \hat{H}\chi + \int_\Sigma d^3x \left[N\nabla_g\sigma_0 \cdot (-\mathrm{i}\hbar\nabla_g) - 2N^k h_k \mathrm{D} \cdot \left(\frac{-\mathrm{i}\hbar\nabla_g\chi}{\chi}\right)\right]\chi$$

where $-2h_i\mathrm{D} \cdot (-\mathrm{i}\hbar\nabla_g) \equiv -2h_{ij}\mathrm{D}_k(-\mathrm{i}\hbar\nabla_g)^{kj}$

The correction terms are $\propto \nabla_g\chi = \mathcal{O}\left(1/M\right)$ and **unitary**:

$\sigma_0 =$ real function, solution of H-J equation (action at $\mathcal{O}\left(M\right)$),
$-\mathrm{i}\hbar\nabla_g =$ conjugate momenta of the gravitational variables.

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Applications in cosmology

Expansion valid for matter fields whose associated particles are of small energy with respect to $m_{Pl}\, c^2$, and small corrections $\mathcal{O}\left(1/M\right)$.

Possible scenarios:

- ★ de Sitter phase of the scalar field ϕ during inflation;
- ★ effects on scalar perturbations during inflation;
- ★ effects on the temperature of the Hawking radiation associated to a black hole.

Application to homogeneous inflaton field

Scalar field $\phi(t)$ in spatially flat FRW background: $R = \frac{6}{c^2}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$, $\Lambda > 0$

Homogeneous case: $N(t) = \frac{\partial T}{\partial t}$ and $p_0 = \mathrm{i}\hbar \frac{\delta}{\delta T}$

Integrating over a fiducial volume, the total Hamiltonian is

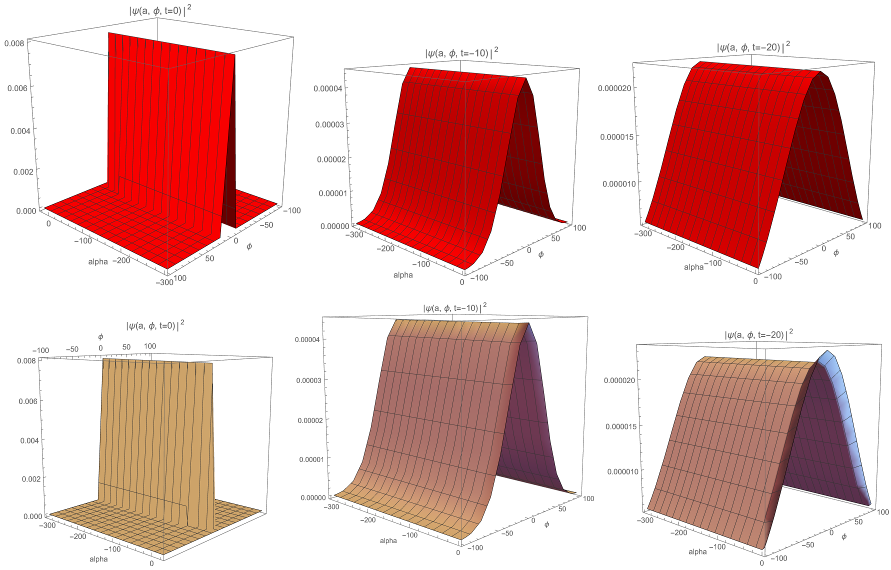
$$N(H_{grav} + H_\phi + H_{kin}) = \frac{1}{2V_0}\,\frac{N}{a^3}\pi_\phi^2 - \frac{2\pi G c^2}{3V_0}\,\frac{N}{a}\pi_a^2 + \frac{V_0}{8\pi G}\,\Lambda N a^3 + N p_0$$

Using the time $d\tau = \frac{dT}{a^3}$, the dynamics for χ in Fourier space $(k = k_a, p = p_\phi)$ is:

$$\mathrm{i}\hbar \frac{\delta \tilde{\chi}}{\delta \tau} = -\frac{\hbar^2}{2}\frac{\delta^2 \tilde{\chi}}{\delta \phi^2} + \hbar\,\frac{k\,(-\tau)^{7/3}}{3(3\Lambda)^{1/6}}\,\tilde{\chi} \rightarrow \tilde{\chi} = e^{-\mathrm{i}\frac{\hbar p^2}{2}\tau + \mathrm{i}\frac{k\,(-\tau)^{7/3}}{7(3\Lambda)^{1/6}}}$$

$\rightarrow$ solution for χ, valid for $-\frac{1}{M} < k < \frac{1}{M}$ (perturbative expansion)

Gaussian wave packet with small $\frac{\delta\psi}{\delta a}$



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Reference frame fixing as incoherent dust

K.V. Kuchař, C.G. Torre (1991) 10.1103/PhysRevD.43.419

$$S = S^{grav} + S^F$$

where S^F fixes the Gaussian reference frame $g^{00} = 1$, $g^{0i} = 0$ with Lagrange multipliers $\mathcal{M}$, $\mathcal{M}_i$.
Parametrized S^F (recovers covariance):

$$S^F = \int_\Sigma d^4x \left[-\frac{1}{2}\mathcal{M}\sqrt{-g}(g^{\alpha\beta}\partial_\alpha T \partial_\beta T - 1) + \mathcal{M}_i\sqrt{-g}(g^{\alpha\beta}\partial_\alpha T \partial_\beta X^i) \right]$$

Source term → Reference system "materializes" as an **incoherent dust**

$$T^{\alpha\beta} = \mathcal{M}\, U^\alpha\, U^\beta \qquad \text{with } U^\alpha = g^{\alpha\beta}\partial_\beta T$$

Dominant energy condition: $\mathcal{M} \geq 0$
Matter dynamics: Schrödinger eq. with time $T \equiv t$ in Gaussian frame
→ the emergent fluid plays the role of clock for matter

WKB expansion with incoherent dust

Applying the WKB expansion (in M) to this scenario, in the minisuperspace:

$$i\hbar\frac{\delta}{\delta t}\chi = H_m\chi + \nabla_g\sigma_0\cdot(-\mathrm{i}\hbar\nabla_g\chi)$$

quantum gravity corrections to the matter dynamics, which are **isomorphic to the ones obtained with kinematical action**!
In minisuperspace:

- consider ADM foliation such that $n^\mu = (1,\vec{0})$
- $y^\mu = (y^0,\vec{0})$ and $\partial_t y^\mu \to \dot{T} = \frac{\delta T}{\delta t} = N = 1$

so the two implementations are equivalent.

Conclusions and perspectives

Summarizing:

- WKB expansion + B-O separation: kinematical action can be used as a clock for the quantum subsystem, gives correct limits and unitary dynamics with gravitational corrections
- Small corrections but in principle detectable

Future perspectives:

- Application to scalar perturbations of the inflaton field
- Equivalence between kinematical action and (Gaussian) reference frame fixing procedure

Thank you for the attention!