Non-unitarity problem in quantum gravity corrections to quantum field theory with Born-Oppenheimer approximation

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Authors: F. Di Gioia\textsuperscript{1}, G. Maniccia\textsuperscript{1,2}, G. Montani\textsuperscript{1,3}, J. Niedda\textsuperscript{1}
Speaker: G. Maniccia, giulia.maniccia@uniroma1.it

\textsuperscript{1}Physics Department, “La Sapienza” University of Rome, 00185 Roma, Italy
\textsuperscript{2}INFN Section of Rome, Roma, Italy
\textsuperscript{3}ENEA, C.R. Frascati, 00044 Frascati (Roma), Italy

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The problem of time in quantum gravity

Canonical quantization approach to gravity with ADM formalism:

Universe wave function: \( \Psi = \Psi(\{h_{ij}\}, \phi_a) \)

\( h_{ij} \) = equivalence class of 3-geometries on \( \Sigma \)

\( \phi_a \) = matter fields

The dynamics of the system is encoded in the Wheeler-deWitt equation:

\[
H \Psi = \left( -\frac{2\hbar^2 \kappa}{\sqrt{h}} \nabla^2_g - \sqrt{h} R(3) \right) \Psi + \left( -\frac{\hbar^2}{2\sqrt{h}} \nabla^2_m + u(h_{ij}, \phi_a) \right) \Psi = 0
\]

Applying the canonical quantization scheme: \( \hat{H} \Psi = i\hbar \frac{\partial}{\partial t} \Psi = 0 \)

→ the Universe wave function does not evolve in time

→ Definition of a relational time
The Wentzel-Kramer-Brillouin expansion

WKB method

\[ \Psi(c, q) = e^{iS/\hbar} \]

Perturbative expansion in parameter \( K \):

\[ S = \sum_{n=0}^{\infty} K^n S_n \]

Decomposition \( S_n = \sigma_n(c) + \eta_n(c, q) \) into a semiclassical part for the background, and a part for the quantum subsystem.

Applied to the WDW equation:

\[ H \Psi = \left( -K \nabla_c^2 + U_c + H_q \right) \Psi = 0 \]

→ identification of a semiclassical background, which can be used to define the time evolution of the quantum subsystem.

What is the dynamics that emerges from expansion?
WKB-expanded gravity-matter systems

  
  Expansion in $\hbar$: only up to $O(\hbar)$, where a Schrödinger eq. for matter fields is found with suitable time.

  Expansion to $O(\hbar^2)$:
  \[
  i\hbar \frac{\partial \chi_2}{\partial \tau} = H_q \chi_2 - \left( 2i\hbar^2 \nabla_c \sigma_1 \cdot \nabla_c + \hbar^2 \nabla_c^2 \right) \chi_2
  \]
  
  $\rightarrow$ non-unitary dynamics for matter fields

- **C. Kiefer, T.P. Singh (1991) 10.1103/PhysRevD.44.1067**
  
  Expansion in $M \propto m_{Pl}^2$ up to $O(1/M)$: non-unitary matter dynamics is found due to quantum gravity corrections.

  
  Redefinition of the quantum state $\chi$ to make $H_{tot}$ Hermitian
  
  Procedure not valid in general due to $[H_{tot}, H_m] \neq 0$
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New proposal for the non-unitarity problem

**WKB expansion:** \( M \equiv \frac{1}{4c^2\kappa} = \frac{cm_p^2}{4\hbar} \)

→ expansion valid for particles with small \( m/\lambda_{Compton} \) ratio

**Born-Oppenheimer separation:** \( \Psi(h_a, \phi, y^\mu) = \psi(h_a)\chi(\phi, y^\mu; h_a) \)

where

\[ \frac{\hat{H}\chi(\phi, y^\mu; h_a)}{\hat{H}\psi(h_a)} = O\left(\frac{1}{M}\right) \rightarrow \psi \text{ satisfies } \hat{H}^g\psi = 0, \hat{H}^g_i\psi = 0 \]

\[ \frac{\delta}{\delta h_a}\chi(\phi, y^\mu; h_a) \simeq O\left(\frac{1}{M}\right) \]

Insertion of the kinematical action

First defined in K.V. Kuchař (1981), “Canonical Methods of quantization” as a way to naturally insert the system constraints into the action.
First used in gravity in 10.1016/S0550-3213(02)00301-2

\[ S^{kin} = \int d^4x (p_\mu \partial_t y^\mu - N^\mu p_\mu) \]

is inserted as a fast component.
The kinematical action reinstates covariance under ADM foliation of the gravitational background (choice of \( N \) and \( N^i \)).
Additional contributions to the total constraints of the system (for \( \Psi \)):

\[ H^{kin} = n^\mu p_\mu \]

\[ H^{kin}_i = b^\mu_i p_\mu \]
Constraints of the model

Gravitational wave function:

\[
-\frac{\hbar^2}{2M} \left( \nabla^2_g + g \cdot \nabla_g \right) + MV \bigg] \psi = 0
\]

\[2i\hbar h_i \mathbf{D} \cdot \nabla_g \psi = 0\]

Total wave function:

\[
\left[ -\frac{\hbar^2}{2M} \left( \nabla^2_g + g \cdot \nabla_g \right) + MV - \hbar^2 \nabla^2_m + U_m \right] \Psi = i\hbar n^\mu \frac{\delta}{\delta y^\mu} \Psi
\]

\[ (2h_i \mathbf{D} \cdot \nabla_g - \partial_i \phi \cdot \nabla_m) \Psi = i\hbar b^\mu_i \frac{\delta}{\delta y^\mu} \Psi \]

Ansatz:

\[\Psi(h_a, \phi, y^\mu) = e^{\frac{i}{\hbar} \left(M\sigma_0 + \sigma_1 + \frac{1}{M} \sigma_2\right)} \cdot e^{\frac{i}{\hbar} \left(\eta_1 + \frac{1}{M} \eta_2\right)}\]
Unitary emerging dynamics for matter field

\[ \mathcal{O}(M^1) \]: H-J equation for gravity sector:

\[
\frac{1}{2} \nabla_g \sigma_0 \cdot \nabla_g \sigma_0 + V = 0
\]

→ correct gravitational limit (Einstein equations).

- \[ \mathcal{O}(M^0) \]: Schrödinger equation for matter sector

\[
i \hbar \frac{\delta}{\delta \tau} \chi_1 = \hat{H} \chi_1 = \int_{\Sigma} d^3x \left( N \mathcal{H}^m + N^i \mathcal{H}_i^m \right) \chi_1
\]

with relational time

\[
i \hbar \frac{\delta}{\delta \tau} \equiv i \hbar \int_{\Sigma} d^3x \left( N n^\mu + N^i b_i^\mu \right) \frac{\delta}{\delta y^\mu}
\]
Unitary emerging dynamics for matter field

$\mathcal{O}(M^{-1})$: matter dynamics with quantum gravity corrections

\[
i\hbar \frac{\delta}{\delta \tau} \chi = \hat{H} \chi + \int_{\Sigma} d^3 x \left[ N \nabla g \sigma_0 \cdot (-i\hbar \nabla g) - 2N^k h_k D \cdot \left( \frac{-i\hbar \nabla g \chi}{\chi} \right) \right] \chi
\]

where $-2h_i D \cdot (-i\hbar \nabla g) \equiv -2h_{ij} D_k (-i\hbar \nabla g)^{kj}$

The correction terms are $\propto \nabla g \chi = \mathcal{O}(1/M)$ and unitary:

$\sigma_0 =$ real function, solution of H-J equation (action at $\mathcal{O}(M)$),

$-i\hbar \nabla g =$ conjugate momenta of the gravitational variables.
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Applications in cosmology

Expansion valid for matter fields whose associated particles are of small energy with respect to $m_{Pl} c^2$, and small corrections $\mathcal{O}(1/M)$.

Possible scenarios:

- de Sitter phase of the scalar field $\phi$ during inflation;
- effects on scalar perturbations during inflation;
- effects on the temperature of the Hawking radiation associated to a black hole.
Application to homogeneous inflaton field

Scalar field $\phi(t)$ in spatially flat FRW background: $R = \frac{6}{c^2} \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$, $\Lambda > 0$

Homogeneous case: $N(t) = \frac{\partial T}{\partial t}$ and $p_0 = i\hbar \frac{\delta}{\delta T}$

Integrating over a fiducial volume, the total Hamiltonian is

$$N(H_{grav} + H_\phi + H_{kin}) = \frac{1}{2V_0} \frac{N}{a^3} \pi_\phi^2 - \frac{2\pi Gc^2}{3V_0} \frac{N}{a} \pi_a^2 + \frac{V_0}{8\pi G} \Lambda N a^3 + N p_0$$

Using the time $d\tau = \frac{dT}{a^3}$, the dynamics for $\chi$ in Fourier space ($k = k_a, p = p_\phi$) is:

$$i\hbar \frac{\delta \tilde{\chi}}{\delta \tau} = -\hbar^2 \frac{\delta^2 \tilde{\chi}}{2 \delta \phi^2} + \hbar \frac{k (\tau)^{7/3}}{3 (3\Lambda)^{1/6}} \tilde{\chi} \rightarrow \tilde{\chi} = e^{\frac{-i}{2} \frac{\hbar p^2}{\tau} + i \frac{k (\tau)^{7/3}}{7 (3\Lambda)^{1/6}}}$$

$\rightarrow$ solution for $\chi$, valid for $-\frac{1}{M} < k < \frac{1}{M}$ (perturbative expansion)
Gaussian wave packet with small $\frac{\delta \psi}{\delta a}$
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Reference frame fixing as incoherent dust

K.V. Kuchař, C.G. Torre (1991) 10.1103/PhysRevD.43.419

\[ S = S^{grav} + S^F \]

where \( S^F \) fixes the Gaussian reference frame \( g^{00} = 1, \ g^{0i} = 0 \) with Lagrange multipliers \( M, M_i \).

Parametrized \( S^F \) (recovers covariance):

\[
S^F = \int_{\Sigma} d^4x \left[ -\frac{1}{2} M \sqrt{-g} (g^{\alpha\beta} \partial_\alpha T \partial_\beta T - 1) + M_i \sqrt{-g} (g^{\alpha\beta} \partial_\alpha T \partial_\beta X^i) \right]
\]

Source term → Reference system “materializes” as an incoherent dust

\[ T^{\alpha\beta} = M \ U^\alpha \ U^\beta \quad \text{with} \quad U^\alpha = g^{\alpha\beta} \partial_\beta T \]

Dominant energy condition: \( M \geq 0 \)

Matter dynamics: Schrödinger eq. with time \( T \equiv t \) in Gaussian frame

→ the emergent fluid plays the role of clock for matter.
Applying the WKB expansion (in $M$) to this scenario, in the minisuperspace:

$$i\hbar \frac{\delta}{\delta t} \chi = H_m \chi + \nabla_g \sigma_0 \cdot (-i\hbar \nabla_g \chi)$$

quantum gravity corrections to the matter dynamics, which are isomorphic to the ones obtained with kinematical action!

In minisuperspace:

- consider $ADM$ foliation such that $n^\mu = (1, \vec{0})$
- $y^\mu = (y^0, \vec{0})$ and $\partial_t y^\mu \rightarrow \dot{T} = \frac{\delta T}{\delta t} = N = 1$

so the two implementations are equivalent.
Conclusions and perspectives

Summarizing:

▶ WKB expansion + B-O separation: kinematical action can be used as a clock for the quantum subsystem, gives correct limits and unitary dynamics with gravitational corrections

▶ Small corrections but in principle detectable

Future perspectives:

▶ Application to scalar perturbations of the inflaton field

▶ Equivalence between kinematical action and (Gaussian) reference frame fixing procedure

Thank you for the attention!