The approach to equilibrium for idealized collisionless self-gravitating systems

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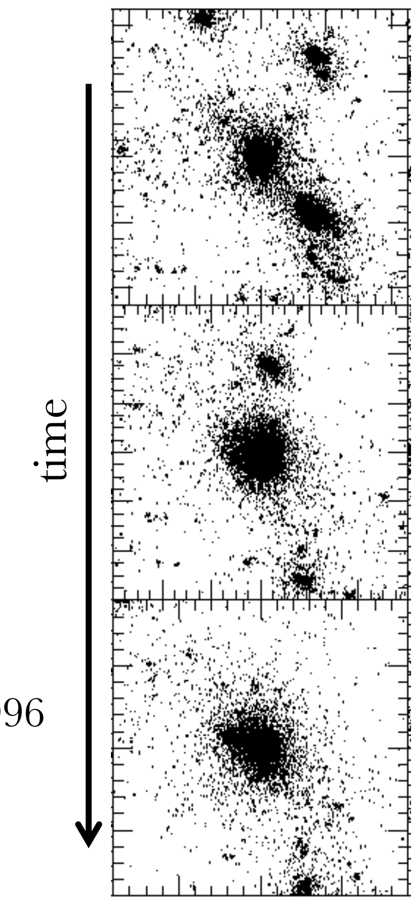
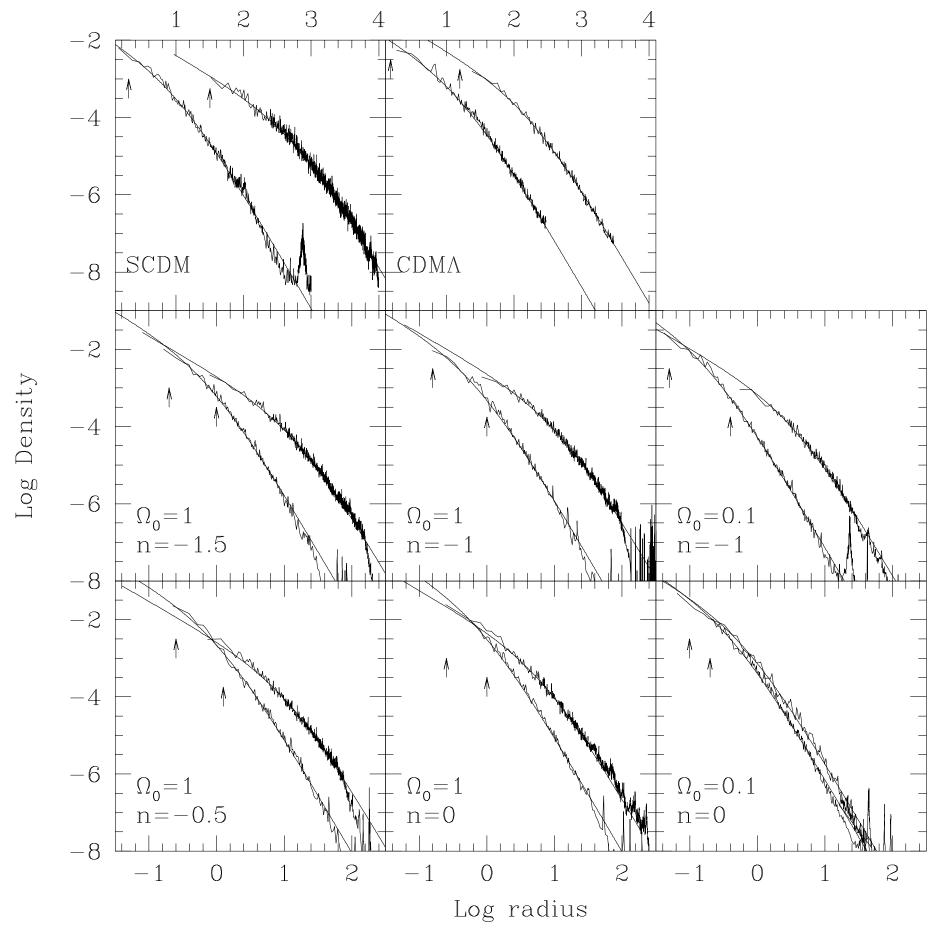
Introduction

Collisionless collapse of self-gravitating systems results in nearly universal mass density profiles of equilibrium halos

NFW

Dubinski & Carlberg 1991
Navarro, Frenk & White 1996
Navarro et al. 2004
Springel et al. 2008
Boylan-Kolchin et al. 2009
Vogelsberger et al. 2014





Navarro, Frenk & White 1997

Introduction

Theoretical explanation of this density profile shape and its universality have been more challenging.

Many attempts were made:

Ogorodnikov 1957
Lynden-Bell 1967
Shu 1978
Madsen 1987
Stiavelli & Bertin 1987
Spergel & Hernquist 1992
Hjorth & Madsen 1991, 1993
Plastino & Plastino 1993
White & Narayan 1987
MacMillan et al. 2006
Kang & He 2011
Barnes & Williams 2012
Pontzen & Governato 2013
Beraldo e Silva et al. 2014, 2017
Levin & Pakter 2014
Chavanis et al. 2015
Wagner 2020

Many authors have noted that

- ❖ large particle number suggests the use of statistical mechanics
- ❖ universality of the equilibrium state may correspond to the most likely state

→ use maximum entropy

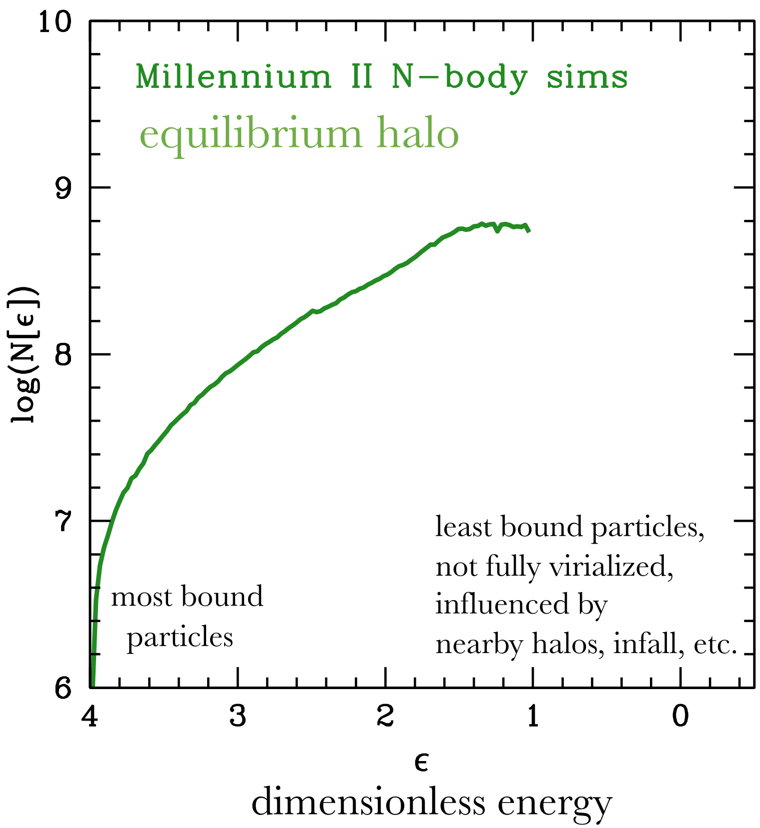
Stat. mech. → energy distribution, n(E)

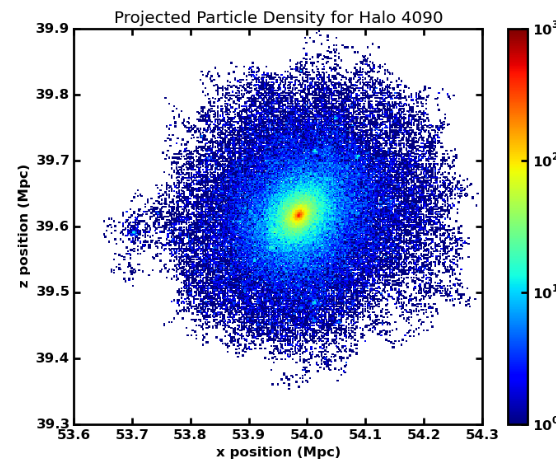
Density profiles are important,
but in statistical mechanics we work with n(E), where E = kinetic + grav. potential

Why n(E)=f(E)g(E), and not the distribution function f(E)?

In an equilibrium collisionless system each particle has a fixed energy
Differential energy distribution, n(E) gives the full description of a system

A successful theory must reproduce n(E) of equilibrium N-body halos





x-y projection of one equilibrium Millennium II halo

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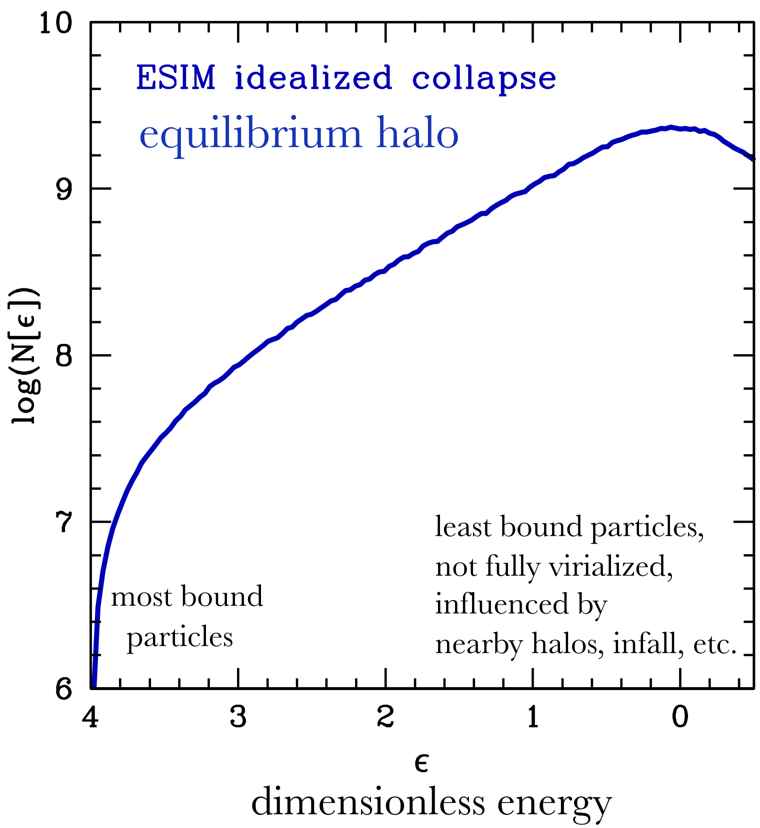
We use a simplified collapse model *(N-body simulations have complex dynamics)*



Extended Secondary Infall Model
(**ESIM**, based on Ryden & Gunn (1987)

-- mean-field model
-- spherically symmetric
-- radial forces only
-- 3D phase-space (1 spatial + 2 velocity)
-- radial, angular actions conserved

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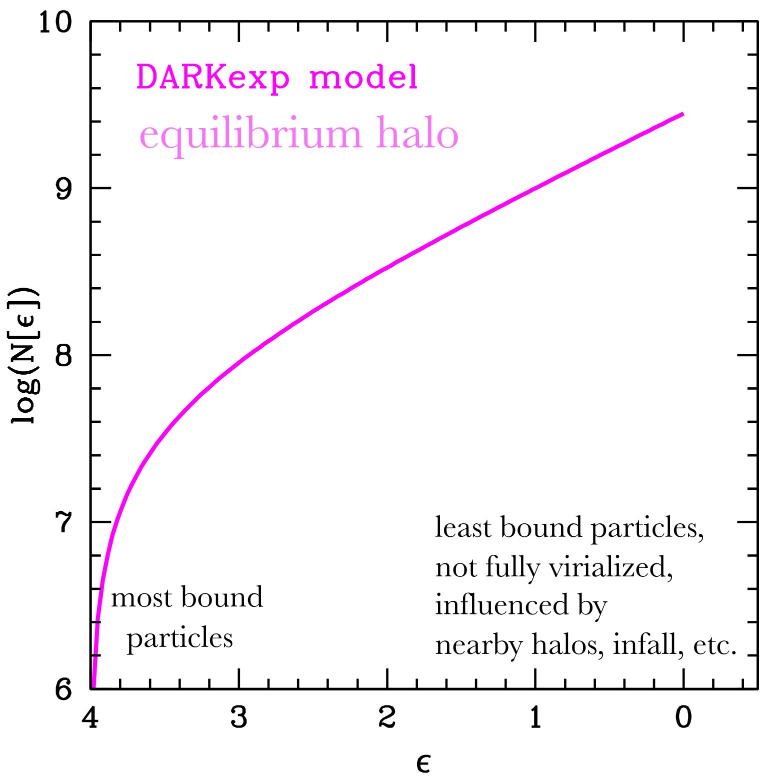
n(E) of our theoretical model, DARKexp



dimensionless energy

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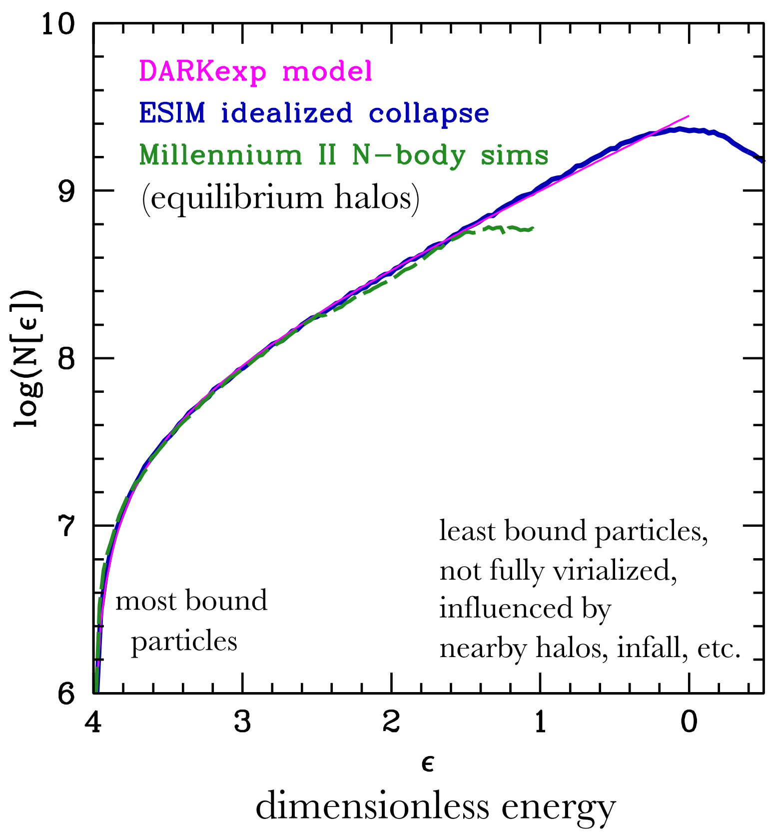
All 3: N-body sims, ESIM simplified collapse, DARKexp model



DARKexp fits equilibrium states of N-body simulated halos and ESIM very well (also density and velocity dispersion profiles)

DARKexp entropy S_D derived

Entropy is related to the # microstates corresponding to a given macrostate

$$W = N! \prod_j \frac{g^{n_j}}{n_j!}$$

fully captured by n(E)

want to derive its shape

n – occupation # of cells in energy space
N – total # of particles
g – degeneracy / cell size
j – energy level label

$$\ln W = \cancel{\ln N!} + \cancel{\sum_j n_j \ln g} - \sum_j \ln\Gamma(n_j+1)$$

normalization irrelevant

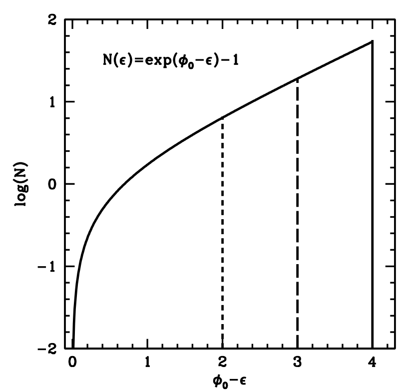
To find the most likely state with a fixed total energy we define ***entropy:***

$$S_D = \ln W - \beta \sum_j n_j E_j \Longrightarrow \boxed{S_D = -\int \ln\Gamma(n(E)+1)\,dE - \beta\int n(E)\,E\,dE}$$

S_D should attain a maximum at equilibrium $\frac{dS_D}{dn} = 0$ $\quad \frac{d^2S_D}{dn^2} < 0$

$$\boxed{n(E) \propto e^{-\beta(E-E_0)} - 1}$$

DARKexp



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S_D *should increase with time, as halo evolves*

$$\boxed{n(E) \propto e^{-\beta(E-E_0)} - 1}$$

DARKexp

DARKexp entropy S_D increases with time

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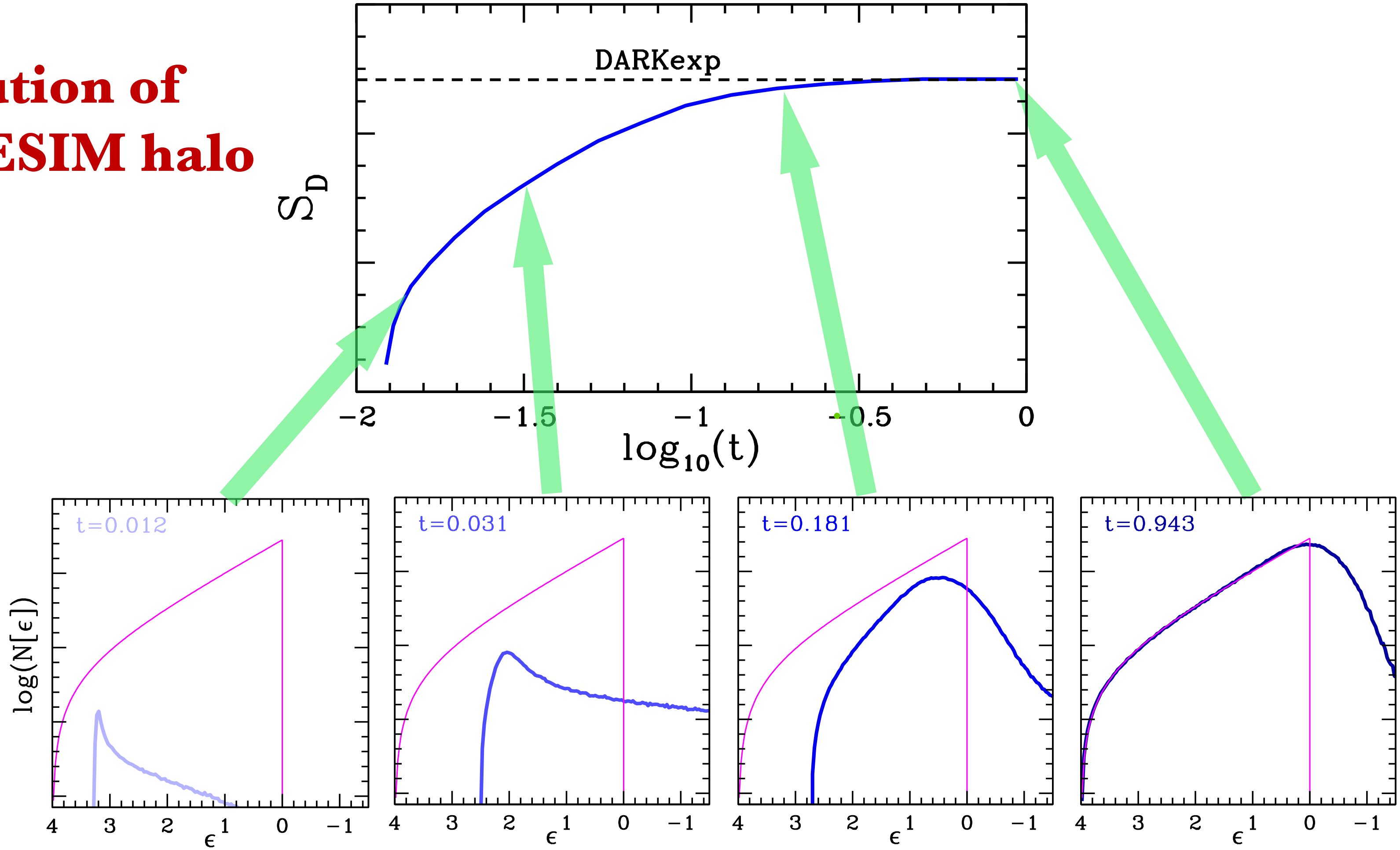
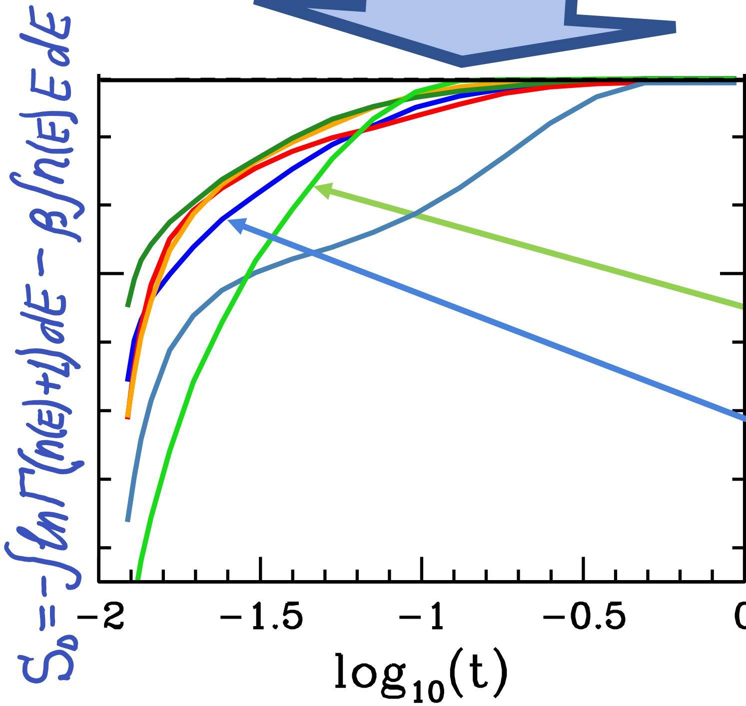
evolution of one ESIM halo

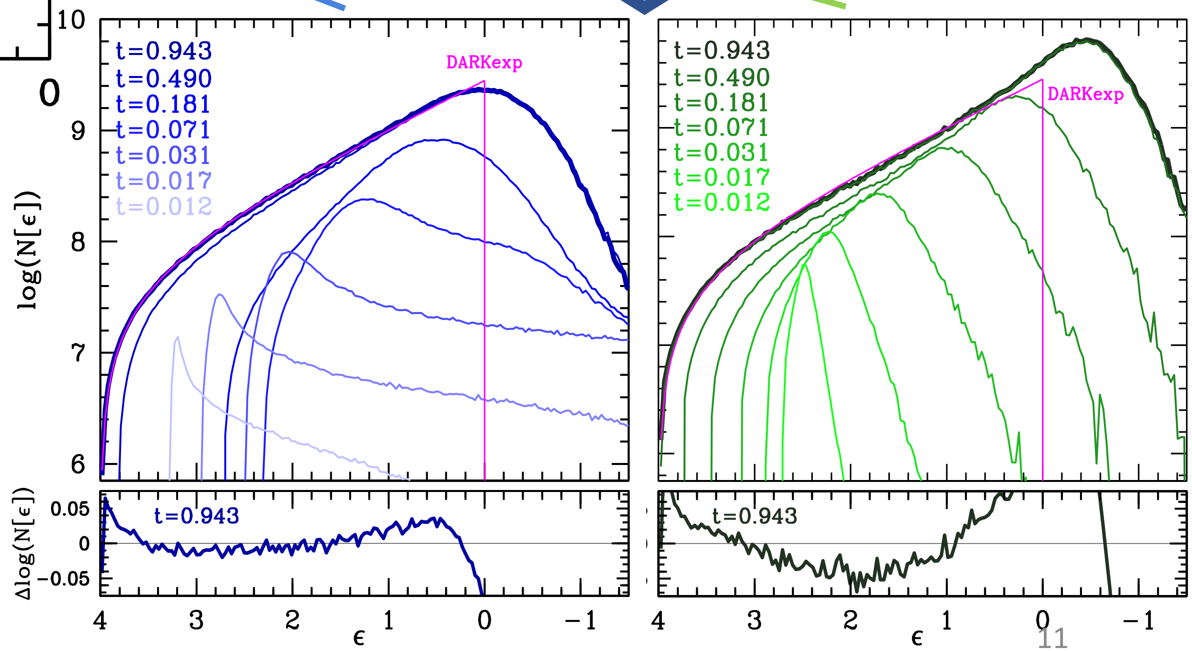


ESIM n(E) shape approaches that of DARKexp

Entropy S_D increases with time



Shape of n(E) approaches DARKexp



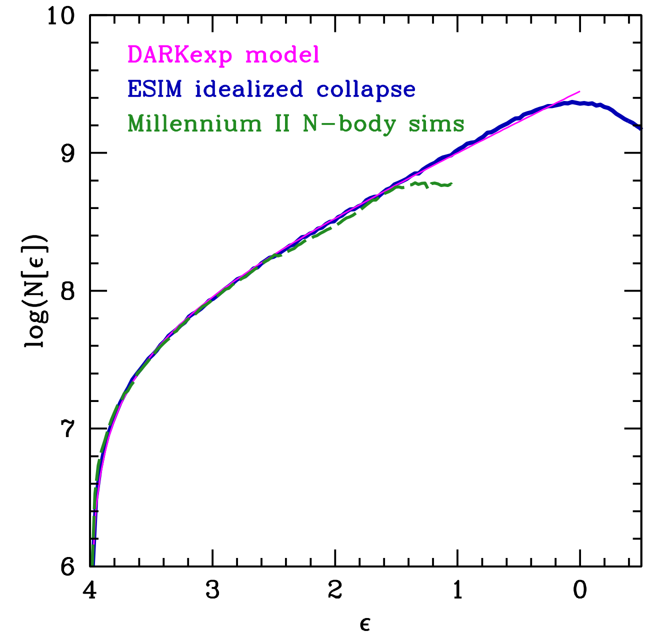
many ESIM halos with different initial conditions

Conclusions

DARKexp:

Derived using principles of statistical mechanics
Derivation is straightforward
(with 1 conceptual difference from Maxwell-Boltzmann stats)
Energy distribution n(E) has no shape parameters,
only a truncation near unbound energies

$$n(E) \propto e^{-\beta(E-E_0)} - 1$$



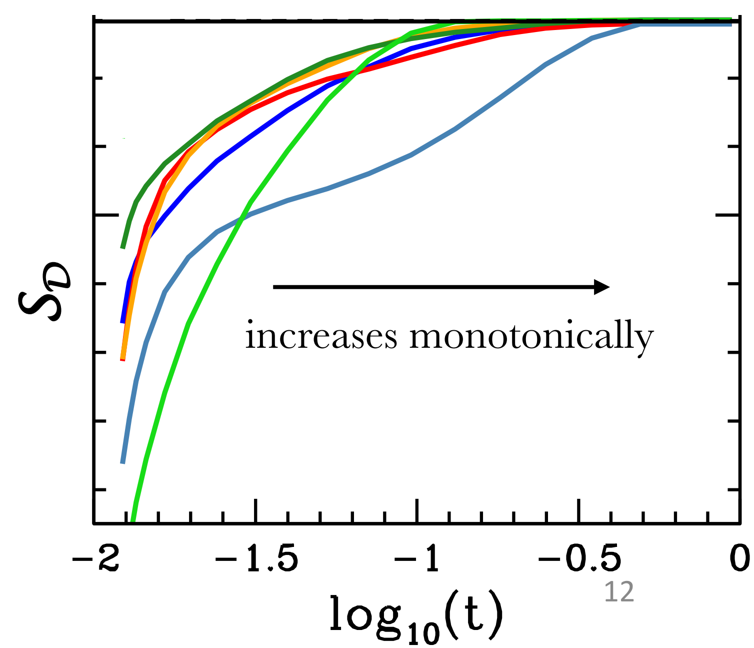
Reproduces n(E), density & velocity dispersion profiles of N-body sims and simplified collapse of ESIM halos

Entropy S_D

$$S_D = -\int \ln\Gamma(n(E)+1)\,dE - \beta\int n(E)\,E\,dE$$

increases monotonically during evolution of ESIM halos and attains a maximum at equilibrium

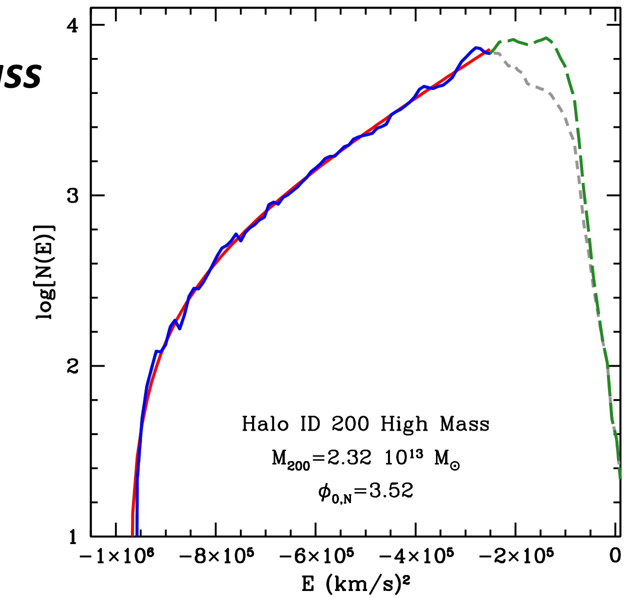
→ ***self-consistent theory***



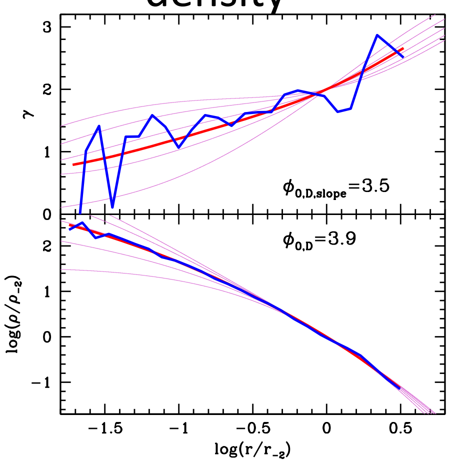
EXTRA SLIDES

Individual Millennium-II halos fitted with DARKexp

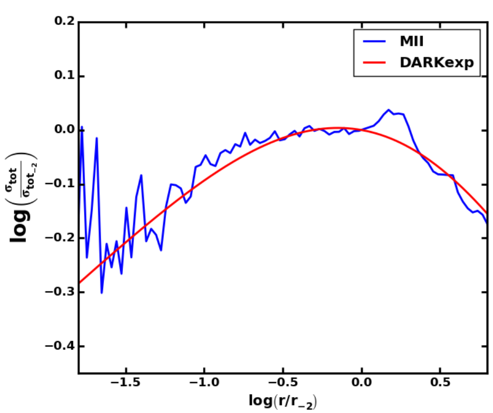
High mass



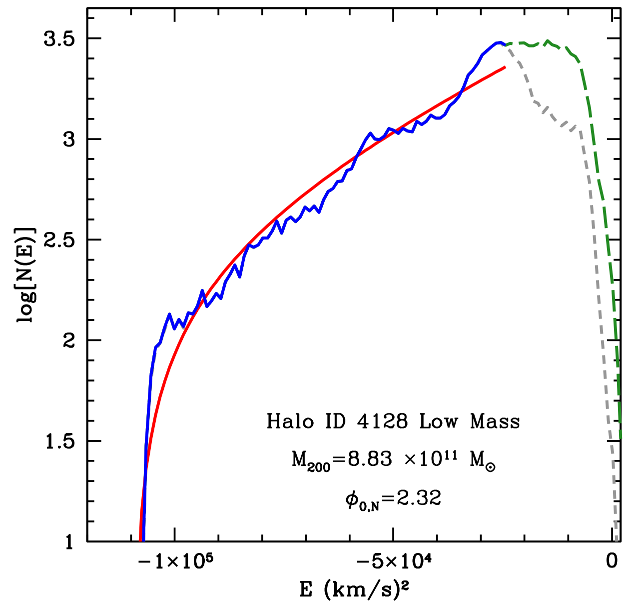
density



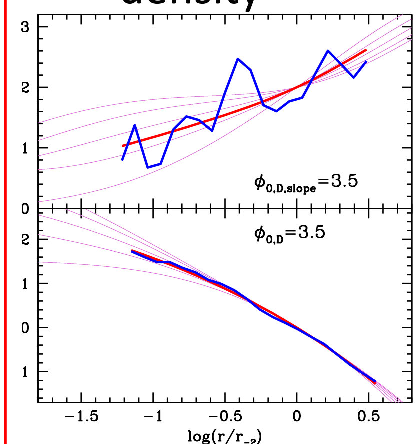
velocity dispersion



Low mass



density



velocity dispersion