Perfect Fluid Warp Drive Solutions with the Cosmological Constant #1000 (talk)

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Abstract:

In this work we examine solutions of the Einstein equations with the Alcubierre warp drive geometry having different matter and field sources. The Alcubierre metric describes a spacetime geometry that allows a massive particle inside a spacetime distortion, called warp bubble, to travel with superluminal global velocities. We advance solutions of the Einstein equations with the cosmological constant for the Alcubierre warp drive metric having the perfect fluid as source and considered the particular case of non interacting particles content, or dust, with the cosmological constant, which generalizes our previous dust solution which led to vacuum solutions connecting the warp drive with shock waves via the Burgers equation.

Motivation

- The original warp drive metric was proposed, there were no solutions for the Einstein equations using the warp drive metric as choice of geometry.
- The original warp drive metric does not satisfy the weak and dominant energy conditions everywhere.
- The hypothesis that the warp drive requires negative energy densities and subsequently negative matter density for it to be a possibility.
- The possibility of warp drive with other geometry, linearized conditions and physical warp drives.

Alcubierre (1994), Krasnikov (1995), Lobo & Visser (1994), Van Den Broeck (1999), Bobrick & Martire (2021)
full references in Santos-Pereira et al. (2020, 2021)

Warp Drive Theory (Alcubierre Warp Drive Metric)

Warp Drive Metric	$ds^2 = -(1-\beta^2)dt^2 - 2\beta(r_s, t)dx\,dt + dx^2 + dy^2 + dz^2$ Alcubierre (1994)
Shift Vector	$\beta = -v_s(t) f\left[r_s(t)\right]$ $\quad v_s(t) = \frac{\mathrm{d}x_s(t)}{\mathrm{d}t}$
Shape Function	$f(r_s) = \frac{\tanh\left[\sigma(r_s + R)\right] - \tanh\left[\sigma(r_s - R)\right]}{2\tanh(\sigma R)}$

Warp Drive Theory

Spacetime Foliation (3+1 Formalism)

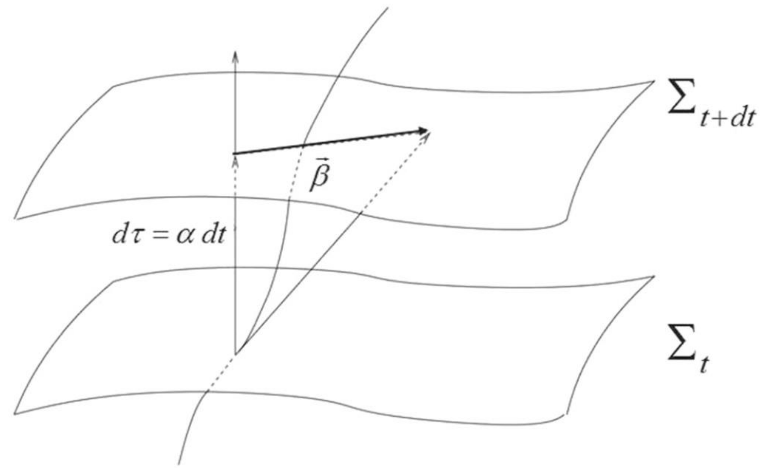
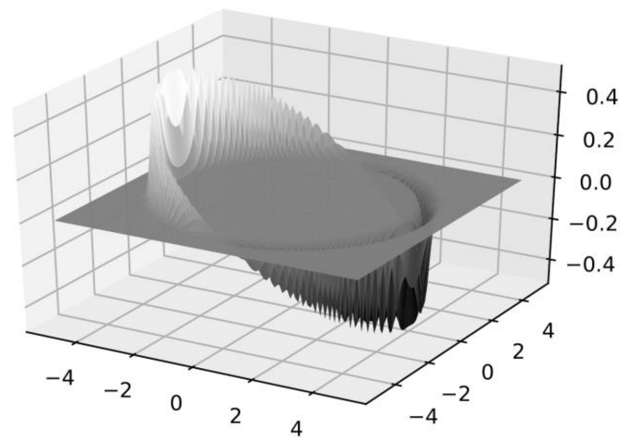


Figure taken from O. L. Santos-Pereira, E. M. C. Abreu and M. B. Ribeiro, Dust content content solutions for the Alcubierre warp drive spacetime. Eur. Phys. J. C, 80 (2020) 786, arXiv:2008.06560.

The shift vector (β) relates spatial coordinates between the foliated hypersurfaces.

The lapse function (α) is the proper time interval between consecutive hypersurfaces.

Warp Bubble Shape Function



$$f(r_s) = \frac{\tanh\left[\sigma\left(r_s + R\right)\right] - \tanh\left[\sigma\left(r_s - R\right)\right]}{2\tanh(\sigma R)}$$

The function dictates the shape of the bubble via the calculation of the expansion volume which is directly proportional to the derivative of the function $f(r_S)$. The parameters σ is proportional to the thickness of the bubble and R is proportional to the radius of the bubble.

Violation of Energy Conditions

$$T_{\alpha\beta}\, u^{\alpha} u^{\beta} = -\frac{v_s^2}{32\pi}\left[\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right]$$

Specific region of spacetime Where the energy conditions are not violated leads to vacum solutions connecting warp drive to shock waves. This will be discussed later on further slides.

Santos-Pereira et al. (2020,2021)

Simple Energy Momentum Distributions

$$T_{\alpha\beta} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Dust

$$T_{\alpha\beta} = \begin{pmatrix} \mu + \beta^2 p & -\beta p & 0 & 0 \\ -\beta p & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Perfect Fluid

$$T_{\alpha\sigma} = \begin{pmatrix} \mu + \beta^2 p & -\beta D & 0 & 0 \\ -\beta D & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & C \end{pmatrix}$$

Anisotropic Fluid

Santos-Pereira et al. (2020,2021)

$$T_{\alpha\nu} = \begin{pmatrix} 4\pi\mu + \frac{1}{2}(E^2 + B^2) & -S_1 & -S_2 & -S_3 \\ -S_1 & -\sigma_{11} & -\sigma_{12} & -\sigma_{13} \\ -S_2 & -\sigma_{21} & -\sigma_{22} & -\sigma_{23} \\ -S_3 & -\sigma_{31} & -\sigma_{32} & -\sigma_{33} \end{pmatrix}$$

Charged Dust

Perfect Fluid and Cosmological Constant Energy Density (positive vs negative valued)

Non-positive energy density	$T_{\alpha\beta}\, u^\alpha u^\beta = -\frac{v_s^2}{32\pi}\left[\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right]$
Energy density with Λ (can be positive)	$T_{\alpha\beta}\, u^\alpha u^\beta = \Lambda - \frac{v_s^2}{32\pi}\left[\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right]$

Santos-Pereira et al. (2020,2021)

First Set of Solutions for the Perfect Fluid with Cosmological Constant

Case	Condition	Results
1) $\frac{\partial \beta}{\partial z} = 0$	1a) $\frac{\partial \beta}{\partial x} = 0$	$\Lambda = 6\pi\left(\mu - \frac{p}{3}\right)$ **State equation that relates the cosmological constant to the dust and the perfect fluid** $\beta = \beta(y, t)$ $\frac{\partial \beta}{\partial y} = \pm\sqrt{4(\Lambda - 8\pi\mu)}$ $\frac{\partial \beta}{\partial y} = \pm\sqrt{\frac{4}{3}(\Lambda - 8\pi p)}$ $\beta\frac{\partial \mu}{\partial x} + \frac{\partial \mu}{\partial t} = 0$ (null divergence)
	1b) $\frac{\partial \beta}{\partial y} = 0$	$\Lambda = 8\pi\mu = 8\pi p = 0$ **Condition for vacum solutions** $\beta = \beta(x, t)$ $\frac{\partial \beta}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}(\beta^2) = h(t)$ **Burgers equation** null divergence is trivially satisfied.

Santos-Pereira et al. (2020,2021)

Second Set of Solutions for the Perfect Fluid with Cosmological Constant

Case	Condition	Results
2) $\frac{\partial \beta}{\partial y} = 0$	2a) $\frac{\partial \beta}{\partial x} = 0$	$\Lambda = 6\pi\left(\mu - \frac{p}{3}\right)$ **State equation that relates the cosmological constant to the dust and the perfect fluid** $\beta = \beta(y,t)$ $\frac{\partial \beta}{\partial z} = \pm\sqrt{4(\Lambda - 8\pi\mu)}$ $\frac{\partial \beta}{\partial z} = \pm\sqrt{\frac{4}{3}(\Lambda - 8\pi p)}$ $\beta\frac{\partial \mu}{\partial x} + \frac{\partial \mu}{\partial t} = 0$ (null divergence)
	2b) $\frac{\partial \beta}{\partial z} = 0$	$\Lambda = 8\pi\mu = 8\pi p = 0$ **Condition for vacum solutions** $\beta = \beta(x,t)$ $\frac{\partial \beta}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}(\beta^2) = h(t)$ **Burgers equation** null divergence is trivially satisfied.

Santos-Pereira et al. (2020,2021)

Conclusions

- We found that the warp drive as geometry is connected to shock waves via Burgers equation as vacuum solutions for Einstein equations.
- The original warp drive metric satisfies the weak and dominant energy conditions in a specific region of spacetime.
- The warp drive requirements of negative energy densities and negative matter density can be weakened with a convenient choice of energy momentum tensor with enough parameters as to balance the fact that the energy density component T_{00} is non-positive.

Santos-Pereira et al. (2020,2021)

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see Santos-Pereira et al. (2020, 2021) for a extended bibliography