# Modelling neutron-star mountains

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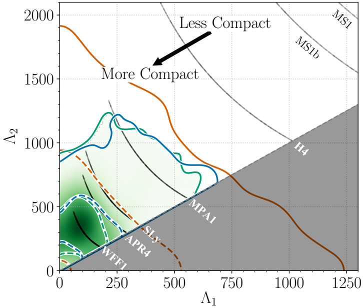
# Neutron stars as gravitational-wave sources

- The wait continues for a detection of gravitational radiation from rotating neutron stars with (non-axially symmetric) deformations known as *mountains*.
- Searches in the data have, so far, only provided upper limits on the size of the deformations.
- There have been theoretical attempts to estimate the *maximum* mountain that a neutron-star crust can support (Ushomirsky *et al.*, 2000; Haskell *et al.*, 2006; Johnson-McDaniel and Owen, 2013).




Marginalised posterior for the tidal deformabilities of the two binary components of GW170817 (Abbott *et al.*, 2018).

# Deformed stellar models

- Traditionally, one encounters the multipole moments, $Q_{\ell m}$, by examining how the exterior gravitational potential of a non-spherical body differs to that of a spherical one. In the Newtonian limit, this is given by

$$\delta\Phi(x^i) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \delta\Phi_{\ell m}(r) Y_{\ell m}(\theta, \phi), \quad \delta\Phi_{\ell m}(r) = -\frac{4\pi G}{2\ell+1} \frac{Q_{\ell m}}{r^{\ell+1}}$$

  for $r \geq R$, where

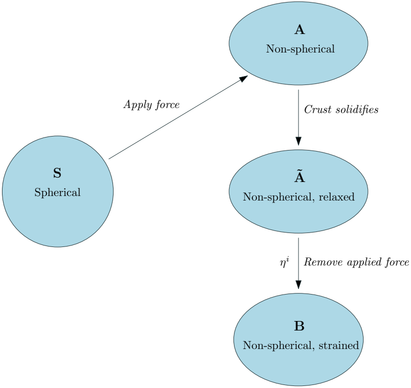
$$Q_{\ell m} \equiv \int_0^R \delta\rho_{\ell m}(r) r^{\ell+2} \, dr.$$

- The dominant multipole in gravitational-wave emission is the quadrupole moment, $Q_{22}$. Thus, we specialise to the $(\ell, m) = (2, 2)$ mode.
- Formally, one can use the law of momentum conservation – the Euler equation – to characterise a stellar model,

$$\begin{aligned} 0 &= -\nabla_i p - \rho \nabla_i \Phi + \nabla^j t_{ij} + f_i \\ &\equiv -H_i + \nabla^j t_{ij} + f_i, \end{aligned}$$

  where $t_{ij}$ is the trace-free, symmetric shear-stress tensor for an elastic solid and $f_i$ is a deforming force.

# Building mountains: the usual approach

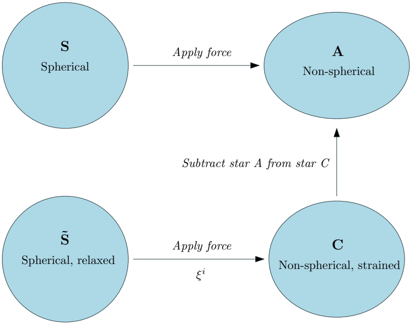



- It would seem that calculating mountains in the usual fashion, via the perturbed Euler equation

$$\delta H_i^{\mathrm{SB}} \equiv H_i^{\mathrm{B}} - H_i^{\mathrm{S}} = \nabla^j t_{ij}(\eta), \tag{1}$$

  has some complications regarding satisfying the boundary conditions of the problem.
- Additionally, in some sense, the fiducial force (which is a proxy for the possibly quite complicated evolutionary history!) is hidden in the calculation.
- To this end, we introduce another scheme which makes explicit use of the deforming force.

# A new mountain scheme



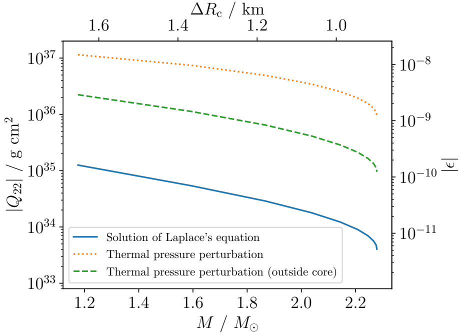

- The difference between stars C and A gives

$$\delta H_i^{\text{AC}} = \nabla^j t_{ij}(\xi). \qquad (2)$$

- It turns out that the mountains calculated using (2) are equivalent to solving the previous perturbed Euler equation (1).

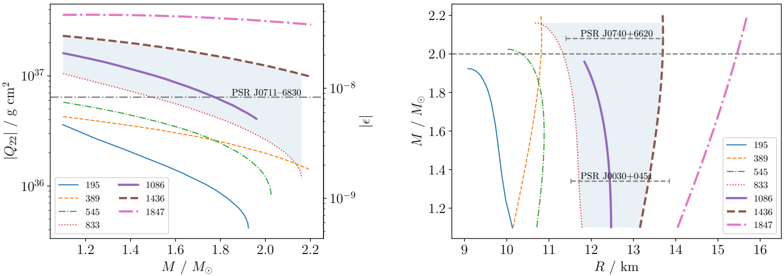
# Examples of the deforming force

- We generated a set of fully relativistic neutron-star models (with a realistic equation of state) that were subjected to a few specific deforming forces. The amplitude of the force on each star was increased until the crust began to fracture. This produced the maximum mountain that each star could support for a given force.




The maximum quadrupole and ellipticity due to the different forces as functions of stellar mass, $M$, and crustal thickness, $\Delta R_c$.

- We also considered the role of the equation of state in supporting the mountains, by implementing a subset of equations of state obtained from chiral effective field theory with a speed-of-sound parametrisation (Tews *et al.*, 2018).




The maximum quadrupole and ellipticity due to thermal pressure perturbations as functions of stellar mass for different chiral effective-field-theory equation-of-state models (left panel) and the corresponding mass-radius diagram traced out by the background stellar models (right panel).

# Summary & future work

- We introduce a new scheme to calculate mountains that explicitly satisfies the necessary boundary conditions. However, this scheme requires the introduction of a deforming force.
- Such a force will depend on the (possibly quite complex) formation history of the star. For this reason, we believe that evolutionary calculations will be necessary to make progress on this problem (Bildsten, 1998; Singh *et al.*, 2020; Osborne and Jones, 2020).
- The neutron-star equation of state plays an important role in supporting the mountains. In particular, the shear modulus of the crust (unsurprisingly) has a significant impact on how large the mountains can be.
- More accurate descriptions of the neutron-star crust may need to take into account plastic deformation.

# References


G. Ushomirsky, C. Cutler, and L. Bildsten, Deformations of accreting neutron star crusts and gravitational wave emission, Mon. Not. R. Astron. Soc. **319**, 902 (2000), astro-ph/0001136.

B. Haskell, D. I. Jones, and N. Andersson, Mountains on neutron stars: accreted versus non-accreted crusts, Mon. Not. R. Astron. Soc. **373**, 1423 (2006), arXiv:astro-ph/0609438 [astro-ph].

N. K. Johnson-McDaniel and B. J. Owen, Maximum elastic deformations of relativistic stars, Phys. Rev. D **88**, 044004 (2013), arXiv:1208.5227 [astro-ph.SR].

B. P. Abbott *et al.*, GW170817: Measurements of Neutron Star Radii and Equation of State, Phys. Rev. Lett. **121**, 161101 (2018), arXiv:1805.11581 [gr-qc].

I. Tews, J. Carlson, S. Gandolfi, and S. Reddy, Constraining the Speed of Sound inside Neutron Stars with Chiral Effective Field Theory Interactions and Observations, Astrophys. J. **860**, 149 (2018), arXiv:1801.01923 [nucl-th].

L. Bildsten, Gravitational Radiation and Rotation of Accreting Neutron Stars, Astrophys. J. Lett. **501**, L89 (1998), astro-ph/9804325.

N. Singh, B. Haskell, D. Mukherjee, and T. Bulik, Asymmetric accretion and thermal ‘mountains’ in magnetized neutron star crusts, Mon. Not. R. Astron. Soc. **493**, 3866 (2020), arXiv:1908.05038 [astro-ph.HE].

E. L. Osborne and D. I. Jones, Gravitational waves from magnetically induced thermal neutron star mountains, Mon. Not. R. Astron. Soc. **494**, 2839 (2020), arXiv:1910.04453 [astro-ph.HE].