Eddington Gravity with Matter

A Thermodynamic perspective

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Outline

- Proposed an alternative action in the spirit of Eddington gravity from which the Einstein's equations with matter can be derived.
- The action has a path-integral description, which follows from thermodynamic nature of gravity.

References

SC and Padmanabhan, PRD 103, 064033 (2021)

SC and Padmanabhan, PRD 101, 064023 (2020)

SC and Padmanabhan, PRD 92, 104011 (2015)

Eddington Gravity

- The action for Eddington gravity is taken to be

$$\mathcal{A} = \int d^4x \sqrt{|\det(R_{(ab)})|}$$

[Eddington, The Mathematical Theory of Relativity (Cambridge)]

- The Ricci tensor R_{ab} is a function of Γ^a_{bc}
- Variation of this action yields, $R_{ab} + \Lambda g_{ab} = 0$.
- Inclusion of matter is problematic, since matter Lagrangian (generally) does not involve Γ^a_{bc}.

Alternative Proposals

- The proposals to include matter is to think of the action as $\mathcal{A} = \mathcal{A}[\Gamma, \Psi, g]$.
- Setting $(\delta\mathcal{A}/\delta g) = 0$, yields $g = g(\Gamma, \Psi)$.
- Substitution in the action itself, becomes $\mathcal{A}[\Gamma, \Psi]$.
- Variation yields the field equations for gravity and matter, differing from Einstein's equations.

[Banados and Ferreira, arXiv: 1006.1769]

[Vollick, PRD 72, 084026 (2005)]

[Delsate and Steinhoff, arXiv:1201.4989]

Eddington gravity with matter

- The following action is proposed

$$\mathcal{A} = \int d^dx \sqrt{\left|\text{det.}\left(R_{(ab)}(\Gamma) - \kappa \bar{T}_{ab}\right)\right|} \; ; \qquad \bar{T}_{ab} = T_{ab} - \frac{1}{(d-2)} T g_{ab} \; ,$$

[SC and Padmanabhan, PRD 103, 064033 (2021)]

- The Ricci tensor depends on the connection alone, $\bar{T}_{ab}$ depends on metric and matter degrees of freedom.

Variation of the Action

- First we map the components of $M_{ab} \equiv R_{(ab)} - \kappa \bar{T}_{ab}$ to that of a matrix $\mathcal{M}^a{}_b$.
- Define an inverse matrix $\mathcal{N}^a_b$, so that $\mathcal{N}^a{}_b \mathcal{M}^b{}_c = \delta^a_c = \mathcal{M}^a{}_b \mathcal{N}^b{}_c$.
- The inverse matrix is mapped to a tensor N^{ab} .

$$
\begin{aligned}
\delta \mathcal{A} &= \frac{1}{2} \int d^d x \, \sqrt{|\mathrm{det.}\,(M)|} \; N^{ba} \left(\nabla_c \delta \Gamma^c_{ab} - \nabla_a \delta \Gamma^c_{bc} \right) \\
&= \frac{1}{2} \int d^d x \, \sqrt{|\mathrm{det.}\,(M)|} \; \delta^{cd}_{ab} \; N^{pa} \nabla_d \delta \Gamma^b_{cp} \; ; \qquad \delta^{ab}_{cd} \equiv \delta^a_c \delta^b_d - \delta^a_d \delta^b_c \; ,
\end{aligned}
$$

Well-posed Boundary value problem

- Einstein-Hilbert action leads to an ill-posed boundary value problem. [Parattu, SC, Majhi and Padmanabhan, GRG 48, 94 (2016)]

[SC and Padmanabhan, PRD 101, 064023 (2020)]

- In this case,

$$\delta\mathcal{A}=\frac{1}{2}\int d^dx\;\partial_d\left[\sqrt{|\mathrm{det.}\,(M)|}\;\delta^{cd}_{ab}\;N^{pa}\delta\Gamma^b_{cp}\right]-\frac{1}{2}\int d^dx\;\nabla_d\left[\sqrt{|\mathrm{det.}\,(M)|}\;\delta^{cd}_{ab}\;N^{pa}\right]\delta\Gamma^b_{cp}\,.$$

- Boundary value problem is well-posed.

Field Equations

- Setting the variation of the action to be zero, we obtain

$$\nabla_c \left[\sqrt{|\text{det.}(M)|} N^{ab} \right] = 0 \ .$$

- The solution being,

$$G_{ab} + \left(\frac{d-2}{2} \right) \lambda g_{ab} \equiv G_{ab} + \Lambda g_{ab} = \kappa T_{ab} \ .$$

- Bianchi identity $\nabla_a G^a_b = 0$ guarantees that $\nabla_a T^a_b = 0$. This yields the matter field equations.

Comments on Result

- Cosmological constant appears naturally.
- We only varied the connection to get the field equations for gravity. The matter equations follow from Bianchi identity.
- Metric behaves as a dynamical variable, without being dynamical.
- Eddington-type action can be constructed for Lovelock theories of gravity as well.

An Alternative Action

- There are other actions from which the Einstein's equations will follow. [SC and Padmanabhan, PRD 103, 064033 (2021)]

$$\mathcal{A}_{\rm eff} = \frac{1}{2}\int \frac{d^4x}{L^4}\sqrt{|\det.(g)|}\ \ln\left(|\det.[L^2(R_{ab}(\Gamma) - \kappa\bar{T}_{ab})]|\right) ,$$

- Here also the connection is the dynamical variable.

A PATH INTEGRAL ORIGIN

- There is a path integral origin of the action.

$$\int \mathcal{D}v^a \;\exp\left[-\int \frac{d^4x}{L^4}\sqrt{|\mathrm{det.}(g)|}\; v^a(L^2 M_{ab})v^b\right] \propto \exp\left[-\frac{1}{2}\int \frac{d^4x}{L^4}\sqrt{|\mathrm{det.}(g)|}\;\ln\left(|\mathrm{det.}(L^2 M_{ab})|\right)\right]$$
$$\propto \exp\left(-\mathcal{A}_{\mathrm{eff}}[M_{ab}]\right)\;,$$

- This suggests the following Lagrangian for gravity at a more fundamental level.

$$L_{\mathrm{grav}} = (R_{ab} - \kappa \bar{T}_{ab})v^a v^b$$

- This is the Lagrangian arising from thermodynamic description of gravity.

ORIGIN OF PATH INTEGRAL ACTION

- The term $\bar{T}_{ab}\ell^a\ell^b$ is the matter heat flux entering a null surface with null normal ℓ^a.
- The term $R_{ab}\ell^a\ell^b$ is related to the rate of change of heat content of the null surface.
- Thus extremization of the combination $(R_{ab}-\kappa\bar{T}_{ab})\ell^a\ell^b$, denotes the thermodynamic equilibrium and the appropriate action for path integral.

[SC and Padmanabhan, PRD 92, 104011 (2015)]

Conclusions

- Provided an alternative way of introducing matter in the Eddington gravity paradigm.
- Variation using the connection yields the Einstein's equations with cosmological constant and Bianchi identity yields the matter equations.
- Action in similar spirit can be obtained from a path integral prescription, with the more fundamental action being originated from thermodynamic description of gravity.

Thank You

Thermodynamic nature of gravity

- Gravitational dynamics is closely connected with the thermodynamic nature of gravity.
- Einstein's equations projected on a null hypersurface yields — (**a**) Damour-Navier-Stokes equations, (**b**) Thermodynamic identity $TdS = dE + PdV$ and (**c**) Clausius relation with dissipation terms.
- The Noether charge associated with diffeomorphism symmetry of general relativity is the heat content of the null boundary.