Teleparallel, f(T) and Torsional Gravity and Cosmology

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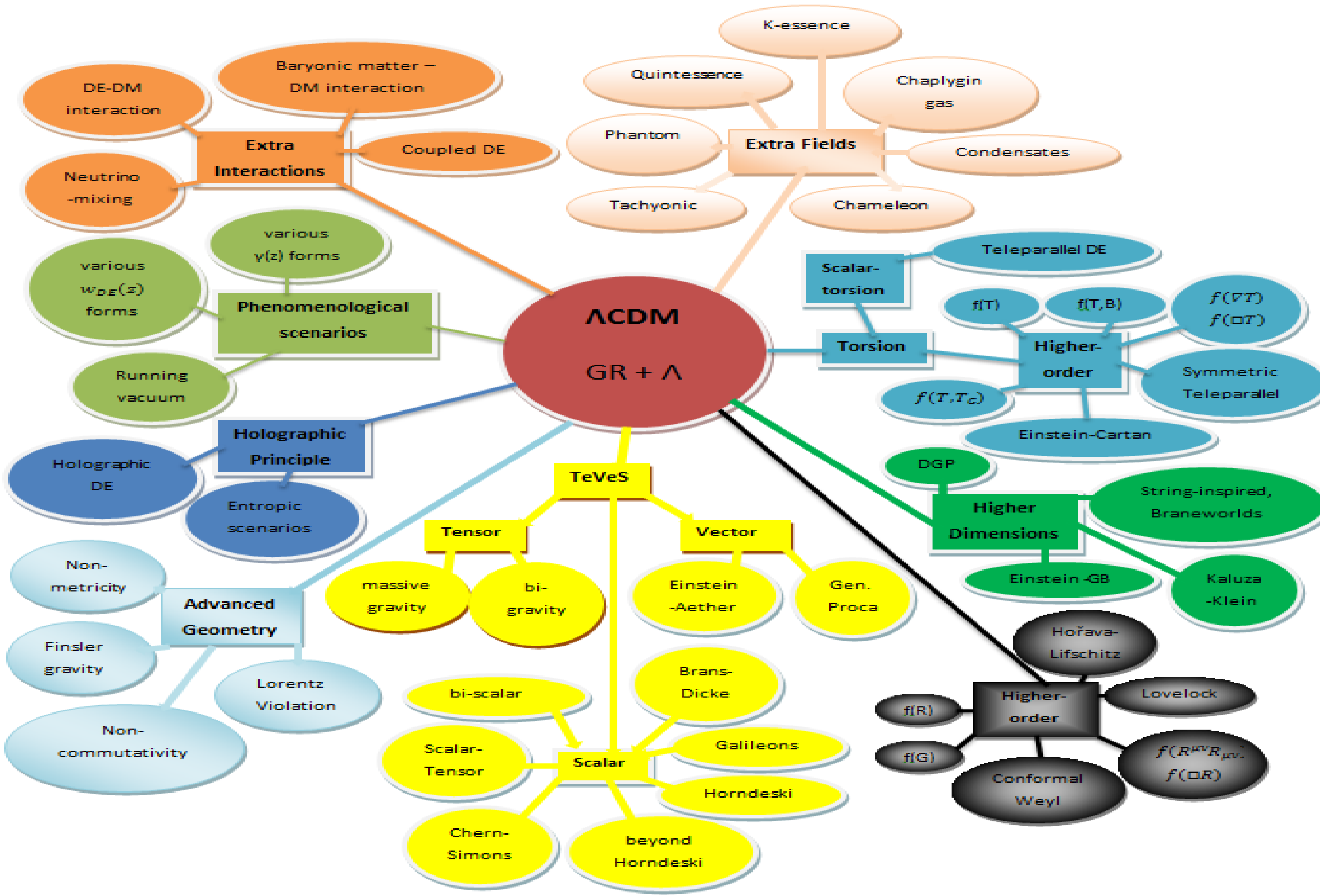
CANTATA
Cosmology and Astrophysics Network for Theoretical Advances and Training Actions

Goal

- We investigate cosmological scenarios arising from torsional modified gravity that can describe the observed Universe as a whole

Modified Gravity



Torsional Gravity

- Einstein 1916: **General Relativity**:
  energy-momentum source of spacetime Curvature
  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**
  Weitzenbock connection: Zero Curvature

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

Teleparallel Equivalent of General Relativity (TEGR)

- Let’s start from the simplest tosion-based gravity formulation, namely TEGR:
- Vierbeins e_A^μ : four linearly independent fields in the tangent space

$$g_{\mu\nu}(x) = \eta_{AB}\, e_\mu^A(x)\; e_\nu^B(x)$$

- Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: $\Gamma_{\nu\mu}^{\lambda\{W\}} = e_A^\lambda \partial_\mu e_\nu^A$
- Torsion tensor:

$$T_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda\{W\}} - \Gamma_{\mu\nu}^{\lambda\{W\}} = e_A^\lambda\left(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A\right)$$

[Einstein 1928], [Pereira: Introduction to TG]

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- Lagrangian (imposing coordinate, Lorentz, parity invariance, and up to 2nd order in torsion tensor)

$$L \equiv T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\rho} T_{\nu}^{\nu\mu}$$

- **Completely equivalent** with GR at the level of equations

[Einstein 1928], [Hayaski,Shirafuji PRD 19], [Pereira: Introduction to TG]

f(T) Gravity and f(T) Cosmology

- f(T) Gravity: Simplest torsion-based modified gravity
- Generalize T to f(T)   (inspired by f(R))

$$S=\frac{1}{16\pi G}\int d^4x\, e\,\left[T+f(T)\right]+S_m$$

[Ferraro, Fiorini PRD 78], [Bengochea, Ferraro PRD 79]
[Linder PRD 82]

- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^\rho S_\rho^{\ \mu\nu}\right)\left(1+f_T\right)-e_A^\lambda T^\rho_{\ \mu\lambda}S_\rho^{\ \nu\mu}+e_A^\rho S_\rho^{\ \mu\nu}\partial_\mu(T)f_{TT}-\frac{1}{4}e_A^\nu\left[T+f(T)\right]=4\pi G e_A^{\ \rho}\mathrm{T}_\rho^{\ \nu\{EM\}}$$

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- f(T) Cosmology: Apply in FRW geometry:

$$e_\mu^A = diag(1, a, a, a) \Rightarrow ds^2 = dt^2 - a^2(t)\delta_{ij} dx^i dx^j$$ (not unique choice)

- Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2 f_T H^2$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT}}$$

- Find easily

$$T = -6H^2$$

f(T) Cosmology: Background

- Effective Dark Energy sector:

$$\rho_{DE} = \frac{3}{8\pi G}\left[-\frac{f}{6}+\frac{T}{3}f_T\right]$$

$$w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1+f_T+2Tf_{TT}][f-2Tf_T]}$$

[Linder PRD 82]

- Interesting cosmological behavior: Acceleration, Inflation etc
- At the background level indistinguishable from other dynamical DE models

Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from $R+f(R)$ one can use $R+\xi R\varphi$^2
- Let's do the same in torsion-based gravity:

$$S=\int d^4x\, e\left[\frac{T}{2\kappa^2}+\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi+\xi T\varphi^2\right)-V(\varphi)+L_m\right]$$

[Geng, Lee, Saridakis, Wu PLB 704]

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- Friedmann equations in FRW universe:

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

with effective Dark Energy sector: $\rho_{DE}=\frac{\dot{\varphi}^2}{2}+V(\varphi)-3\xi H^2\varphi^2$

$$p_{DE}=\frac{\dot{\varphi}^2}{2}-V(\varphi)+4\xi H\varphi\dot{\varphi}+\xi\left(3H^2+2\dot{H}\right)\varphi^2$$

- **Different** than non-minimal quintessence!

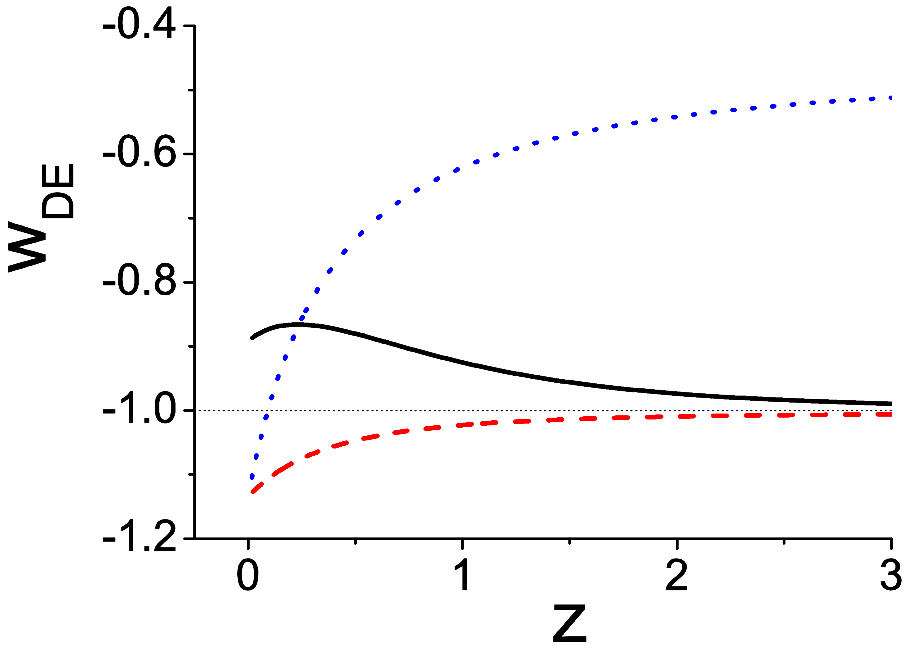
 (no conformal transformation in the present case)

[Geng, Lee, Saridakis,Wu PLB 704]

[Hohmann, Pfeifer, PRD 98]

Non-minimally coupled scalar-torsion theory

- Main advantage: Dark Energy may lie in the phantom regime or/and experience the phantom-divide crossing
- Teleparallel Dark Energy:



[Geng, Lee, Saridakis, Wu PLB 704]

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

- In curvature-based gravity, one can use higher-order invariants like the Gauss-Bonnet one $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}$
- Let's do the same in torsion-based gravity:
- Similar to $e\overline{R} = -eT + 2\left(eT_{\nu}^{\nu\mu}\right)_{,\mu}$ we construct $e\overline{G} = eT_G + tot.diverg$ with

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$$T_G = \left(K^{a_1}_{\ \ ea_2} K^{ea_2}_{\ \ b} K^{a_3}_{\ \ fc} K^{fa_4}_{\ \ d} - 2K^{a_1a_2}_{\ \ \ a} K^{a_3}_{\ \ eb} K^{e}_{\ fc} K^{fa_4}_{\ \ d} + 2K^{a_1a_2}_{\ \ \ a} K^{a_3}_{\ \ eb} K^{ea_4}_{\ \ f} K^{f}_{\ cd} + +2K^{a_1a_2}_{\ \ \ a} K^{a_3}_{\ \ eb} K^{ea_4}_{\ \ f} K^{f}_{\ c,d}\right)\delta^{abcd}_{a_1a_2a_3a_4}$$

- $f(T, T_G)$ gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, \{T + f(T, T_G)\} + S_m$$

[Kofinas, Saridakis, PRD 90a]

[Kofinas, Saridakis, PRD 90b]

[Kofinas, Leon, Saridakis, CQG 31]

- **Different** from $f(R,G)$ and $f(T)$ gravities

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

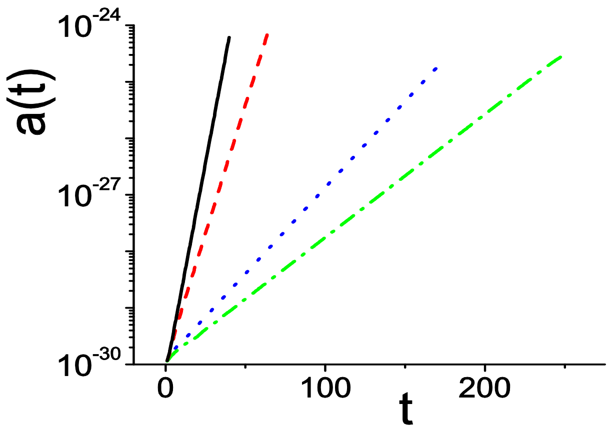
- Cosmological application:

$$\rho_{DE} = -\frac{1}{2\kappa^2}\left[f - 12H^2 f_T - T_G f_{T_G} + 24H^3 \dot{f}_{T_G}\right]$$

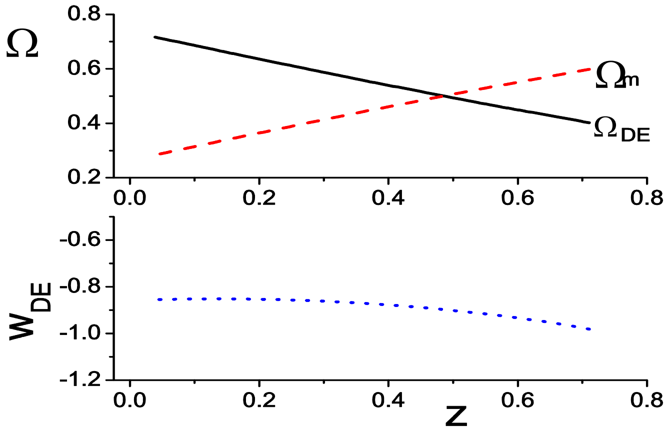
$$p_{DE} = \frac{1}{2\kappa^2}\left[f - 4\left(\dot{H} + 3H^2\right) f_T - 4H\dot{f}_T - T_G f_{T_G} + \frac{2}{3H} T_G \dot{f}_{T_G} + 8H^2 \ddot{f}_{T_G}\right]$$

$$T = 6H^2$$

$$T_G = 24H^2\left(\dot{H} + H^2\right)$$



$f(T,T_G) = \alpha_1 T^2 + \alpha_2 T\sqrt{|T_G|}$



$f(T,T_G) = \beta_1\sqrt{T^2 + \beta_2 T_G}$

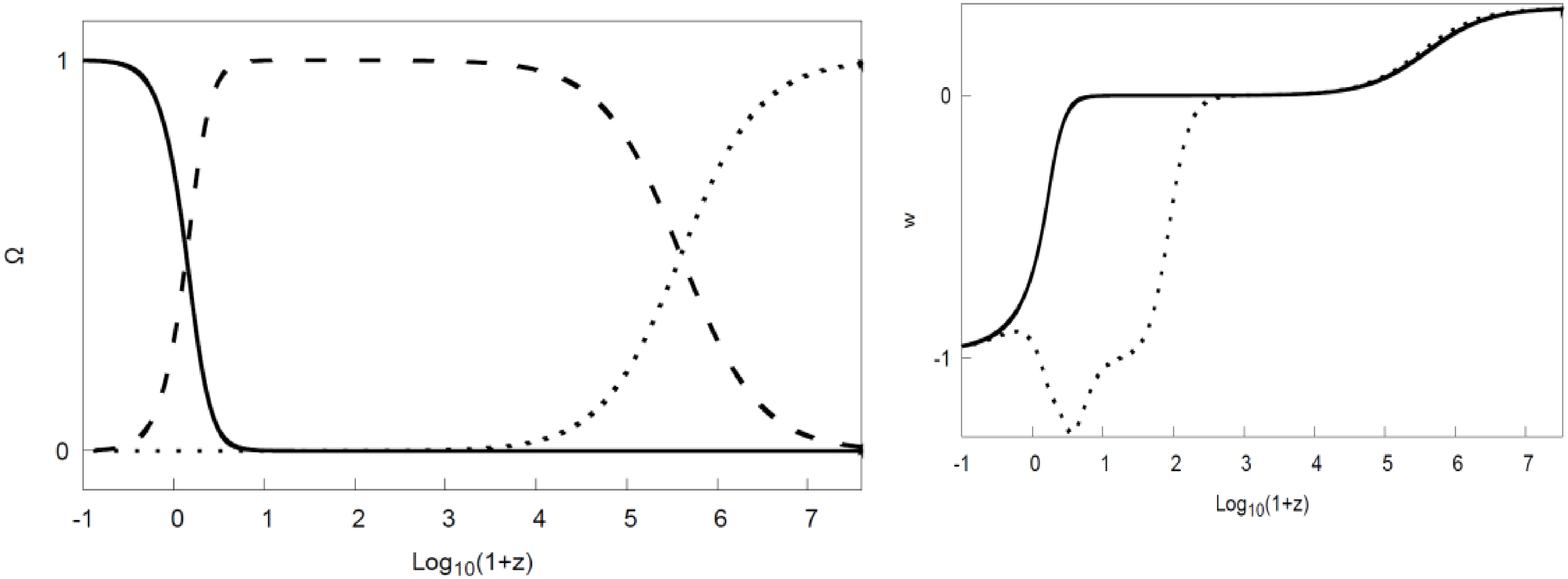
[Kofinas, Saridakis, PRD 90a]

[Kofinas, Saridakis, PRD 90b]

[Kofinas, Leon, Saridakis, CQG 31]

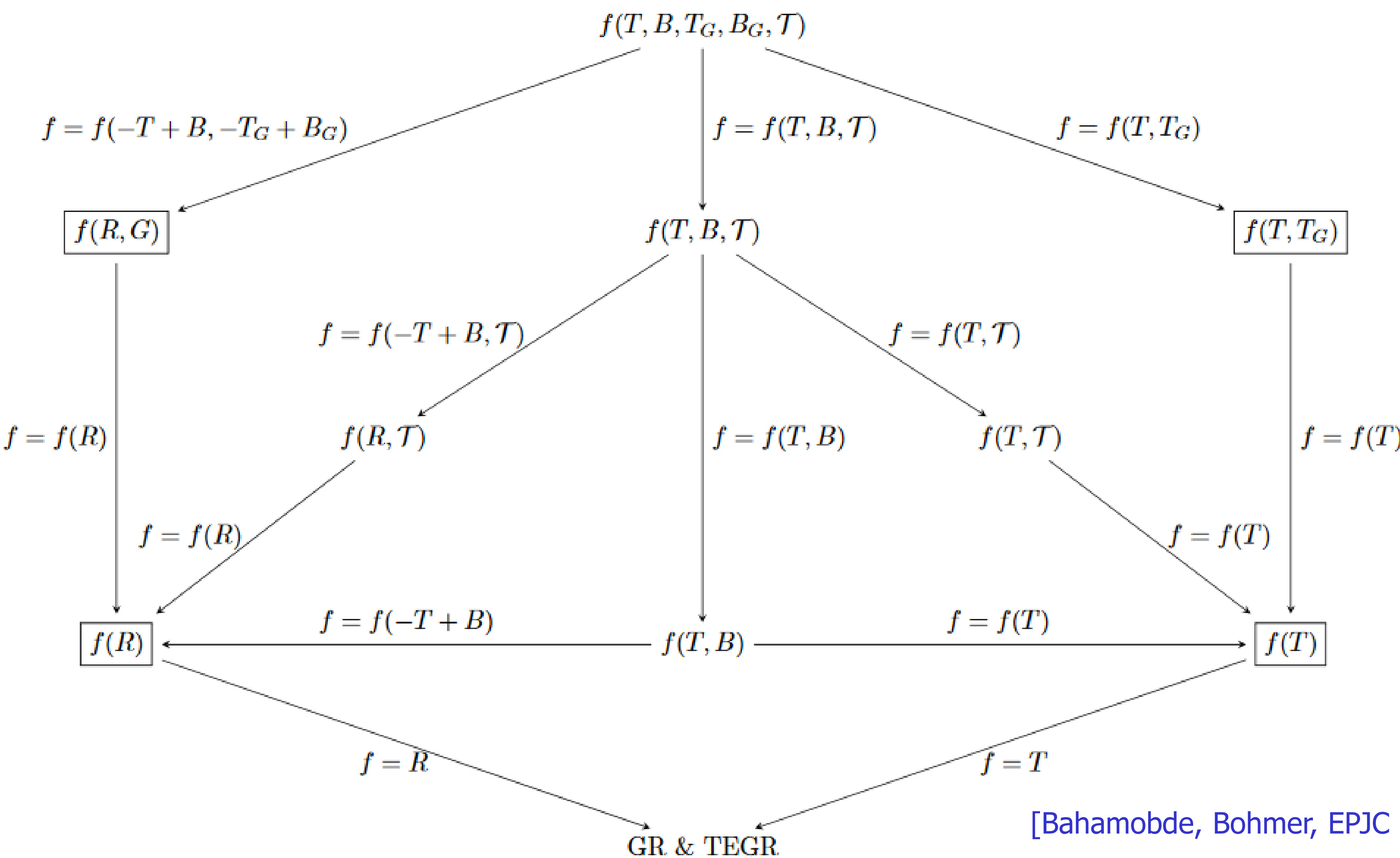
Torsional Gravity with higher derivatives

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, F\big(T, (\nabla T)^2, \Diamond T\big) + S_m(e^A_\mu, \Psi_m)$$



[Otalora, Saridakis, PRD 94]

Torsional Modified Gravity



[Bahamobde, Bohmer, EPJC 76]

Metric-Affine Modified Gravity

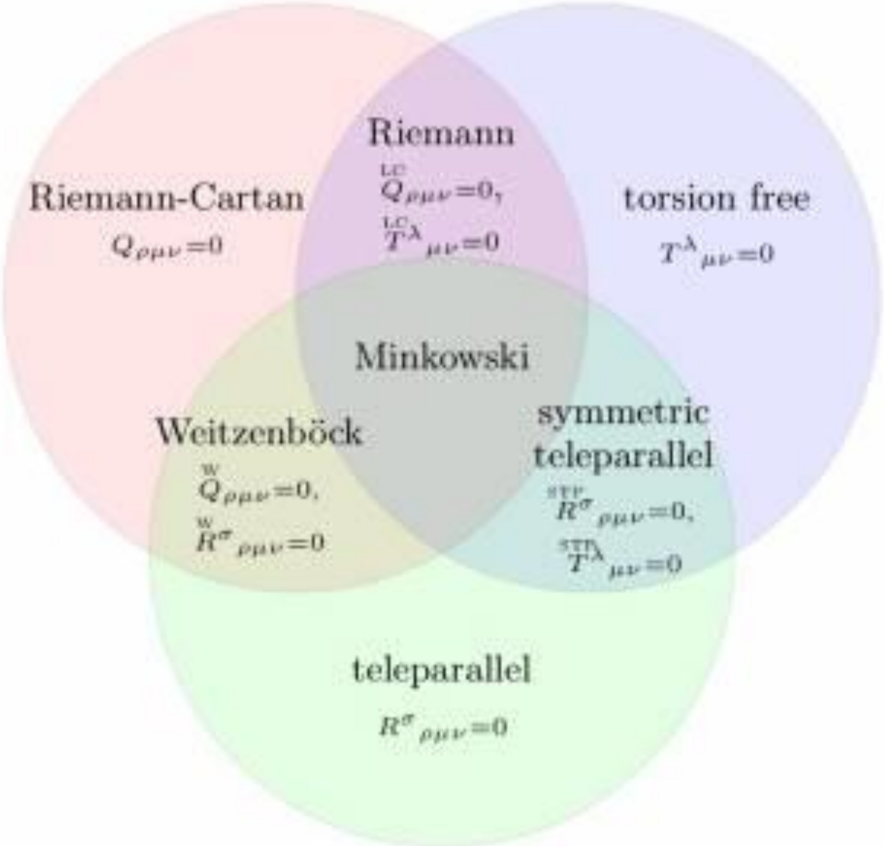


FIG. 1. Subclasses of metric-affine geometry, depending on the properties of connection.

Growth-index constraints on f(T) gravity

- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\,\rho_m\delta_m$, clustering growth rate: $\frac{d\ \ln\delta_m}{d\ \ln a} = \Omega_m^{\gamma}(a)$
- γ(z): Growth index. $G_{eff} = \frac{1}{1+f'(T)}$

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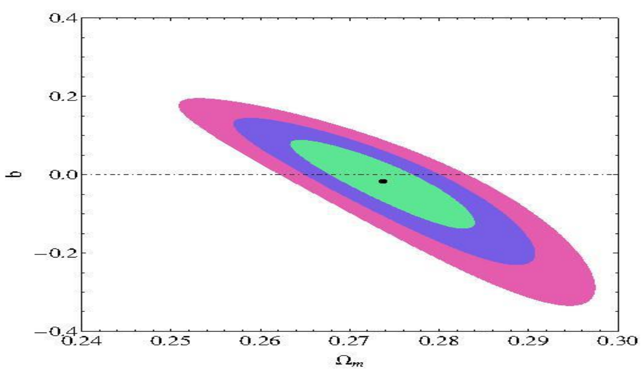
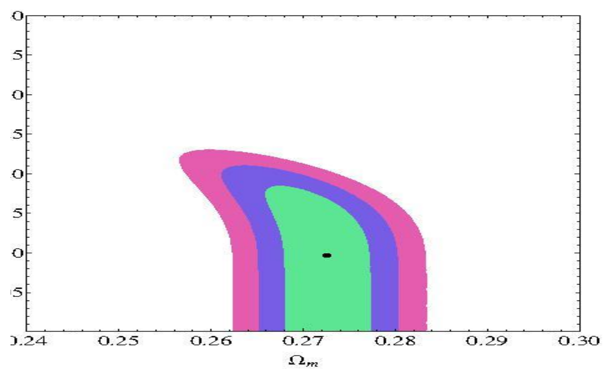
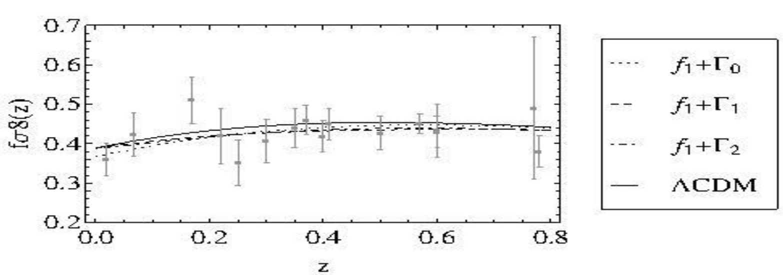
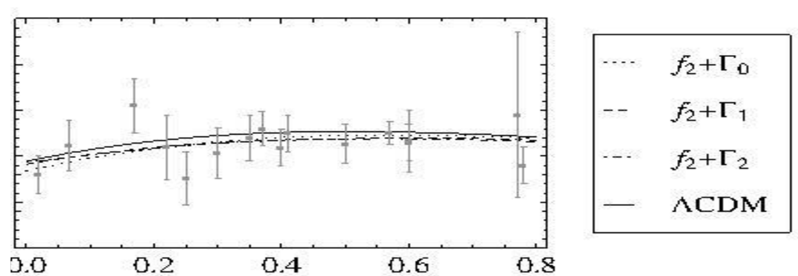






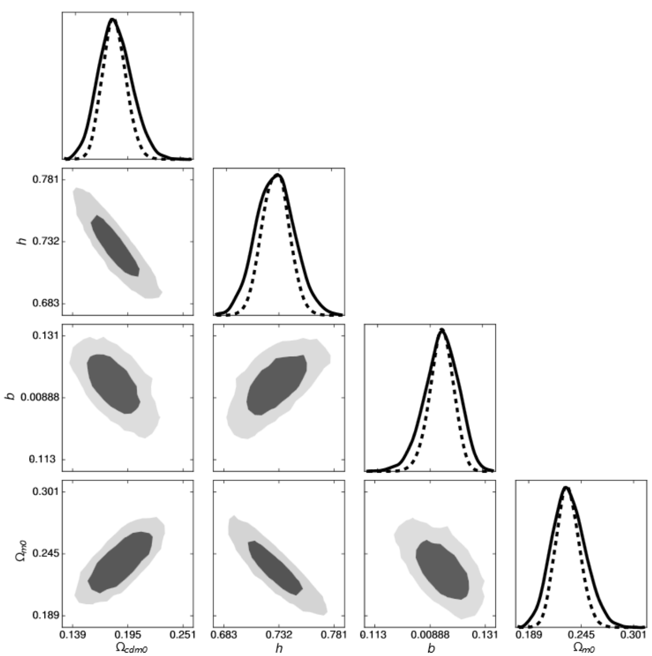


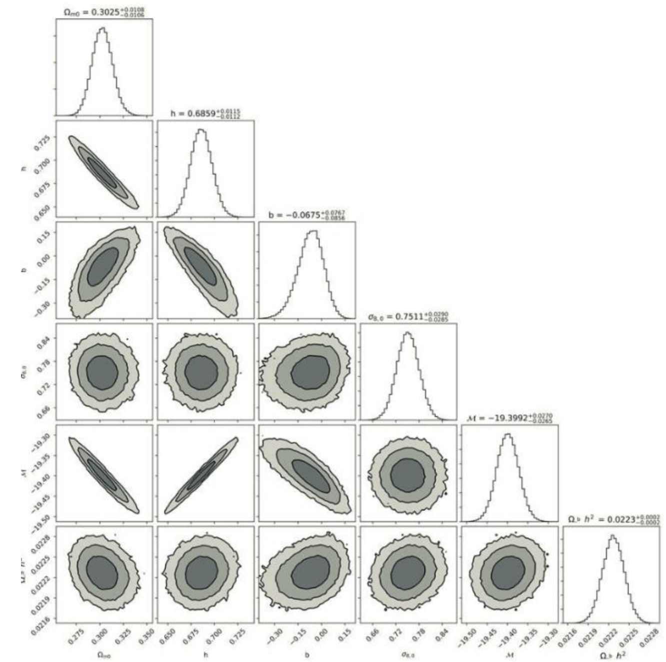
- Viable f(T) models are practically indistinguishable from ΛCDM.

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

[Nunes, Pan, Saridakis, JCAP 1608]

Observational Constraints on f(T) gravity





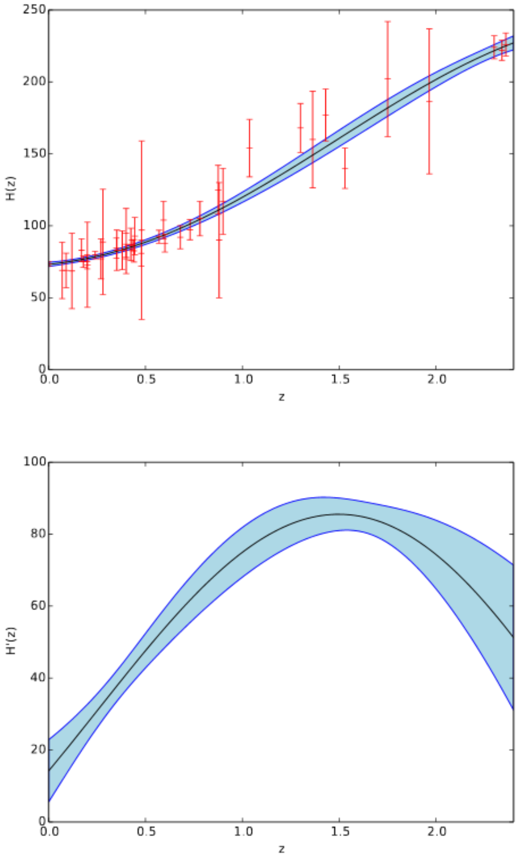
[Anagnostopoulos, Basilakos, Nesseriss, Saridakis JCAP08]

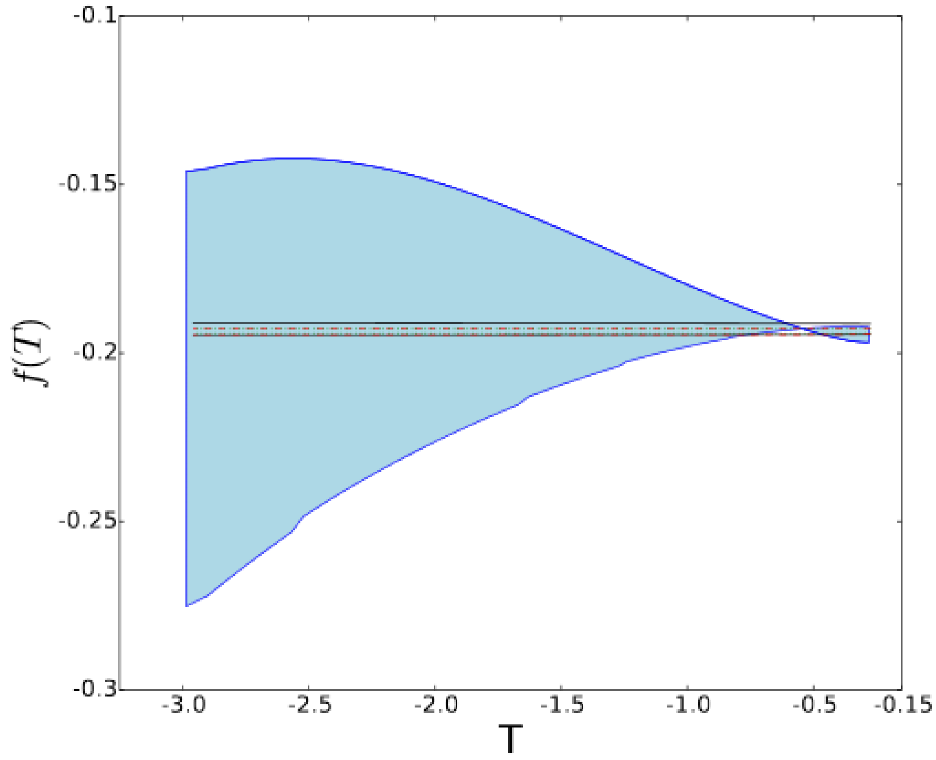
[Nunes, Pan, Saridakis, JCAP08]

[Anagnostopoulos, Basilakos, Saridakis PRD 100]

[Nunes, Bonilla, Pan, Saridakis, EPJC77]

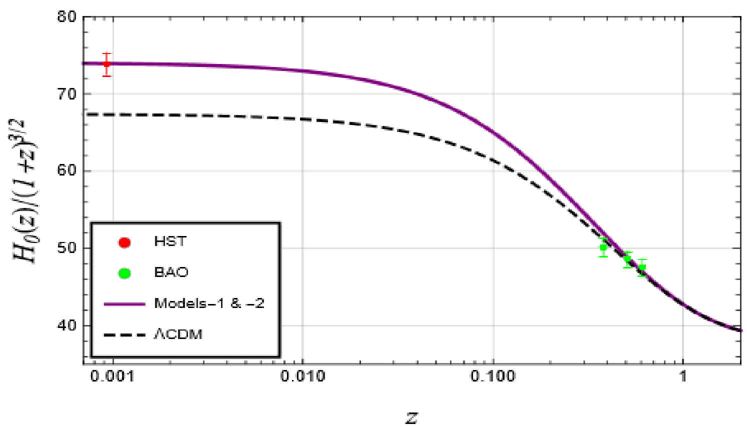
Gaussian Process constraints on f(T)

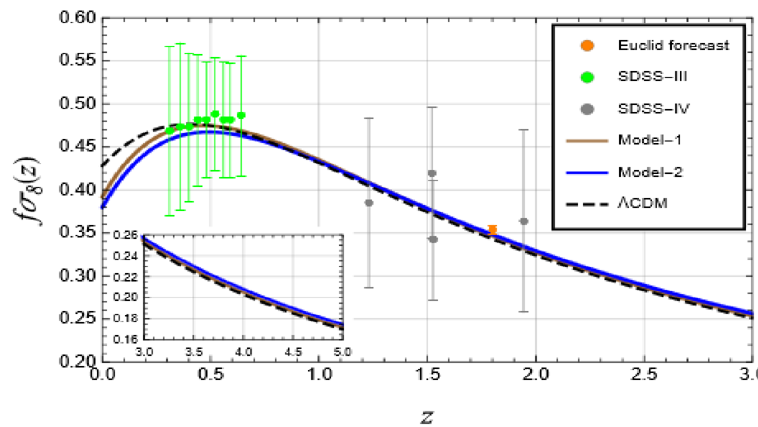




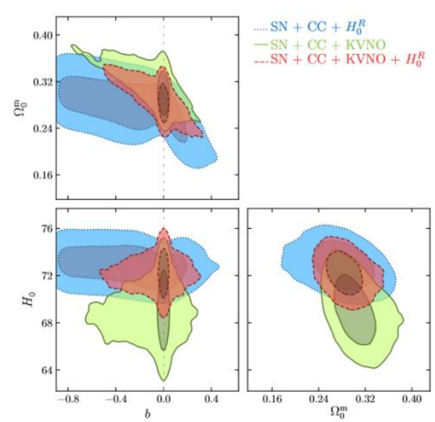
[Cai, Khurshudyan, Saridakis, Astroph. J 888]

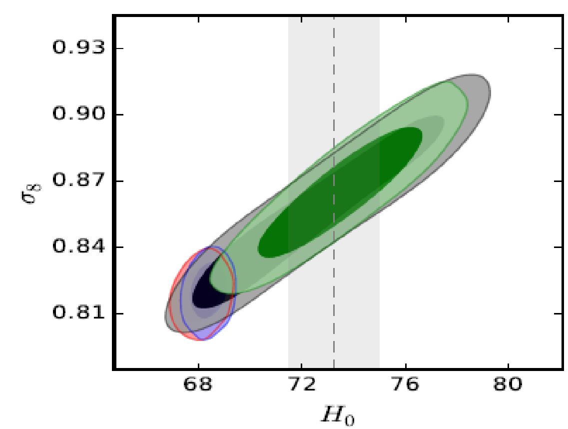
H0 and σ8 tension can be alleviated





Parameter	CMB + BAO	CMB + BAO + H_0
$10^2\omega_b$	$2.235^{+0.013}_{-0.013}$	$2.235^{+0.013}_{-0.013}$
ω_{cdm}	$0.1181^{+0.001}_{-0.001}$	$0.118^{+0.001}_{-0.001}$
$100\theta_s$	$1.041^{+0.00027}_{-0.00027}$	$1.041^{+0.00030}_{-0.00027}$
$\ln 10^{10}A_s$	$3.078^{+0.023}_{-0.023}$	$3.08^{+0.022}_{-0.022}$
n_s	$0.9678^{+0.0039}_{-0.0039}$	$0.9684^{+0.0039}_{-0.0044}$
τ_{reio}	$0.073^{+0.012}_{-0.013}$	$0.075^{+0.012}_{-0.012}$
n	$0.0043^{+0.0033}_{-0.0039}$	$0.0054^{+0.0020}_{-0.0020}$
$\log\alpha$	$10.00^{+0.081}_{-0.12}$	$10.03^{+0.06}_{-0.06}$
Ω_{F0}	$0.73^{+0.021}_{-0.028}$	$0.738^{+0.015}_{-0.015}$
H_0	$72.4^{+3.3}_{-4.1}$	$73.5^{+2.1}_{-2.1}$
σ_8	$0.855^{+0.023}_{-0.033}$	$0.866^{+0.02}_{-0.02}$
$\chi^2_{min}/2$	6480.48	6482.27

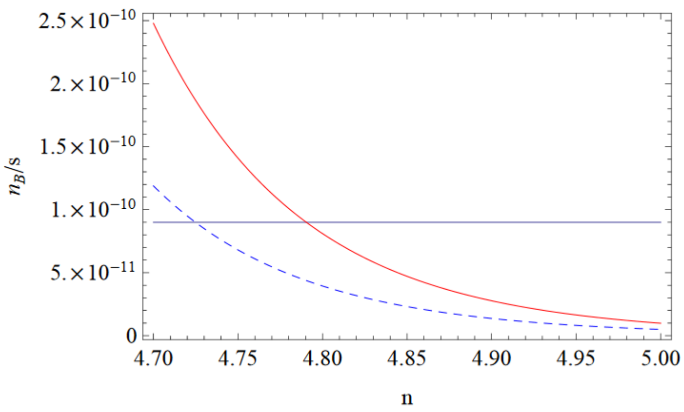




[Yang,Zhang,Chen,Cai,Li,Saridakis,Xue, PRD 101] [Said, Mifsud, Parkinson, Saridakis, Sultana, 2005.05368] [Nunes, JCAP 1805]

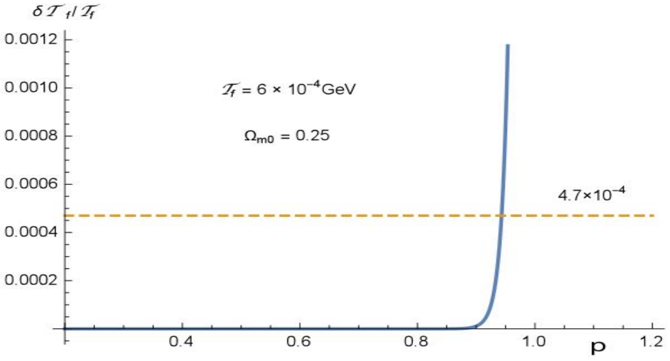
Baryogenesis and BBN constraints on f(T) gravity

- Baryon-anti-baryon asymmetry through CP violating term: $\frac{1}{M_*^2}\int d^4x\, e\left[\partial_\mu f(T)\right]J^\mu$



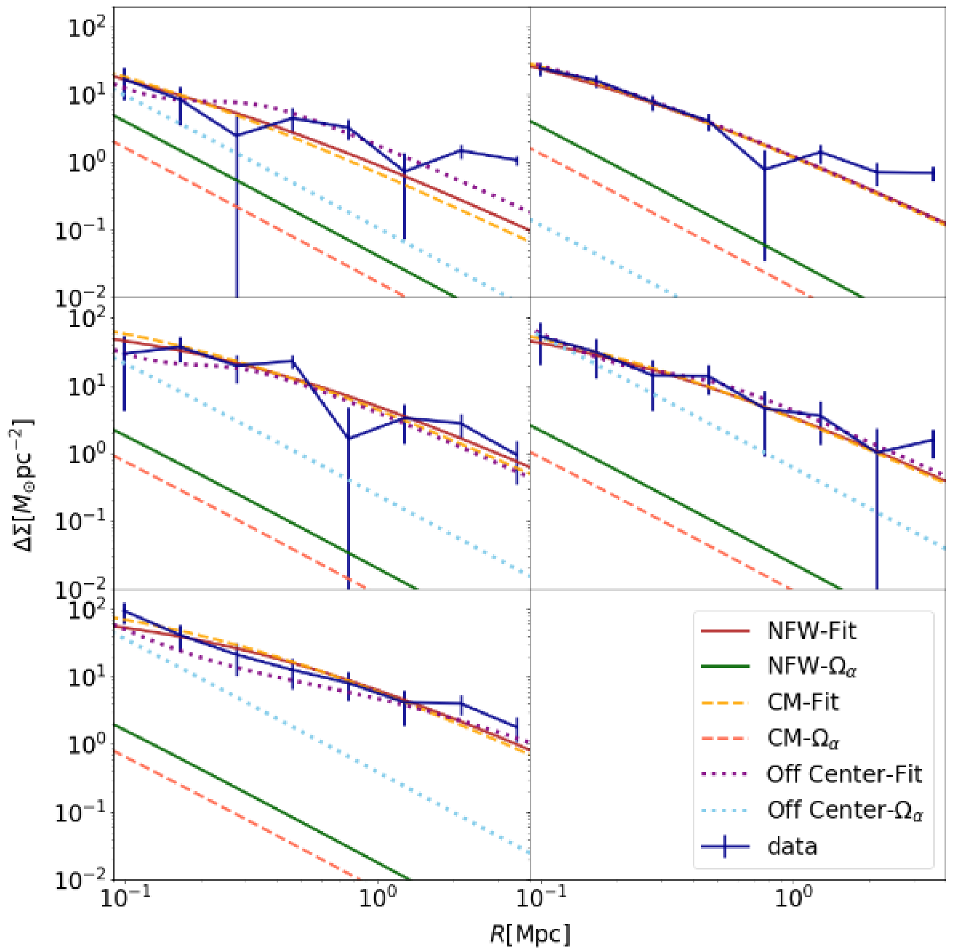
[Oikonomou, Saridakis, PRD 94]

- BBN constraints: $\frac{\delta T_f}{T_f} \approx \frac{\rho_T}{\rho}\frac{H_{GR}}{10qT_f^5}$



[Capozziello, Lambiase, Saridakis, EPJC77]

Galaxy-Galaxy lensing constraints on f(T) gravity

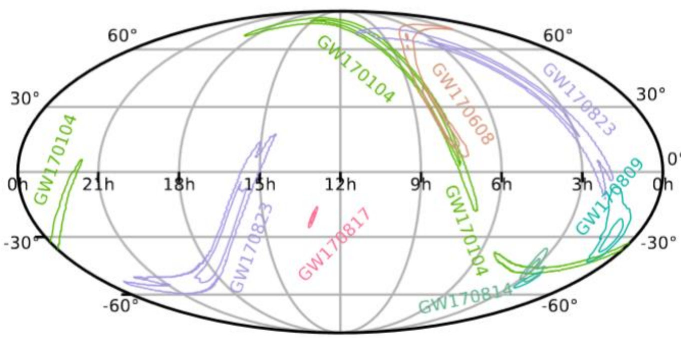


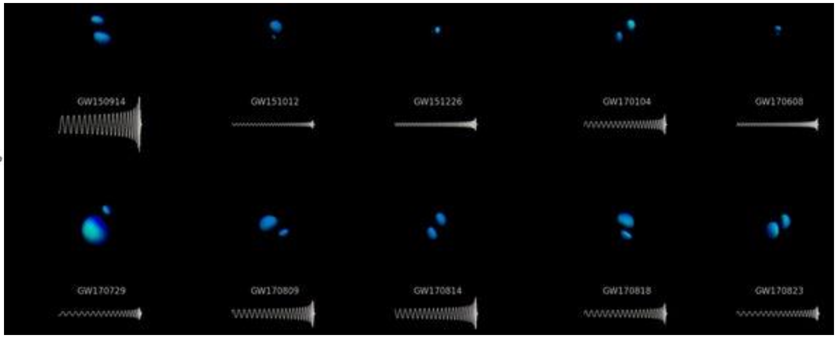
$$f(T) = -T - 2\Lambda/M_P^2 + \alpha T^\beta$$

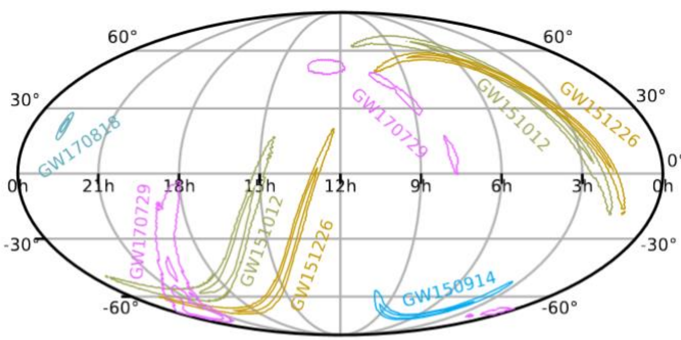
[Chen. Luo, Cai, Saridakis, PRD 102]

Gravitational waves - Observations

- Observations: 43 up to now (34 BH-BH, 3 NS-NS, 2 NS-BH, 4 uncertain, 19-85 Msun, 320-2800 MPc)







Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$

[LIGO-Virgo Collaborations 1811.12907]

Designation
150914+09:50:45UTC
151226+03:38:53UTC
151012+09:54:43UTC
151019+00:23:16UTC
150928+10:49:00UTC
151218+18:30:58UTC
160103+05:48:36UTC
151202+01:18:13UTC
160104+03:51:51UTC
151213+00:12:20UTC
150923+07:10:59UTC
151029+13:34:39UTC
151206+14:19:29UTC
151202+15:32:09UTC
151012+06:30:45UTC
151116+22:41:48UTC
151121+03:34:09UTC
150922+05:41:08UTC
151008+14:09:17UTC
151127+02:00:30UTC

- Expectations: Many thousands in the next years

Gravitational waves in f(T) gravity

- For tensor perturbations: $g_{00} = -1\ ,\quad g_{0i} = 0\ ,$ i.e. $\bar{e}^0_\mu = \delta^0_\mu\ ,$

$$g_{ij} = a^2\big(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\big)$$

$$\bar{e}^a_\mu = a\delta^a_\mu + \frac{a}{2}\delta^i_\mu\delta^{aj}h_{ij} + \frac{a}{8}\delta^i_\mu\delta^{ja}h_{ik}h_{kj}\ ,$$

$$\bar{e}^\mu_0 = \delta^\mu_0\ ,$$

$$\bar{e}^\mu_a = \frac{1}{a}\delta^\mu_a - \frac{1}{2a}\delta^{\mu i}\delta^j_a h_{ij} + \frac{1}{8a}\delta^{i\mu}\delta^j_a h_{ik}h_{kj}$$

- We obtain:

$$^{(3)}R \approx -\frac{1}{4}a^{-2}(\partial_i h_{kl}\partial_i h_{kl})\ ,$$

$$K^{ij}K_{ij} \approx 3H^2 + \frac{1}{4}\dot{h}_{ij}\dot{h}_{ij}\ ,$$

$$K \approx 3H\ ,$$

$$\boxed{T = T^{(0)} + O(h^2) = 6H^2 + O(h^2)}$$

- And finally: $$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[\frac{f_T}{4}\big(a^{-2}\vec{\nabla}h_{ij}\cdot\vec{\nabla}h_{ij} - \dot{h}_{ij}\dot{h}_{ij}\big) + 6H^2 f_T - 12H\dot{f}_T - T^{(0)}f_T + f(T^{(0)})\Big]$$

[Cai, Li, Saridakis, Xue, PRD 97] [Li, Cai, Cai, Saridakis, JCAP 1810]

Gravitational waves in f(T) gravity

- Varying the action and going to Fourier space we get the equation for GWs:

$$\ddot{h}_{ij} + 3H(1-\beta_T)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

with $\beta_T \equiv -\frac{\dot{f}_T}{3Hf_T}$

$$h_{\mu\nu}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\gamma_1^{(1)1} & {B_1}^2\exp(ip_\mu x^\mu) & 0 \\ 0 & {B_1}^2\exp(ip_\mu x^\mu) & -2\gamma_1^{(1)1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

- An immediate result: The speed of GWs is equal to the speed of light!
- GW170817 constraints that

$$|c_g/c - 1| \leq 4.5 \times 10^{-16}$$

are trivially satisfied.

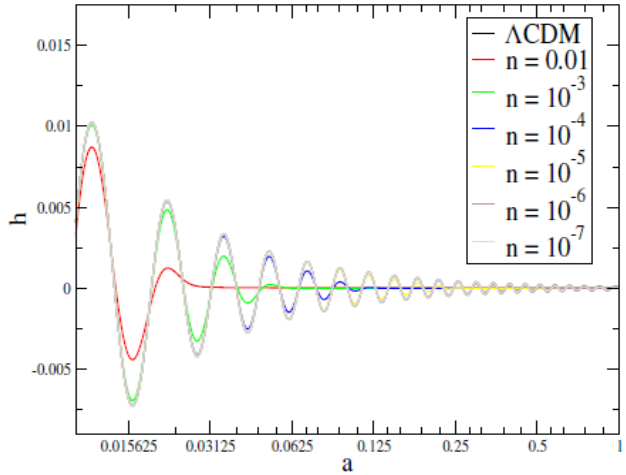
[Cai, Li, Saridakis, Xue, PRD 97]

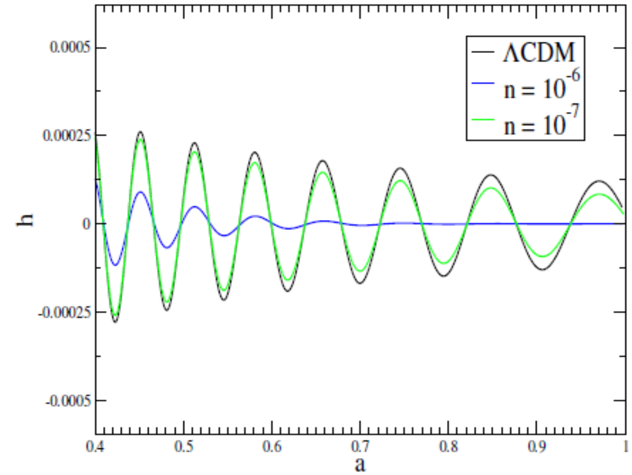
Gravitational waves in f(T) gravity

- Gw's propagation at cosmological scales: $h = e^{-\mathcal{D}} e^{-ik\Delta T} h_{GR}$

$\mathcal{D} = \frac{1}{2}\int \nu \mathcal{H} d\tau'$ (affects amplitude) $\Delta T = \int \left(1 - c_T - \frac{a^2\mu^2}{2k^2}\right) d\tau'$ (affects phase)

- In f(T) gravity:





[Cai, Li, Saridakis, Xue PRD 97]

[Farrugia, Said, Gakis, Saridakis, PRD 97]

[Soudi, Farrugia, Gakis, Said, Saridakis, PRD 100]

[Nunes, Pan, Saridakis, PRD98]

Gravitational Waves in f(T,B) gravity

$$[\delta e^A{}_\mu] := \begin{bmatrix} \varphi & a(\partial_i\beta + \beta_i) \\ \delta^I{}_i(\partial^i b + b^i) & a\delta^{Ii}\left(-\psi\delta_{ij} + \partial_i\partial_j h + 2\partial_{(i}h_{j)} + \frac{1}{2}h_{ij} + \epsilon_{ijk}\left(\partial^k\sigma + \sigma^k\right)\right) \end{bmatrix}$$

$$[\delta g_{\mu\nu}] = \begin{bmatrix} -2\varphi & a\left(\partial_i(b-\beta) + (b_i - \beta_i)\right) \\ a\left(\partial_i(b-\beta) + (b_i-\beta_i)\right) & 2a^2\left(-\psi\delta_{ij} + \partial_i\partial_j h + 2\partial_{(i}h_{j)} + \frac{1}{2}h_{ij}\right) \end{bmatrix}$$

We get:

$$\ddot{h}_{ij} + (3+\nu)H\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

with $\nu = \dfrac{1}{H}\dfrac{\dot{f}_T}{f_T}$ $\qquad c_T^2 = 1$

Stability conditions: $f_T < 0$

$f_{BB} < 0$

[Bahamonde, Gakis Kiorpelidi, Koivisto,Said, Saridakis, EPJC81]

Conclusions - Outlook

- Torsional modified gravity is theoretically robust and leads to very efficient cosmology at both background and perturbation levels.

- f(T) gravity, f(T,TG), f(T,B), f(Q) gravity, Symmetric teleparallel gravity, and modified teleparallel gravity in general, currently under investigation

  [Saridakis, Cai, Capozziello, Said, Bahamonde, Koivisto, Ren, Zhao, Wong, Ilyas, Zhu, Zheng, Yan, Zhang, Chen, Zhang, Luo, Khurshudyan, Marciano, Krssak, Odintsov, Nojiri, Nunes, Toporensky, Basilakos, Anagnostopouos, Kofinas, Dialektopoulos, Gakis, Palikaris, Iosifidis, Kiorpelidi, Chatzifotis, Asimakis]

- vi) Get prepared for the huge flow of data that will come!

“Those that do not know geometry are not allowed to enter”.
Front Door of Plato’s Academy

THANK YOU!

Corfu Schools and Conferences

http://www.physics.ntua.gr/corfu2021/

Curvature and Torsion

- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space

  $g_{\mu\nu}(x) = \eta_{AB}\, e_\mu^A(x)\; e_\nu^B(x)$
- Connection: ω_{ABC}
- Curvature tensor: $R^A_{B\mu\nu} = \omega^A_{B\nu,\mu} - \omega^A_{B\mu,\nu} + \omega^A_{C\mu}\omega^C_{B\nu} - \omega^A_{C\nu}\omega^C_{B\mu}$
- Torsion tensor: $T^A_{\mu\nu} = e^A_{\nu,\mu} - e^A_{\mu,\nu} + \omega^A_{B\mu} e^B_\nu - \omega^A_{B\nu} e^B_\mu$
- Levi-Civita connection and Contorsion tensor: $\omega_{ABC} = \Gamma_{ABC} + K_{ABC}$

  $K_{ABC} = \frac{1}{2}\left(T_{CAB} - T_{BCA} - T_{ABC}\right) = -K_{BAC}$
- Curvature and Torsion Scalars: $R = \overline{R} + T - 2\left(T_\nu^{\ \nu\mu}\right)_{;\mu}$

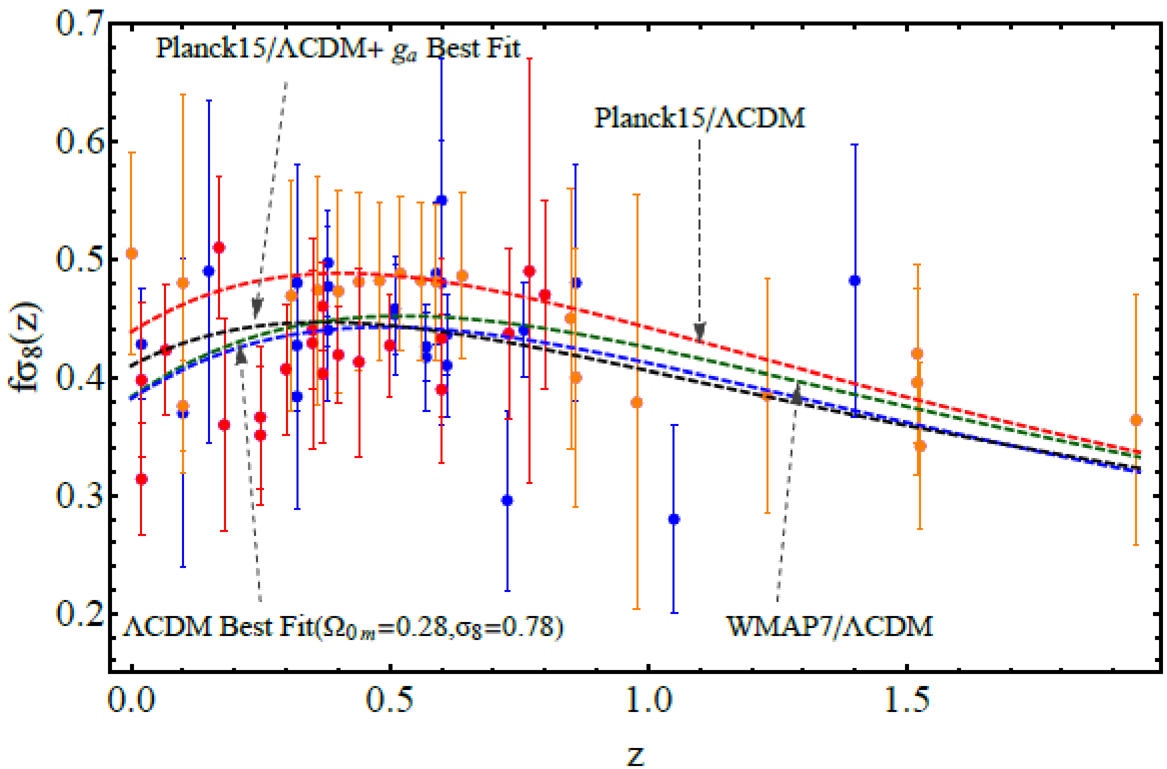
$R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu} R^\rho_{\mu\rho\nu}$

$T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - \mathrm{T}^\rho_{\rho\mu} \mathrm{T}^{\nu\mu}_\nu$

Tension1 – fσ8

- Tension between the data and Planck/ΛCDM. The data indicate a lack of "gravitational power" in structures on intermediate-small cosmological scales.

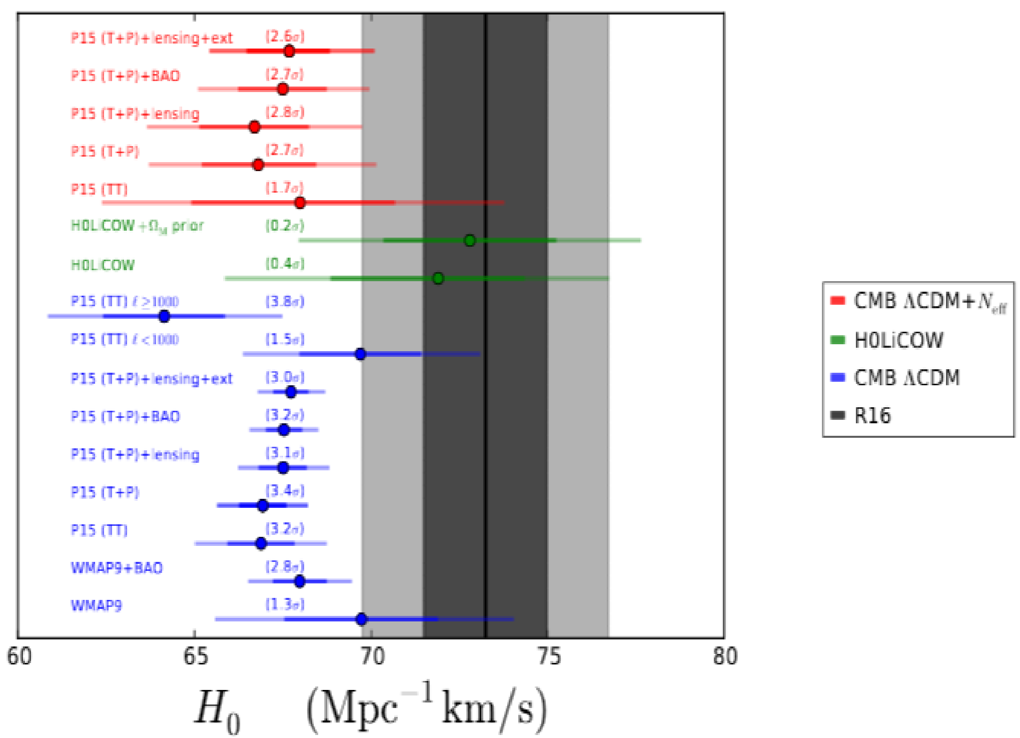
Parameter	Planck15/ΛCDM [12]	WMAP7/ΛCDM [45]
$\Omega_b h^2$	0.02225 ± 0.00016	0.02258 ± 0.00057
$\Omega_c h^2$	0.1198 ± 0.0015	0.1109 ± 0.0056
n_s	0.9645 ± 0.0049	0.963 ± 0.014
H_0	67.27 ± 0.66	71.0 ± 2.5
Ω_{0m}	0.3156 ± 0.0091	0.266 ± 0.025
w	-1	-1
σ_8	0.831 ± 0.013	0.801 ± 0.030



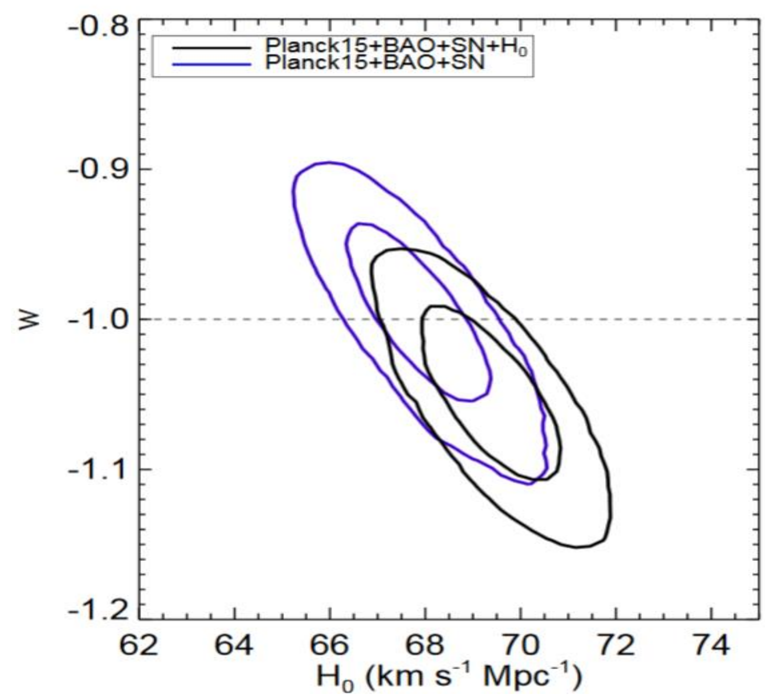
[Kazantzidis, Perivolaropoulos, PRD97]

Tension2 – H0

- Tension between the data (direct measurements) and Planck/ΛCDM (indirect measurements). The data indicate a lack of "gravitational power".



[Bernal, Verde, Riess, JCAP1610]



[Riess et al, Astrophys.J 826]