Late time cosmology with derivatives of matter Lagrangian

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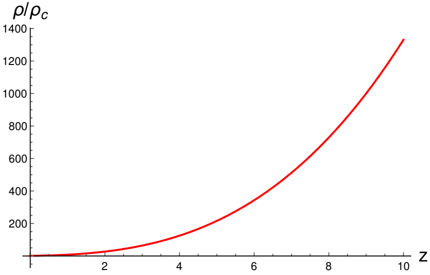
Motivations

- The accelerated expansion of the Universe is one of the most interesting findings of modern cosmology.
- It is mainly explained by the presence of the cosmological constant (CC).
- There are issues on ΛCDM model:
  CC problem, Hubble tension, σ_8 tension,...
- It would then be promising to search for a late time gravity theory to relax or compensate the effect of *CC*.

MOTIVATIONS

- Our idea: dealing with derivatives of the matter Lagrangian.
- The evolution of the non-relativistic matter energy density wrt the redshift



- Roughly speaking, the density around $z \approx 0$ is nearly flat.
- So, derivatives of the matter density is nearly constant, which can play the role of the cosmological constant.

THE MODEL

- The action (SS, Zahra Haghani, PDU 30 (2020) 100683.)

$$S=\int d^4x\sqrt{-g}\Big[\kappa^2(R-2\Lambda)+\alpha\,\nabla_\mu f\,\nabla^\mu f\Big]+S_m,$$

- $f(L_m)$ is an arbitrary function of the matter Lagrangian with mass dimension 1.
- Λ is the CC.
- α is a dimensionless constant.
- The α term is a kinetic term for scalar field f.
- The theory is not equivalent with $f(R, L_m)$ theory.

THE MODEL

- The equation of motion is

$$\kappa^2(G_{\mu\nu} + \Lambda g_{\mu\nu}) + \alpha f'\Box f\left(T_{\mu\nu} - g_{\mu\nu}L_m\right)$$
$$+\alpha\left(\nabla_\mu f\nabla_\nu f - \frac{1}{2}g_{\mu\nu}\nabla_\alpha f\nabla^\alpha f\right) = \frac{1}{2}T_{\mu\nu},$$

- Prime denotes derivative with respect to the argument.
- The second line is the equation of motion of the scalar field $f(L_m)$.
- In the first line, we have a new term relating to the true nature of $f(L_m)$.

Conservation of energy

- (non-)Conservation of the energy momentum tensor:

$$\nabla^{\mu} T_{\mu\nu} = 2(T_{\mu\nu} - L_m g_{\mu\nu})\nabla^{\mu}\mathrm{Log}\left[1 - \alpha f' \Box f\right].$$

- When $\alpha = 0$, the conservation equation restores.
- In the case of perfect fluid, One obtains

$$\nabla^{\mu} T_{\mu\nu} = -2(\rho + p) h_{\mu\nu} \nabla^{\mu}\mathrm{Log}\left[1 - \alpha f' \Box f\right].$$

- $h_{\mu\nu} = u_{\mu} u_{\nu} + g_{\mu\nu}$ is the projection tensor.
- In the case of FRW space-time, everything depends on time and derivatives are orthogonal to $h_{\mu\nu}$, so the energy-momentum tensor becomes conserved.

Newtonian limit

- The metric in the Newtonian limit

$$ds^2 = -(1+2\Phi(\vec{x}))dt^2 + (1-2\Phi(\vec{x}))d\vec{x}^2.$$

- The only non-zero component of the energy momentum tensor in this limit is $T_{00} = \rho$.
- If f is a smooth function so that

$$f = \sum_{i=0} \alpha_i \rho^i.$$

- f appears with at least two derivatives in eom. The derivative coupling term do not contribute to the linear level.
- As a result we have

$$\vec{\nabla}^2\Phi(\vec{x}) = 4\pi G\rho(\vec{x}),$$

- Assume $f = \gamma(-L_m/\rho_c)^n$.
- The Hubble parameter becomes

$$h^2 = a^2(r + \Omega_{\Lambda 0}) + \frac{1}{9}\beta r^{2(n-1)}\dot{r}^2,$$

- We have defined

$$H = H_0 h, \quad \bar{\rho}_m = \frac{\rho_m}{\rho_c}, \quad \bar{\rho}_r = \frac{\rho_r}{\rho_c}, \quad r \equiv \bar{\rho}_m + \bar{\rho}_r,$$

$$\rho_c = 6\kappa^2 H_0^2, \qquad \beta = \frac{3\gamma^2 \alpha n^2}{2\kappa^2}, \qquad \Omega_{\Lambda 0} = \frac{\Lambda}{3H_0^2}.$$

- Note the matter sources are conserved

$$\bar{\rho}_m = \frac{\Omega_{m0}}{a^3}, \qquad \bar{\rho}_r = \frac{\Omega_{r0}}{a^4}$$

- Transforming to the redshift coordinates and neglecting radiation

$$(1+z)^2 h^2 = \frac{(1+z)^3 \Omega_{m0} + \Omega_{\Lambda 0}}{1 + \beta (1+z)^{6n} \Omega_{m0}^{2n}}.$$

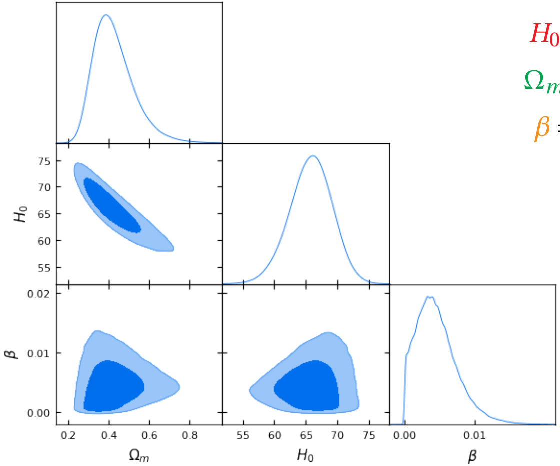
- In the case of non-vanishing CC: the CC can be obtained as

$$\Omega_{\Lambda 0} = 1 - \Omega_{m0} + \beta \Omega_{m0}^{2n}.$$

- In the case of vanishing CC: one obtains

$$\beta = (\Omega_{m0} - 1) \Omega_{m0}^{-2n}.$$

Parameters - Non-vanishing CC

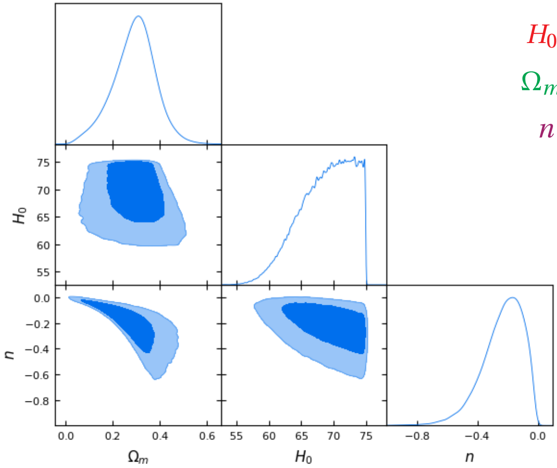


$$H_0 = 65.91^{+3.23}_{-3.45},$$

$$\Omega_{m0} = 0.41^{+0.11}_{-0.08},$$

$$\beta = 0.004^{+0.003}_{-0.002}.$$

$$\Omega_{\Lambda 0} = 0.59.$$



$$H_0 = 69.32^{+3.85}_{-4.82},$$

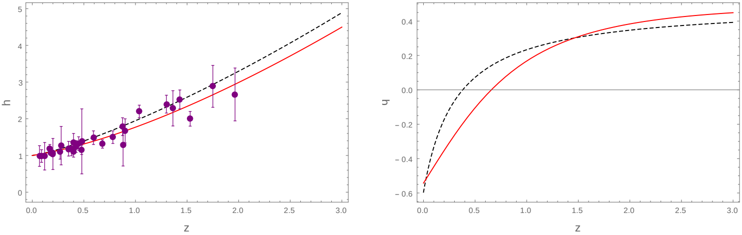
$$\Omega_{m0} = 0.29^{+0.07}_{-0.09},$$

$$n = -0.21^{+0.11}_{-0.15},$$

$$\beta = -0.42.$$

HUBBLE AND DECELERATION PARAMETERS - WITHOUT CC

- The evolution of the Hubble and deceleration parameters as a function of redshift:



- Red line is the ΛCDM curve.
- Bars are the observational data.

(H. Boumaza and K. Nouicer, PRD 100 (2019) 124047.)

Dynamical system - Vanishing CC

- Dynamical variables are

$$\Omega_m = \frac{\bar{\Omega}_m a^2}{h^2}, \quad \Omega_r = \frac{\bar{\Omega}_r a^2}{h^2}, \quad \Omega_h = \frac{h^2}{a^2}.$$

- From the Friedmann equation, one gets

$$\Omega_h = \left[\sqrt{\frac{1-\Omega_m-\Omega_r}{-\beta}} \frac{3}{(\Omega_m+\Omega_r)^{n-1}(3\Omega_m+4\Omega_r)} \right]^{\frac{1}{n}}.$$

- We have 2 dynamical dof.
- $\beta = -0.42$ at best fit.
- $\Omega_m, \Omega_r \geq 0$.
- Also we have $\Omega_m + \Omega_r \leq 1$.

Dynamical system - Non-vanishing CC

- The dynamical variables are

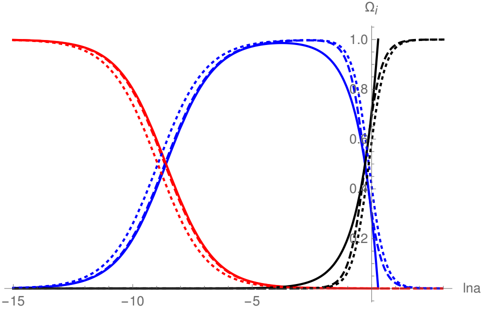
$$\Omega_m = \frac{\bar{\Omega}_m a^2}{h^2}, \quad \Omega_r = \frac{\bar{\Omega}_r a^2}{h^2}, \quad \Omega_h = \frac{h^2}{a^2}, \quad \Omega_\Lambda = \frac{\Omega_{\Lambda 0} a^2}{h^2}$$

$$\Omega_\Lambda = 1 - \Omega_m - \Omega_r + \frac{1}{9}\beta\Omega_h^{2n}(\Omega_m + \Omega_r)^{2(n-1)}(3\Omega_m + 4\Omega_r)^2.$$

- In this case the system has 3 dynamical dof.

MATTER ABUNDANCE EVOLUTION

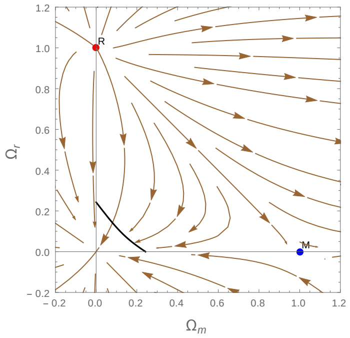
- The evolution of Ω_m, Ω_r and $\Omega_{dark} \equiv 1 - \Omega_m - \Omega_r$ as a function of $N \equiv \ln a$



- Dashed curves: ΛCDM model.
- Dotted curves: Non-vanishing CC.
- Solid curves: Vanishing CC.
- Red: radiation, Blue: matter, Black: dark energy.

Vanishing CC - Dynamical analysis

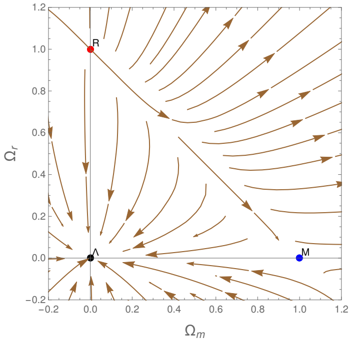
	(Ω_m, Ω_r)	Type	ω_{eff}	Status
R	$(0,1)$	Radiation	$1/3$	unstable
M	$(1,0)$	matter	0	saddle



- Stable manifold passing $(0,0)$.
- Closer to $(0,0)$, more acceleration we get: $\omega_{eff} < 0$.
- The black solid curve: de Sitter expansion $\omega_{eff} = -1$.
- The universe starts from radiation point, meet the matter dominated saddle point and falls to acceleration phase at late times.

NON-VANISHING CC - DYNAMICAL ANALYSIS

	$(\Omega_m, \Omega_r, \Omega_h)$	Type	ω_{eff}	Status
R	$(0,1,0)$	Radiation	$1/3$	unstable
M	$(1,0,0)$	matter	0	saddle
Λ	$(0,0,x)$	de Sitter	-1	stable



- x is arbitrary.
- Λ is a de Sitter fixed line.
- Stable point at $(0,0)$.
- The universe starts from the radiation point, meet the matter dominated saddle pointand falls to acceleration phase at late times.

Conclusions

- Derivatives of the matter Lagrangian could be considered as an alternative to ΛCDM model, or at least compensate some percent of CC.
- Higher order terms, Galileon like ...
- Also, we can consider derivatives of the energy-momentum tensor itself. The cosmology does not altered much.
- One advantage of the derivative matter coupling is that the matter becomes conserved at least at the background level.
- Other $f(R,T)$ theories does not have this property; should put it by hand.
- We should think more about the early cosmology in this model...

Thanks for your attention!