LATE TIME COSMOLOGY WITH DERIVATIVES OF MATTER LAGRANGIAN

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In collaboration with Zahra Haghani.
Why derivatives?
A model - with and without cosmological constant
Background cosmology
Parameter estimation
Dynamical system analysis
Conclusions...
The accelerated expansion of the Universe is one of the most interesting findings of modern cosmology. It is mainly explained by the presence of the cosmological constant (CC).

There are issues on ΛCDM model:
- CC problem, Hubble tension, $\sigma_8$ tension,...
- It would then be promising to search for a late time gravity theory to relax or compensate the effect of CC.
Motivations

- Our idea: dealing with derivatives of the matter Lagrangian.
- The evolution of the non-relativistic matter energy density wrt the redshift

Roughly speaking, the density around $z \approx 0$ is nearly flat.

So, derivatives of the matter density is nearly constant, which can play the role of the cosmological constant.
The model (SS, Zahra Haghani, PDU 30 (2020) 100683.)

\[
S = \int d^4 x \sqrt{-g} \left[ \kappa^2 (R - 2\Lambda) + \alpha \nabla_{\mu} f \nabla^{\mu} f \right] + S_m,
\]

- \( f(L_m) \) is an arbitrary function of the matter Lagrangian with mass dimension 1.
- \( \Lambda \) is the CC.
- \( \alpha \) is a dimensionless constant.
- The \( \alpha \) term is a kinetic term for scalar field \( f \).
- The theory is not equivalent with \( f(R, L_m) \) theory.
The equation of motion is

\[ \kappa^2 (G_{\mu\nu} + \Lambda g_{\mu\nu}) + \alpha f' \Box f \left( T_{\mu\nu} - g_{\mu\nu} L_m \right) + \alpha \left( \nabla_{\mu} f \nabla_{\nu} f - \frac{1}{2} g_{\mu\nu} \nabla_{\alpha} f \nabla^{\alpha} f \right) = \frac{1}{2} T_{\mu\nu}, \]

- Prime denotes derivative with respect to the argument.
- The second line is the equation of motion of the scalar field \( f(L_m) \).
- In the first line, we have a new term relating to the true nature of \( f(L_m) \).
(non-)Conservation of the energy momentum tensor:

\[ \nabla^\mu T_{\mu\nu} = 2(T_{\mu\nu} - L_m g_{\mu\nu})\nabla^\mu \text{Log}[1 - \alpha f' \Box f]. \]

When \( \alpha = 0 \), the conservation equation restores.

In the case of perfect fluid, one obtains

\[ \nabla^\mu T_{\mu\nu} = -2(\rho + p) h_{\mu\nu} \nabla^\mu \text{Log}[1 - \alpha f' \Box f]. \]

\( h_{\mu\nu} = u_\mu u_\nu + g_{\mu\nu} \) is the projection tensor.

In the case of FRW space-time, everything depends on time and derivatives are orthogonal to \( h_{\mu\nu} \), so the energy-momentum tensor becomes conserved.
Newtonian limit

- The metric in the Newtonian limit

\[ ds^2 = -(1 + 2\Phi(\vec{x}))dt^2 + (1 - 2\Phi(\vec{x}))d\vec{x}^2. \]

- The only non-zero component of the energy momentum tensor in this limit is \( T_{00} = \rho. \)

- If \( f \) is a smooth function so that

\[ f = \sum_{i=0}^{\infty} \alpha_i \rho^i. \]

- \( f \) appears with at least two derivatives in eom. The derivative coupling term do not contribute to the linear level.

- As a result we have

\[ \nabla^2 \Phi(\vec{x}) = 4\pi G\rho(\vec{x}). \]
Assume \( f = \gamma (-L_m / \rho_c)^n \).

The Hubble parameter becomes

\[
h^2 = a^2 (r + \Omega_{\Lambda 0}) + \frac{1}{9} \beta r^{2(n-1)} j^2 ,
\]

We have defined

\[
H = H_0 h, \quad \bar{\rho}_m = \frac{\rho_m}{\rho_c}, \quad \bar{\rho}_r = \frac{\rho_r}{\rho_c}, \quad r \equiv \bar{\rho}_m + \bar{\rho}_r,
\]

\[
\rho_c = 6 \kappa^2 H_0^2, \quad \beta = \frac{3 \gamma^2 \alpha n^2}{2 \kappa^2}, \quad \Omega_{\Lambda 0} = \frac{\Lambda}{3 H_0^2}.
\]
COSMOLOGY

- Note the matter sources are conserved

\[ \bar{\rho}_m = \frac{\Omega_{m0}}{a^3}, \quad \bar{\rho}_r = \frac{\Omega_{r0}}{a^4} \]

- Transforming to the redshift coordinates and neglecting radiation

\[ (1 + z)^2 h^2 = \frac{(1 + z)^3 \Omega_{m0} + \Omega_{\Lambda0}}{1 + \beta (1 + z)^6 \Omega_{m0}^{2n}}. \]

- In the case of non-vanishing CC: the CC can be obtained as

\[ \Omega_{\Lambda0} = 1 - \Omega_{m0} + \beta \Omega_{m0}^{2n}. \]

- In the case of vanishing CC: one obtains

\[ \beta = (\Omega_{m0} - 1) \Omega_{m0}^{-2n}. \]
PARAMETERS - NON-VANISHING CC

\[ H_0 = 65.91^{+3.23}_{-3.45}, \]
\[ \Omega_{m0} = 0.41^{+0.11}_{-0.08}, \]
\[ \beta = 0.004^{+0.003}_{-0.002}. \]

\[ \Omega_{\Lambda 0} = 0.59. \]
PARAMETERS - VANISHING CC

\[ H_0 = 69.32^{+3.85}_{-4.82}, \]
\[ \Omega_{m0} = 0.29^{+0.07}_{-0.09}, \]
\[ n = -0.21^{+0.11}_{-0.15}, \]
\[ \beta = -0.42. \]
The evolution of the Hubble and deceleration parameters as a function of redshift:

- Red line is the $\Lambda$CDM curve.
- Bars are the observational data.

(H. Boumaza and K. Nouicer, PRD 100 (2019) 124047.)
Dynamical variables are

\[ \Omega_m = \frac{\bar{\Omega}_m a^2}{h^2}, \quad \Omega_r = \frac{\bar{\Omega}_r a^2}{h^2}, \quad \Omega_h = \frac{h^2}{a^2}. \]

From the Friedmann equation, one gets

\[ \Omega_h = \left[ \sqrt{\frac{1 - \Omega_m - \Omega_r}{-\beta}} \right]^{\frac{1}{n}} \frac{3}{(\Omega_m + \Omega_r)^{n-1}(3\Omega_m + 4\Omega_r)}. \]

We have 2 dynamical dof.

\( \beta = -0.42 \) at best fit.

\( \Omega_m, \Omega_r \geq 0. \)

Also we have \( \Omega_m + \Omega_r \leq 1. \)
The dynamical variables are

\[ \Omega_m = \frac{\bar{\Omega}_m a^2}{h^2}, \quad \Omega_r = \frac{\bar{\Omega}_r a^2}{h^2}, \quad \Omega_h = \frac{h^2}{a^2}, \quad \Omega_{\Lambda} = \frac{\Omega_{\Lambda_0} a^2}{h^2} \]

\[ \Omega_{\Lambda} = 1 - \Omega_m - \Omega_r + \frac{1}{9} \beta \Omega_h^{2n} (\Omega_m + \Omega_r)^{2(n-1)} (3\Omega_m + 4\Omega_r)^2. \]

In this case the system has 3 dynamical dof.
The evolution of $\Omega_m$, $\Omega_r$ and $\Omega_{\text{dark}} \equiv 1 - \Omega_m - \Omega_r$ as a function of $N \equiv \ln a$.

- Dashed curves: $\Lambda$CDM model.
- Dotted curves: Non-vanishing CC.
- Solid curves: Vanishing CC.
- Red: radiation, Blue: matter, Black: dark energy.
Vanishing CC - Dynamical analysis

<table>
<thead>
<tr>
<th>$(\Omega_m, \Omega_r)$</th>
<th>Type</th>
<th>$\omega_{\text{eff}}$</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>(0, 1)</td>
<td>Radiation</td>
<td>1/3</td>
</tr>
<tr>
<td>$M$</td>
<td>(1, 0)</td>
<td>matter</td>
<td>0</td>
</tr>
</tbody>
</table>

- Stable manifold passing $(0, 0)$.
- Closer to $(0, 0)$, more acceleration we get: $\omega_{\text{eff}} < 0$.
- The black solid curve: de Sitter expansion $\omega_{\text{eff}} = -1$.
- The universe starts from radiation point, meet the matter dominated saddle point and falls to acceleration phase at late times.
Non-vanishing CC - Dynamical analysis

<table>
<thead>
<tr>
<th>$\Omega_m, \Omega_r, \Omega_\Lambda$</th>
<th>Type</th>
<th>$\omega_{\text{eff}}$</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Radiation</td>
<td>1/3</td>
<td>unstable</td>
</tr>
<tr>
<td>$M$</td>
<td>matter</td>
<td>0</td>
<td>saddle</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>de Sitter</td>
<td>$-1$</td>
<td>stable</td>
</tr>
</tbody>
</table>

$x$ is arbitrary.

$\Lambda$ is a de Sitter fixed line.

Stable point at $(0, 0)$.

The universe starts from the radiation point, meet the matter dominated saddle point and falls to acceleration phase at late times.
CONCLUSIONS

- Derivatives of the matter Lagrangian could be considered as an alternative to \textit{ACDM model}, or at least compensate some percent of CC.
- Higher order terms, Galileon like ...
- Also, we can consider derivatives of the energy-momentum tensor itself. The cosmology does not altered much.
- One advantage of the derivative matter coupling is that the matter becomes \textit{conserved} at least at the background level.
- Other $f(R, T)$ theories does not have this property; should put it by hand.
- We should think more about the \textit{early cosmology} in this model...
Thanks for your attention!