Tolman-Oppenheimer-Volkoff conditions beyond spherical symmetry

José Pedro Mimoso

Collaboration with
Alan Maciel (U. Federal ABC, S. Paulo, Brasil)
Morgan Le Delliou (U. Lanzhou, China)

Instituto de Astrofísica e Ciências do Espço & Dep. Física – Faculty of Science, University of Lisbon

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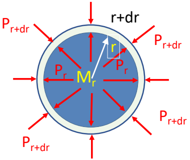
General Aim

- In the present work we aim at showing that a generalised TOV equation also characterises the equilibrium of models endowed with other symmetries besides spherical.

[A. Maciel, M. Le Delliou, JPM, Class. Quantum Grav. (2020) 37, 125005]

“Hydrostatic” Equilibrium

The Tolman-Oppenheimer-Volkov (TOV) equation appears as the relativistic counterpart of the classical condition for hydrostatic equilibrium

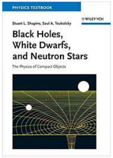


and establishes that static equilibrium requires the negative pressure gradient

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\rho + P}{1 - \frac{2GM}{r}} \left(\frac{M}{r^2} + 4\pi P r \right) . \tag{1}$$

$(\rho + P)$ accounts for the relativistic inertia, $(1 - 2GM/r)^{-1}$ conveys the spatial curvature, and $M = M(r) = 4\pi \int \rho(r')\, r'^2 \,\mathrm{d}r'$ is the Misner-Sharp-Hernandez (MSH) gravitational mass.

Overview of Literature



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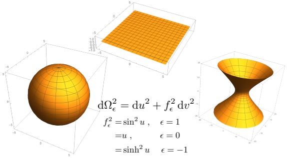
Class of space-times

We envisage spacetime environments characterised by metrics that have a codimension-two maximally symmetric foliation, that can be written as

$$\mathrm{d}s^2 = N_{ab}\mathrm{d}x^a\mathrm{d}x^b + Y^2(x^c)\left(\mathrm{d}\theta^2 + S_\epsilon^2\mathrm{d}\phi^2\right) \;, \tag{2}$$

where

$$\mathrm{d}\Omega_\epsilon^2 = \left(\mathrm{d}\theta^2 + S_\epsilon^2\mathrm{d}\phi^2\right) \tag{3}$$



with

$$S_\epsilon = \begin{cases} \sin\theta,\ \text{for } \epsilon = 1 \\ 1,\ \text{for } \epsilon = 0 \\ \sinh\theta,\ \text{for } \epsilon = -1 \end{cases} \;,$$

So $\epsilon = 0, \pm 1$ distinguishes the 3 possible curvatures: $\epsilon = 0$ corresponds to flat spatial hypersurfaces, $\epsilon = +1$ corresponds to closed spatial hypersurfaces, and $\epsilon = -1$ corresponds to open spatial hypersurfaces, endowed with negative curvature.

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Double-null formalism

- We divide the tangent space $\mathcal{T}$ at each event in two orthogonal subspaces $\mathcal{T} = \mathcal{N} \oplus \mathcal{S}$. Here $\mathcal{S}$ is the subspace generated by the orbits of (θ, ϕ) and $\mathcal{N}$, the subspace of $\mathcal{T}$ orthogonal to $\mathcal{S}$.
- We define an orthonormal two dimensional basis $(n^a\,, e^a)$ for $\mathcal{N}$, whose induced metric is N_{ab}, according to Eq. (2). This basis satisfies

$$-n^a n_a = e^a e_a = 1\,, \quad n^a e_a = n^a s_{ab} = e^a s_{ab} = 0\,. \tag{4}$$

  where $s_{ab} = Y^2 \gamma_{ab}$ the induced metric in each leaf of the foliation where $Y(x^c)$ is the warp factor.
- We further introduce a dual null basis for the same subspace from n^a and e^a by

$$\begin{aligned} k^a &= \frac{1}{2}\left(n^a + e^a\right)\,, \quad l^a = \frac{1}{2}\left(n^a - e^a\right)\,, \\ n^a &= k^a + l^a\,, \quad e^a = k^a - l^a\,, \end{aligned} \tag{5}$$

  which satisfies

$$k^a k_a = l^a l_a = 0\,, \quad k^a l_a = -\frac{1}{2}\,. \tag{6}$$

Double-null formalism

- So, the metrics can be cast as

$$g_{ab} = \frac{2}{k^c l_c} k_{(a} l_{b)} + s_{ab} \,. \tag{7}$$

- We associate the null expansion for each null vector as follows

$$\Theta_k = \frac{1}{2} s^{ab} \mathcal{L}_k s_{ab} = \frac{1}{2} Y^{-2} \gamma^{ab} \mathcal{L}_k Y^2 \gamma_{ab} = \frac{2}{Y} k^a \partial_a Y \,. \tag{8}$$

- We may define the mean curvature form $\mathcal{K}_a = \partial_a \ln Y^2$, such that, we obtain for the two-expansion $\Theta_{(u)}$ of any vector u^a in $\mathcal{N}$

$$\Theta_{(u)} = u^a \mathcal{K}_a \,. \tag{9}$$

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Double-null formalism

- The field equations then read

$$\mathcal{L}_k\Theta_{(k)} = \nu_k\Theta_{(k)} - \frac{\Theta^2_{(k)}}{2} - 8\pi T_{ab}k^a k^b\,, \tag{10a}$$

$$\mathcal{L}_l\Theta_{(l)} = \nu_l\Theta_{(l)} - \frac{\Theta^2_{(l)}}{2} - 8\pi T_{ab}l^a l^b\,, \tag{10b}$$

$$\begin{aligned}\mathcal{L}_k\Theta_{(l)} + \mathcal{L}_l\Theta_{(k)} &= -\Theta_{(l)}\nu_k - \Theta_{(k)}\nu_l - \\ 2\,\Theta_{(k)}\,\Theta_{(l)} &+ \epsilon\frac{2\,k^a l_a}{Y^2} + 16\pi\,T_{ab}k^a l^b\,,\end{aligned} \tag{10c}$$

where we included the inaffinities ν_k and ν_l, defined as

$$\nu_k = \frac{1}{k^c l_c} l^b k^a \nabla_a k_b \qquad \nu_l = \frac{1}{k^c l_c} k^b l^a \nabla_a l_b\,. \tag{11}$$

- In this work we adapt our vector basis to a fluid source, such that n^a gives its flow. By construction, the flow n^a is always orthogonal to the surfaces of symmetry and will be characterized by two quantities

$$\mathcal{A} = e^a\dot{n}_a = e^a n^b\nabla_b n_a\,, \qquad \mathcal{B} = e^a n'_a = e^a e^b\nabla_b n_a\,. \tag{12}$$

The scalar $\mathcal{A}$ gives us the acceleration of the flow. The scalar $\mathcal{B}$ gives the change of direction of n^a as we travel along e^a.

Double-Null Formalism

- We now assume that our metric has a Killing vector orthogonal to maximally symmetric surfaces
- If a spacetime is described by a metric of the form (2) and admits an orthogonal Killing vector $\chi^a \in \mathcal{N}$, then $\Theta_\chi = 0$.
- This follows from the assumption of the existence of a Killing vector field

$$\begin{aligned} 0 = Y^{-2}\gamma^{ab}\mathcal{L}_\chi g_{ab} &= Y^{-2}\gamma^{ab}\mathcal{L}_\chi N_{ab} + 2\Theta_\chi \\ &= -Y^{-2}N^{ab}\mathcal{L}_\chi \gamma_{ab} + 2\Theta_\chi \,. \end{aligned} \tag{13}$$

  and from

$$\mathcal{L}_\chi \gamma_{ab} = 0\,, \tag{14}$$

  since χ^a does not admit components in $\mathcal{S}$ and γ_{ab} doesn't depend on coordinates along $\mathcal{N}$.
- This implies that if $\mathrm{d}Y$ is spacelike, then χ_a is timelike and vice-versa. If $\mathrm{d}Y$ is null, the Killing vector will also be null.

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Mass-Energy

- Our symmetry assumptions imply that the only non-vanishing optical scalar on the leaves Σ is the null expansion (shear and vorticity vanish). Therefore, the Hawking-Hayward mass-energy is reduced to

$$M_{\Sigma} = \frac{1}{8\pi}\sqrt{\frac{A}{16\pi}} \int_{\Sigma} \left[\mathcal{R} - \frac{1}{k^a l_a} \Theta_{(k)} \Theta_{(l)} \right] \mathrm{d}\Sigma \tag{15}$$

Since we assume that Σ is maximally symmetric, we have $\mathcal{R} = \dfrac{2\epsilon}{Y^2}$. We also have

$$\Theta_{(k)}\Theta_{(l)} = k^a \partial_a \ln Y^2 \, l^b \partial_b \ln Y^2 = k^a l^b \partial_a \ln Y^2 \partial_b \ln Y^2 =$$

$$= k^{(a} l^{b)} \partial_a \ln Y^2 \partial_b \ln Y^2 = \frac{k^c l_c}{2} g^{ab} \partial_a \ln Y^2 \partial_b \ln Y^2 =$$

$$\frac{1}{2} k^c l_c ||\mathrm{d} \ln Y^2||^2 = k^c l_c \frac{2}{Y^2} ||\mathrm{d}Y||^2 \Rightarrow$$

$$\frac{\Theta_{(k)}\Theta_{(l)}}{k^c l_c} = \frac{2}{Y^2} ||\mathrm{d}Y||^2 \,, \tag{16}$$

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Mass-Energy

- We thus obtain

$$||\mathrm{d}Y||^2 = \epsilon - \frac{\kappa\mu(Y)}{2Y}\,. \tag{17}$$

- For the spherical case $\epsilon = 1$ and $A = 4\pi Y^2$, we obtain the known interpretation of $||\mathrm{d}Y||$ in terms of the Misner-Sharp mass-energy, which coincides with the Hawking-Hayward one

$$M_\Sigma = \frac{Y}{2}\left(1 - ||\mathrm{d}Y||^2\right) \Leftrightarrow ||\mathrm{d}Y||^2 = 1 - \frac{2M}{Y}\,. \tag{18}$$

  In the planar and hyperbolic cases ($\epsilon = 0$ and $\epsilon = -1$, respectively), the Hawking-Hayward mass is not conveniently defined for the integration domain set by our preferred foliation, as it requires a closed compact surface.

An alternative route can be obtained by computing the HH mass-energy in a finite domain, symmetric with respect to the central plane or wire, $Y = 0$, and taking the limit where the domain tends to be the whole surface. The finite integration domain consist of the union of

1. a subset of the Σ_Y, that we denote Γ_r, bounded by a circle γ_r of radius r on the (θ, ϕ) coordinate plane and
2. a compact surfaces given by the surfaces Δ_r defined by γ_r transported along Y orbits.

It forms a closed surface, corresponding to a part of a cylinder bounded by $Y =$ constant surfaces in the space of coordinates (Y, θ, ϕ). Therefore, the HH mass-energy enclosed by those surfaces will by finite, and given by

$$M_r = \frac{1}{8\pi}\sqrt{\frac{A_r}{16\pi}}\left(\int_{\Gamma_r}(\dots)S_\epsilon^2 \mathrm{d}\theta\mathrm{d}\phi + \int_{\Delta_r}(\dots)\mathrm{d}\Delta\right) \tag{19}$$

where (...) replaces the integrand of the HH mass. In the limit $r \to \infty$, the first integral in Eq. (19) scales as r^2 while the second one scales as r. This means that, in the limit $r \to \infty$we obtain

$$\frac{M_r}{A_r} \to \frac{\mu(Y)}{4\pi Y^2} \tag{20}$$

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Mass-Energy

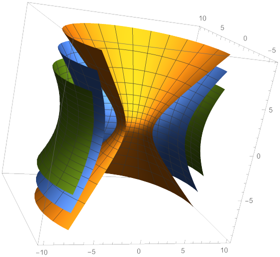

Figure: Hyperbolic Foliation Case $\epsilon = -1$

Mass-Energy

We then define the quasi-local mass-energy parameter $\mu(Y)$ by

$$M_\Sigma = \frac{\mu(Y)}{4\pi} \int S^2_\epsilon(\theta)\, \mathrm{d}\theta \mathrm{d}\phi\,, \tag{21}$$

and we write

$$\frac{Y}{4\pi\kappa}\left[\mathcal{R} - \frac{\Theta_{(k)}\Theta_{(l)}}{k^c l_c}\right]\int_\Sigma \mathrm{d}\Sigma = \tag{22}$$

$$\frac{Y}{4\pi\kappa}\left[2\epsilon - 2||\mathrm{d}Y||^2\right]\int S_\epsilon(\Theta)\mathrm{d}\theta\mathrm{d}\phi\,. \tag{23}$$

We eliminate the improper area integral on both sides

$$||\mathrm{d}Y||^2 = \epsilon - \frac{\kappa\mu(Y)}{2Y}\,. \tag{24}$$

So we realise that for $\epsilon = 0, -1$ we have $\mu < 0$ violating the weak energy condition.

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Generalised TOV equation

Taking the source to be a perfect fluid, then the energy momentum tensor is

$$T_{ab} = \rho n_a n_b + P(e_a e_b + s_{ab})\,. \tag{25}$$

Contracting the conservation of the energy-momentum tensor with e_b we get

$$e_b \nabla_a T^{ab} = (\rho + P)\dot{n}^b e_b + e^a \nabla_a P = 0 \Rightarrow$$
$$\mathcal{A} = -\frac{e^a \partial_a P}{\rho + P}\,. \tag{26}$$

Since $\Theta_{(n)} = 0$, this implies that e^a is proportional to ∂_Y, and as e^a is normalized, we have $e_a = \frac{1}{||\mathrm{d}Y||}\partial_a Y$. Imposing $e^a e_a = 1$ we obtain

$$e^a = ||\mathrm{d}Y||(\partial_Y)^a\,, \tag{27}$$

which gives us

$$\mathcal{A}\Theta_e = -||\mathrm{d}Y||^2\,\frac{2}{Y}\,\frac{\partial_Y P}{\rho + P}\,. \tag{28}$$

and so we derive is the unified TOV equation

$$\frac{\partial_Y P}{\rho + P} = -\left(\frac{\mu(Y)}{Y^2} + 4\pi P Y\right)\left(\epsilon - \frac{2\mu(Y)}{Y}\right)^{-1}, \tag{29}$$

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Notice that to determine $\mu(Y)$ we can use as usually

$$\partial_Y \mu = 4\pi\rho Y^2\,, \tag{30}$$

which looks like the mass-energy equation of spherical symmetry.

Example: Incompressible fluid solutions

By choosing a timelike coordinate T along the flow we consider the following line element in the (T, Y) coordinates:

$$\mathrm{d}s^2 = -\alpha^2(Y)\mathrm{d}T + \frac{\mathrm{d}Y^2}{\epsilon - \frac{2\mu(Y)}{Y}} + Y^2\mathrm{d}\Omega_\epsilon\,, \tag{31}$$

where $\mathrm{d}\Omega_\epsilon = \left(\mathrm{d}\theta^2 + S_\epsilon^2\mathrm{d}\phi^2\right)$ and the functions α and μ will be given by solving Einstein equations.
We apply our unified treatment to find the analogs of Schwarzschild interior solution, that is, we will use case study with the equation of state of an incompressible fluid $\rho = \rho_0$ constant. It is important to note that as the static solutions with $\epsilon \neq 1$ violate the WEC, we should take $\rho_0 < 0$ in those cases.
The generalised Euler equation implies

$$\frac{\alpha'}{\alpha} = -\frac{P'}{\rho + P} \Rightarrow \alpha = \frac{c_0}{\rho_0 + P}\,, \tag{32}$$

where c_0 is an integration constant that can be set by rescaling the time coordinate and the prime denotes Y differentiation.
Equation (30) gives us

$$\mu(Y) = \frac{4\pi\rho_0 Y^3}{3}\,, \tag{33}$$

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Incompressible fluid

Substitution into the generalised TOV equation yields

$$P(Y) = \rho_0 \left(\frac{2\sqrt{|\epsilon - \frac{Y_s}{Y_g}|}}{3\sqrt{|\epsilon - \frac{Y_s}{Y_g}|} - \sqrt{|\epsilon - \frac{Y_s Y^2}{Y_s^3}|}} - 1 \right) . \tag{34}$$

where Y_g is the analog of the radius of the object and is the least positive number that satisfy $P(Y_g) = 0$, $Y_s = \frac{8\pi\rho_0 Y_g^3}{3}$ is the analog of the Schwarzschild radius, although it can not be interpreted as a location when it will be a negative number. T
his gives

$$\alpha = \frac{1}{2}\left(3\sqrt{\left|\epsilon - \frac{Y_s}{Y_g}\right|} - \sqrt{\left|\epsilon - \frac{Y_s Y^2}{Y_g^3}\right|}\right) \tag{35}$$

which has a similar form to the interior Schwarzschild solution. Of course the physical properties are very distinct, since the solutions violate the WEC.

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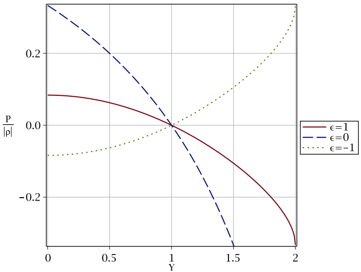
Incompressible fluid

The following figure summarises the differences between the choices of ϵ



the hyperbolic case presents $P' > 0$

Summary and Conclusions

- We analysed spacetimes with a two-dimensional maximally symmetric foliation sourced by a perfect fluid.
- if there is a Killing vector orthogonal to the leaves, its two-expansion vanishes.
- the geometric meaning of the mass-energy in such spacetimes, and our procedure matches the traditional mass parameter found in those cases by usual methods of integration of Einstein equations .
- the only static fluid solutions that satisfy the WEC are the spherical ones, as the other two cases require a negative energy density
- We found that, besides the known case of spherically symmetric spacetimes, we obtain a static interior fluid configuration only in the case of planar symmetric spacetimes. In the hyperbolic case, the static configuration is an exterior solution that can surround an inner vacuum region.

Thanks for listening!

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