Causality in Spinfoams

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Based on work with Pierre Martin-Dussaud (Penn State)

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PennState
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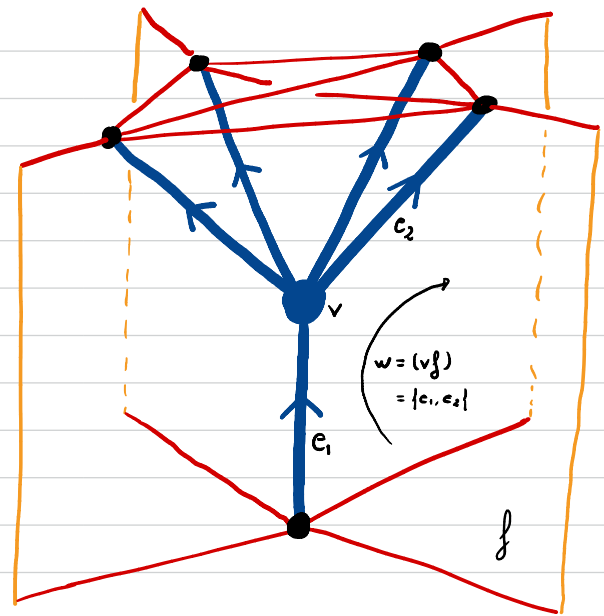
QISS | THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

Causality in Spinfoams

- Current Lorentzian Spinfoams are formulated in terms of:
  - a two-complex $\mathcal{C} = \{v, e, f\}$
  - spins on faces j_f
  - $SU(2)$ intertwiners on edges i_e



- This talk: Add
  - Causal structure on wedges $w = (vf) = \{e_1, e_2\}$

$$\varepsilon_w = \begin{cases} +1 & \text{co-chronal edges} \\ -1 & \text{anti-chronal edges} \end{cases}$$

$\Rightarrow$ Causal Vertex Amplitude

$$A_v(j_f, i_e, \varepsilon_w)$$

Causality in Spinfoams

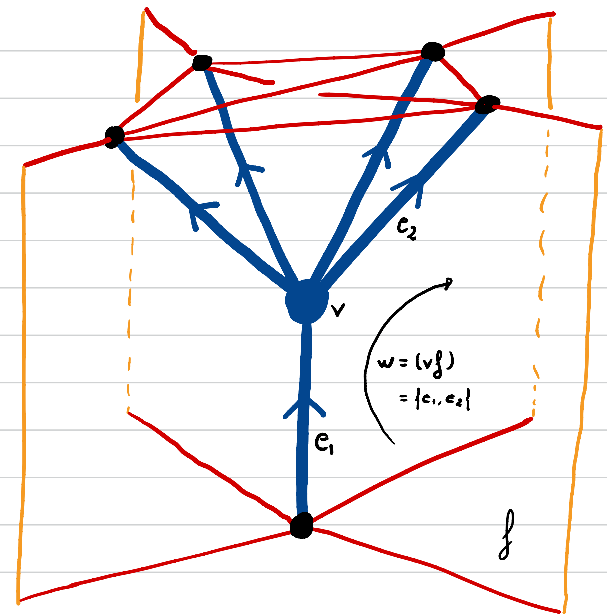
* Previous work. We are building on:

causality

- Markopoulou & Smolin (1997) "Causal evolution..."
- Livine & Oriti (2002) "Implementing Causality..." [BC model]
- Cortês & Smolin (2014) "Energetic causal sets"
- Wieland (2014) "New Action..."

γ-simplicity

- Engle (2011), "...proper vertex.." [Euclidean EPRL]
  Engle, Vilensky & Zipfel (2015) [Lorentzian EPRL]
- Rovelli & Wilson-Ewing (2012) "Discrete symmetries..."
- Donà, Gozzini & Nicotra (2021) "A Wick rotation..."



Plan:

1. Geometry of a Lorentzian 4-simplex

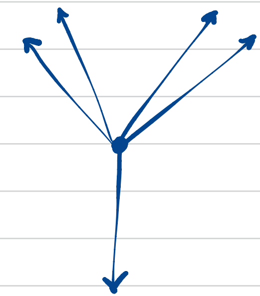
2. Causal vertex amplitude

3. Semiclassical limit

$\boxed{1}$ Geometry of a Lorentzian 4-simplex

Data:

- Minkowski Space $(\mathbb{R}^4, \eta_{IJ})$ $(-+++)$
- Five time-like vectors N_a^I with 4d closure $(a, b = 1, .., 5)$

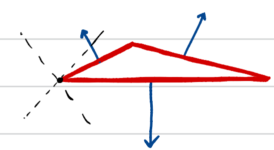


$$
\left\{
\begin{array}{l}
\mathrm{span}\{N_1^I, \ldots, N_5^I\} = \mathbb{R}^4 \\[2ex]
\eta_{IJ} N_a^I N_a^J = - v_a{}^2 \\[2ex]
\sum\limits_{a=1}^{5} N_a^I = 0
\end{array}
\right.
$$

Reconstruction: (cf. Minkowski Thm)

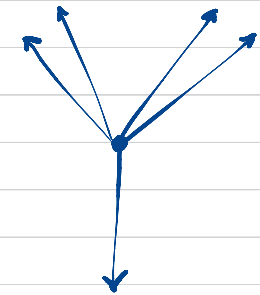
- Lorentzian 4-simplex with spacelike boundary tetrahedra

$$
\left\{
\begin{array}{l}
1 \longrightarrow 4 \\
2 \longrightarrow 3 \\
3 \longrightarrow 2 \\
4 \longrightarrow 1
\end{array}
\right.
$$



1 Geometry of a Lorentzian 4-simplex in terms of 4-normals N_a^I

- 4-volume $\quad V = \left(\frac{9}{4} \left| \frac{1}{4!} \epsilon_{IJKL} N_1^I N_2^J N_3^K N_4^L \right| \right)^{1/3}$
- 3-volume of tetrahedra $\quad v_a = \sqrt{-\eta_{IJ} N_a^I N_b^J}$



$\Rightarrow \quad N_a^I = v_a \hat{N}_a^I$ unit 4-normal

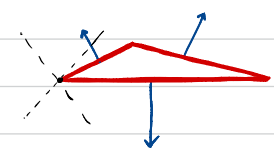
- 2-area of triangles $\quad A_{ab} = \frac{3}{4} \frac{v_a v_b}{V} \sqrt{\left(-\eta_{IJ} \hat{N}_a^I \hat{N}_b^J\right)^2 - 1}$
- scalar product of unit normals

$$\boxed{-\eta_{IJ} \hat{N}_a^I \hat{N}_b^J = \varepsilon_{ab} \cosh \beta_{ab}}$$

$\beta_{ab} \geq 0 \quad$ boost from 3-plane a to 3-plane b

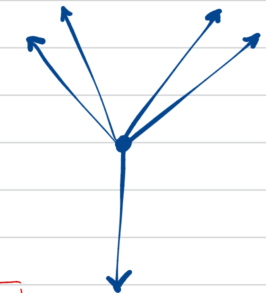
$$\varepsilon_{ab} = \begin{cases} +1 & \text{co-chronal 4-normals} \\ -1 & \text{anti-chronal 4-normals} \end{cases}$$

causal structure



$\boxed{1}$ Lorentzian Regge Action in terms of 4-normals N_a^I

$$\boxed{S_R = \sum_{a<b} \frac{A_{ab}}{8\pi G}\, \varepsilon_{ab}\, \beta_{ab}}$$



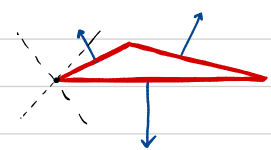
* Strategies for computing $\varepsilon_{ab}\beta_{ab}$ from 4-normals:

(a) Use $\cosh\beta \pm \sinh\beta = e^{\pm\beta}$

$$\varepsilon_{ab}\,\beta_{ab} = \log\left| -\eta_{IJ}\,\hat{N}_a^I\,\hat{N}_b^J + \sqrt{(-\eta_{IJ}\,\hat{N}_a^I\,\hat{N}_b^J)^2 - 1}\,\right|$$

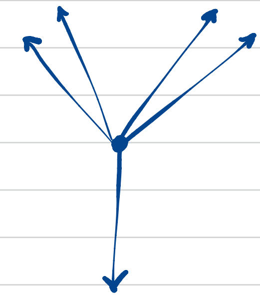
check:

$$\log|\varepsilon\cosh\beta + \sinh\beta| = \log|\cosh\beta + \varepsilon\sinh\beta| = \log|e^{\varepsilon\beta}| = \varepsilon\beta$$



1 Lorentzian Regge Action in terms of 4-normals N_a^I and causal structure ε_{ab}

$$\boxed{S_R = \sum_{a<b} \frac{A_{ab}}{8\pi G}\, \varepsilon_{ab}\, \beta_{ab}}$$



* Strategies for computing $\varepsilon_{ab}\beta_{ab}$ from 4-normals:

(b) Use knowledge of causal structure ε_{ab}

together with

$$\log\left|\mathrm{Eigenvals}\left[\hat{N}_a^I \hat{N}_b^J \sigma_I \sigma_J\right]\right| = \pm\, \varepsilon_{ab}\, \beta_{ab} \qquad \sigma_I = (\mathbb{1}, \vec{\sigma})$$

to select eigenval with + sign.

Note: in spinfoam asymptotics, the Regge action arises via a mechanism analogous to (b).

$\Longrightarrow$ Proposal: use causal structure ε_{ab} as additional data in spinfoams

Engle, Pereira, Rovelli & Livine (2007)

Freidel & Krasnov (2007)

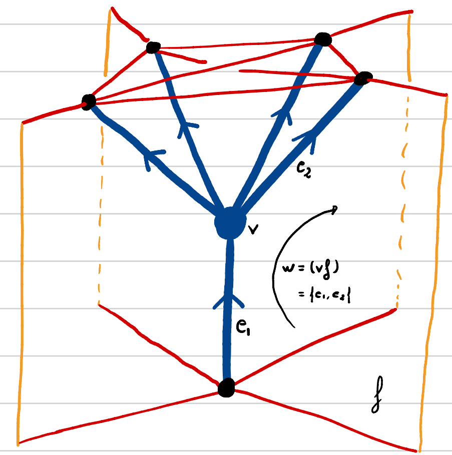
Barrett, Dowdall, Fairbairn, Gomes & Hellmann (2009)

Kaminski, Kisielowski & Lewandowski (2009)

→ Donà & Speziale (2020) "Asympt. of lowest unitary..."

$\boxed{2}$ Lorentzian EPRL Vertex Amplitude $A_v^{EPRL}(j_f, i_e)$

- Vertex dual to a 4-simplex → boundary graph Γ_5
- Boundary data: $\begin{cases} \bullet \text{ spins (on faces) } & j_{ab} \\ \bullet \text{ spinors (on half-wedges) } & \zeta_{ab},\ \zeta_{ba} \end{cases}$



- Vertex Amplitude

$$A_v^{EPRL}(\{j_{ab}, \zeta_{ab}\}) = \int_{SL(2,\mathbb{C})} \prod_{a=2}^{4} dh_a \prod_{a<b} K^{EPRL}(h_a, h_b; j_{ab}, \zeta_{ab}, \zeta_{ba})$$

- Wedge Amplitude

$$K^{EPRL}(h_a, h_b; \underbrace{j_{ab}, \zeta_{ab}, \zeta_{ba}}_{\text{boundary data}}) = [j_{ab}, \zeta_{ba}|\, D^{(\gamma j_{ab}, j_{ab})}(h_b^{-1} h_a)\, |j_{ab}, \zeta_{ab}\rangle$$

* New Proposal

Engle, Pereira, Rovelli & Livine (2007)

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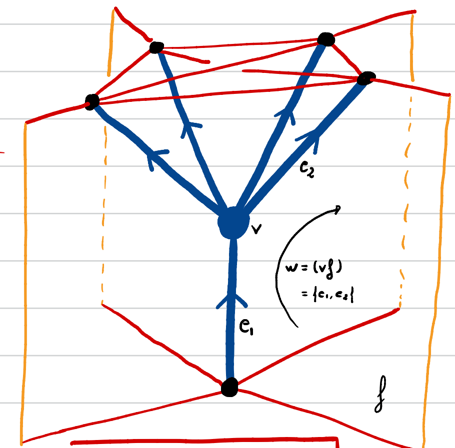
2 Causal EPRL Vertex Amplitude $A_v(j_f, i_e, \varepsilon_w)$

- Vertex dual to a 4-simplex → boundary graph Γ_5
- Boundary data:
  - spins (on faces) j_{ab}
  - spinors (on half-wedges) ζ_{ab}, ζ_{ba}
  - causal structure (on wedges) $\varepsilon_{ab} = \pm 1$



- Causal Vertex Amplitude

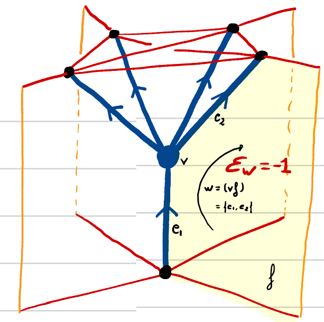
$$A_v(\{j_{ab}, \zeta_{ab}, \varepsilon_{ab}\}) = \int_{SL(2,\mathbb{C})} \prod_{a=2}^{4} dh_a \prod_{a<b} K(h_a, h_b; j_{ab}, \zeta_{ab}, \zeta_{ba}, \varepsilon_{ab})$$

- Causal Wedge Amplitude

$$\boxed{K(\underbrace{h_a, h_b; j_{ab}, \zeta_{ab}, \zeta_{ba}, \varepsilon_{ab}}_{\text{boundary data}}) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi i} \frac{\varepsilon_{ab}}{\tau - \gamma - i\,\varepsilon_{ab}\, 0^+} [j_{ab}, \zeta_{ba}| D^{(\tau j_{ab}, j_{ab})}(h_b^{-1} h_a) | j_{ab}, \zeta_{ab}\rangle}$$

* Feynman $i\varepsilon$ prescription for unitary irrep of $SL(2,\mathbb{C})$

* New Proposal



$\boxed{2}$ Causal EPRL Vertex Amplitude $A_v(j_f, i_e, \varepsilon_w)$

$$\boxed{K(h_a, h_b; \underbrace{j_{ab}, \varsigma_{ab}, \varsigma_{ba}, \varepsilon_{ab}}_{\text{boundary data}}) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi i} \frac{\varepsilon_{ab}}{\tau - \gamma - i\varepsilon_{ab} 0^+} [j_{ab}, \varsigma_{ba}| D^{(\tau j_{ab}, j_{ab})}(h_b^{-1} h_a) | j_{ab}, \varsigma_{ab}\rangle}$$

- **Remarks:**

i) EPRL recovered as unconstrained sum $K^{EPRL}(\dots) = \sum_{\varepsilon_{ab} = \pm 1} K(\dots, \varepsilon_{ab})$

ii) Linear in boundary state $\Rightarrow$ A_v can be expressed in any basis (or as operator)

iii) The Feynman $i\varepsilon$ integral is well defined and residue results in sum of ${}_2F_1(z)$ analogous to EPRL

iv) Beyond single vertex: 2-complex C with causal edge orientation ε_a $\Rightarrow$ wedge orientation $\varepsilon_{ab} = \varepsilon_a \varepsilon_b$

note: reverse, $W_C^{EPRL} = \sum_{\varepsilon_w = \pm 1} W_C(\varepsilon_w) = \sum_{\varepsilon_w \in \mathrm{Causal}(C)} W_C(\varepsilon_w) + \sum_{\varepsilon_w \notin \mathrm{Causal}(C)} W_C(\varepsilon_w)$

v) Relation to Engle's Proper Vertex for IIb+ : $\varepsilon_{ab}^{(\mathrm{IIb}+)} = f(\text{all 4-simplex boundary data } \{j_{cd}\}, \{\varsigma_{cd}\})$

Here: analogous expression, but local on additional wedge data ε_{ab}

- **To do:**

a) Numerical implementation of the $i\varepsilon$ $\rightsquigarrow$ note (iii) expression analog to booster functions [Speziale (2017), Donà-Fanizza-Sarno-Speziale (2019)]

b) Interpretation of $i\varepsilon$ prescription as Feynman Green function for γ-simplicity : $(\vec{K} + \gamma \vec{L}) G_F(h_a, h_b) = 0$

c) Large spin asymptotics

3 Large spin asymptotics & semiclassical limit

Barrett, Dowdall, Fairbairn, Gomes & Hellmann (2009)
→ Donà & Speziale (2020) "Asympt. of lowest unitary..."

- Causal Wedge Amplitude as integral over $\mathbb{C}P^1$

$$K(h_a, h_b; \underbrace{j_{ab}, \zeta_{ab}, \zeta_{ba}, \varepsilon_{ab}}_{\text{boundary data}}) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi i} \frac{\varepsilon_{ab}}{\tau - \gamma - i\varepsilon_{ab} 0^+} [j_{ab}, \zeta_{ba}| D^{(\tau j_{ab}, j_{ab})}(h_b^{-1} h_a) | j_{ab}, \zeta_{ab}\rangle$$

$$= \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi i} \frac{\varepsilon_{ab}}{\tau - \gamma - i\varepsilon_{ab} 0^+} \quad \frac{2j_{ab}+1}{\pi} \int_{\mathbb{C}P^1} \frac{i}{2} \frac{[z_{ab}|dz_{ab}\rangle \wedge [\bar{z}_{ab}|d\bar{z}_{ab}\rangle}{\|h_a^\dagger z_{ab}\|^2 \, \|h_b^\dagger z_{ab}\|^2} \cdot$$

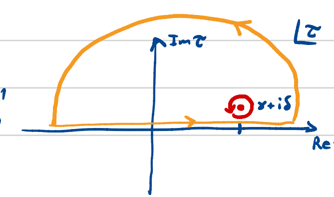
$$\cdot \exp\left[j_{ab} \log \frac{\langle \zeta_{ab}|h_a^\dagger z_{ab}\rangle^2 \langle h_b^\dagger z_{ab}|\zeta_{ba}]^2}{\|h_a^\dagger z_{ab}\|^2 \, \|h_b^\dagger z_{ab}\|^2} + i\tau j_{ab} \log \frac{\|h_b^\dagger z_{ab}\|^2}{\|h_a^\dagger z_{ab}\|^2} \right]$$

use:

$$\boxed{\lim_{\delta \to 0^+} \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi i} \frac{\varepsilon_{ab}}{\tau - \gamma - i\varepsilon_{ab}\delta} e^{i\tau x} = \theta(\varepsilon_{ab} x)\, e^{i\gamma x}}$$

eg

$\varepsilon_{ab} = +1$
$x > 0$



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$\boxed{3}$ Large spin asymptotics & semiclassical limit

- Causal Wedge Amplitude as integral over $\mathbb{C}P^1$

$$K(h_a, h_b; \underbrace{j_{ab}, \zeta_{ab}, \zeta_{ba}, \varepsilon_{ab}}_{\text{boundary data}}) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi i} \frac{\varepsilon_{ab}}{\tau - \gamma - i\varepsilon_{ab} 0^+} [j_{ab}, \zeta_{ba}| D^{(\tau j_{ab}, j_{ab})}(h_b^{-1} h_a) | j_{ab}, \zeta_{ab}\rangle$$

$$= \frac{2j_{ab}+1}{\pi} \int_{\mathbb{C}P^1} \frac{i}{2} \frac{[z_{ab}|dz_{ab}\rangle \wedge [\bar{z}_{ab}|d\bar{z}_{ab}\rangle}{\|h_a^\dagger z_{ab}\|^2 \, \|h_b^\dagger z_{ab}\|^2} \cdot \Theta\left(\varepsilon_{ab} \, j_{ab} \log \frac{\|h_b^\dagger z_{ab}\|^2}{\|h_a^\dagger z_{ab}\|^2}\right) \cdot$$

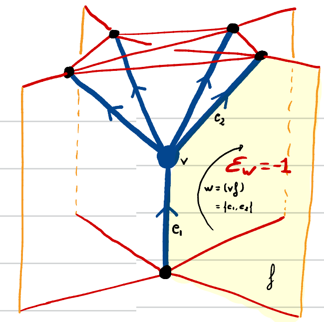
$$\cdot \exp\left[j_{ab} \log \frac{\langle \zeta_{ab}| h_a^\dagger z_{ab}\rangle^2 \langle h_b^\dagger z_{ab} | \zeta_{ba}]^2}{\|h_a^\dagger z_{ab}\|^2 \, \|h_b^\dagger z_{ab}\|^2} + i\gamma \, j_{ab} \log \frac{\|h_b^\dagger z_{ab}\|^2}{\|h_a^\dagger z_{ab}\|^2} \right]$$

- For Lorentzian Regge boundary data $\{j_{ab}, \zeta_{ab}, \varepsilon_{ab}\}$ $\Longrightarrow$ N_a^I 4-simplex

Single saddle point in integrals over h_a, z_{ab} $\Rightarrow$ $\boxed{A_v(\lambda j_{ab}, \zeta_{ab}, \varepsilon_{ab}) = \frac{1}{\lambda^{12} N} e^{+i\lambda S_R} + O(\lambda^{-13})}$

where $S_R = \sum_{ab} \gamma j_{ab} \, \varepsilon_{ab} \beta_{ab}$

Causal EPRL Vertex Amplitude $\quad A_v(j_f, i_e, \varepsilon_w)$



$$\boxed{K(h_a, h_b; \underbrace{j_{ab}, \zeta_{ab}, \zeta_{ba}, \varepsilon_{ab}}_{\text{boundary data}}) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi i} \frac{\varepsilon_{ab}}{\tau - \gamma - i\,\varepsilon_{ab}\, 0^+} [j_{ab}, \zeta_{ba}|\, D^{(\tau j_{ab}, j_{ab})}(h_b^{-1} h_a)\, | j_{ab}, \zeta_{ab}\rangle}$$

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✓ c) Large spin asymptotics $\Rightarrow$ e^{+iS_R} for Lorentzian boundary data (Euclidean and vector geoms cut by θ)

d) Analyze bubble ξ^{∞} divergences following Christodoulou, Långvik, Riello, Röken & Rovelli (2013) "Divergences and orientation ..." e^{+iS_R} in Ponzano-Regge