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Exploring alternatives to the Hamiltonian formulation of the AOS black hole model

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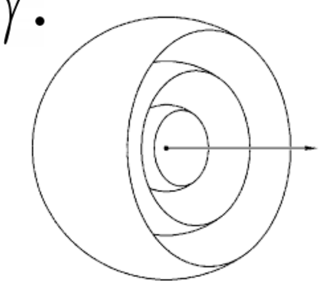
Introduction

- ✓ System: interior region of a Schwarzschild black hole.
- ✓ Canonical formalism: Ashtekar-Barbero variables + constraints.
- ✓ Dynamical information contained in two canonical pairs:

$$\{b, p_b\} = G\gamma, \qquad \{c, p_c\} = 2G\gamma.$$

Radial sector — Angular sector



- ✓ Variables bound by the Hamiltonian constraint:

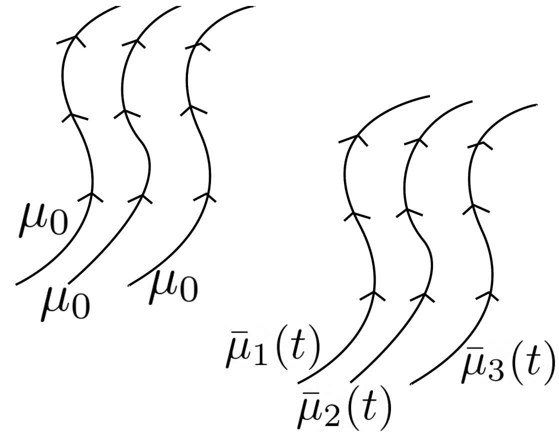
$$N = \frac{\gamma\delta_b\sqrt{|p_c|}}{\sin\delta_b b}, \qquad NH = \frac{L_o}{G}(O_b - O_c),$$

$$O_b = -\frac{1}{2\gamma}\left(\frac{\sin\delta_b b}{\delta_b} + \frac{\gamma^2\delta_b}{\sin\delta_b b}\right)\frac{p_b}{L_o}, \qquad O_c = \frac{1}{\gamma}\frac{\sin\delta_c c}{\delta_c}\frac{p_c}{L_o}.$$

- ✓ Partial Hamiltonians: functions of dynamical variables and polym. param.

$$\delta_b, \delta_c \to 0.$$

Selection of parameters

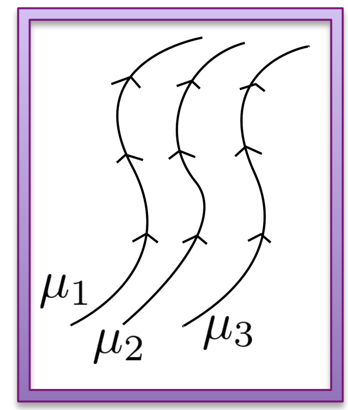


- ✓ How to choose these parameters?

  1. Constants on the whole phase space.

  2. Arbitrary functions of phase space.

  1.5. Constant along dynamical trajectories.



- ✓ AOS → intermediate path (1.5).

- ✓ How to define the polym. parameters as Dirac observables?
  → Functions of constants of motion.

- ✓ The partial Hamiltonians turn out to be constants of motion:

$$O_b\big|_{\text{on-shell}} = O_c\big|_{\text{on-shell}} = m.$$

- ✓ AOS → $\delta_i = g_i(m)$ → in practice, their Poisson brackets are ignored.

Parameters as constants of motion

- ✓ Bodendorfer, Mele, Münch → $\delta_i = g_i(O_i)$.
- ✓ Hamiltonian description where the Poisson brackets of the parameters are taken into account in the derivation of the dynamical equations?
- ✓ The contributions of each partial Hamiltonian cannot be told apart:

$$\delta_i = f_i(O_b, O_c).$$

- ✓ The radial and angular sectors no longer decouple!

$$\partial_t i = C_i \left[s_i \frac{L_o}{G} \{i, p_i\} \frac{\partial O_i}{\partial p_i} \right], \quad \partial_t p_i = C_i \left[-s_i \frac{L_o}{G} \{i, p_i\} \frac{\partial O_i}{\partial i} \right],$$

- ✓ A phase space dependent factor multiplies the AOS dynamical equations.

$$C_i = \frac{1 - \Delta_{jj} - \Delta_{ji}}{(1 - \Delta_{ii})(1 - \Delta_{jj}) - \Delta_{ij}\Delta_{ji}}, \quad j \neq i, \quad \Delta_{ij} = \frac{\partial O_i}{\partial \delta_i} \frac{\partial f_i}{\partial O_j}.$$

Time redefinitions

- ✓ These factors can be reabsorbed through time redefinitions:

$$d\tilde{t}_i = C_i dt.$$

- ✓ These redefinitions are sector dependent!

- ✓ Dynamical equations adopt the same form as AOS when written in terms of two a priori different time variables.

- ✓ ¿Can the off-shell freedom be used to set $\tilde{t}_b|_{\text{on-shell}} = \tilde{t}_c|_{\text{on-shell}}$?

$$\boxed{C_b|_{\text{on-shell}} = C_c|_{\text{on-shell}}}$$

$$1 - F_c(p_c)\frac{\partial f_c(m,m)}{\partial m} = 1 - F_b(p_b)\frac{\partial f_b(m,m)}{\partial m}.$$

- ✓ The answer is in the negative, unless we consider constant param. on the whole phase space.

Large black hole mass limit

- ✓ The two time variables coincide at the lowest order in an asymptotic expansion:

$$\lim_{m\to\infty} \tilde{t}_b = \lim_{m\to\infty} \tilde{t}_c = \tilde{t}.$$

- ✓ In the asymptotic limit, the equations of motion of the AOS model can be recovered.

- ✓ In this asymptotic limit, the solutions of the AOS model are solutions of our dynamical equations, up to subdominant contributions.

- ✓ Indeed, an asymptotic expansion reveals that

$$\lim_{m\to\infty} F_i(p_i)\frac{\partial f_i(m,m)}{\partial m} = 0.$$

- ✓ Hence, the equality condition that could not be made work unless the parameters were taken as constants does hold in the limit of large masses.

Relation between time variables

- ✓ We can obtain the exact relation between the two time variables by integrating

$$\frac{d\tilde{t}_b}{C_b} = \frac{d\tilde{t}_c}{C_c}.$$

- ✓ In the limit of large black hole masses, we find that

$$\tilde{t}_c = \tilde{t}_b - \frac{1}{9}\gamma^2 \left(-3\tilde{t}_b + 3\sinh\tilde{t}_b + \cosh\tilde{t}_b - 1\right)\delta_b^2 + o\left(\delta_b^2\right).$$

$\delta_i \sim m^{-1/3}$

- ✓ The leading order reproduces the result discussed in the previous slide.

Conclusions

- ✓ Our objective is to explore alternatives that had not been explored before in order to see whether other proposals can be reconciled with the results of the original model.
- ✓ The effect of considering polymerisation parameters that are Dirac observables is encoded in two phase space dependent factors in the dynamical equations. These can be interpreted as local time redefinitions: two distinct time variables appear if the parameters are chosen as constants of motion.
- ✓ These two variables are approximately similar to each other in the asymptotic limit of large black hole masses, recovering the dynamical equations of the AOS model.
- ✓ In this limit, the original results of the AOS model can be reconciled with our formalism to a certain extent, since the spacetime geometry is modified.
- ✓ This might lead to an alleviation of certain criticisms of the original model (asymptotic behaviour…).

References

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Thank you

for your attention