
Title: A solution of the cosmological constant and DE and arrow of time, using Model of a nonsingular universe from Rosen from Volume (56) Ettore Majorana International science series, physics, 1991.

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Abstract:

We reduplicate the Book “Dark Energy” by M. Li, X-D. Li, and Y. Wang, given zero-point energy calculation with an unexpected “length’ added to the ‘width’ of a graviton wave via the ideas given in the text by Rosen

1. Introduction

What we are doing is to try to confirm if we can apply the techniques of the following reference to the problem of DE and the arrow of time, and heavy gravity. After work I did in [1] was allegedly not credible, due to people having doubts as to the existence of a multiverse and equating two first integrals as I did, via early pre Planckian space-time, the following reference was accessed [2]. And then applied to [3] and the work on heavy gravity in [4]. In doing so we will keep in mind the t’Hooft memorandum as to the arrow of time, which is in [5] as a basic organizational principle for our discussion, i.e. formation of our program is assuming initial conditions for using [4] in the expansion of the universe say after 10^{-42} seconds

$$m_g = \frac{\hbar\sqrt{\Lambda}}{c} \quad (1)$$

Whereas we ask for initial conditions for the arrow of time, and Λ and DE formation

2. Methods for defining DE and heavy gravity

We will first start off with the redone calculation as to the Vacuum energy as given in [3] and how we rescale them to be in sync as to the observed experimentally given value for vacuum energy which is of the present era. This methodology is consistent with the zero point energy calculation, we start off with the following as given by [3]

$$\frac{1}{2} \cdot \sum_i \omega_i \equiv V(\text{volume}) \cdot \int_0^{\hat{\lambda}} \sqrt{k^2 + m^2} \frac{k^2 dk}{4\pi^2} \approx \frac{\hat{\lambda}^4}{16\pi^2}$$

$$\xrightarrow{\hat{\lambda}=M_{\text{Planck}}} \rho_{\text{boson}} \approx 2 \times 10^{71} \text{ GeV}^4 \approx 10^{119} \cdot \left(\rho_{\text{DE}} = \frac{\Lambda}{8\pi G} \right) \quad (2)$$

In stating this we have to consider that $\rho_{\text{DE}} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{\text{DE}}^4}$, so then

that the equation we have to consider is a wavelength $\lambda_{\text{DE}} \approx 10^{30} \ell_{\text{Planck}}$

which is about 10^{30} times a Plank length radius of a space-time bubble. That would mean we have after 10^{-42} seconds.

$$\lambda_{\text{DE}} \approx 10^{30} \ell_{\text{Planck}} \quad (3)$$

We then have to consider how to reach the experimental conditions for when

$$\rho_{DE} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{DE}^4} \quad (4)$$

a nonsingular expansion point for Cosmology, will after 10^{-42} seconds lead to

Eq. (4). That means a discussion of what Rosen and Israelit did in [2]. Our point to applying [2] to Eq. (3) is that we have a factor of 10^{30} expansion as to where we can at least measure the onset of DE, for reasons which will be in the next section so Eq. (4) has a value of roughly DE in magnitude as given in [3]

2.1 Looking now at Rosen and Israelit, in terms of Thermodynamics of a non-singular universe

[2] will be relevant for several reasons.

A. We will be able to come up with an initial temperature of 10^{-180} Kelvin, at a radius of about Planck length, in value. Almost absolute zero

B. The temperature of space-time will be of the order of Planck Temperature after expansion of about 10^{30} times from the initial nonsingular configuration

C. For making effective use of [3] we will be looking at Eq. (1) to Eq.(4) as being measured after 10^{-42} seconds, which is roughly Planck Time, in this model. I.e.

the convention is that we will be using is that Eq.(1) to Eq.(4) will be what [2] calls the pre-matter radiation transition point, in the history of the universe, i.e. go to pages 153 to 154, of [2] and one sees that what I am doing is specifying the formation of Eq. (1) to Eq. (4) at the time the acceleration of the universe stops in its earliest phase, with the formation of DE, and cosmological Constant. We should also keep in mind that A. and B. and C will allow an arrow of time forming due to the reasons brought up in [5] whereas we have the following Entropy value of [6]

$$S \sim 3 \cdot [1.66\sqrt{g_*}]^2 T^3 \quad (5)$$

Whereas we have that [7] gives us a value $g_* \approx 100 - 110$. Hence it is time to do the treatment of the temperature values, of what that says about Entropy, and the arrow of time

2.2 Underlying thermodynamics of the Rosen-Israelit nonsingular model

In this section we outline temperature values T at beginning of expansion, at the end of expansion up to when DE is formed and answer if [5] criteria as to forming the arrow of time can be formed according to [5].

While noting the issue of casuality and causal relations, in the actual

context of the arrow of time[8] where we take into consideration the following, namely that

Quote

The **causal sets** program[8] is an approach to [quantum gravity](#). Its founding principles are that [spacetime](#) is fundamentally discrete (a collection of discrete spacetime points, called the elements of the causal set) and that spacetime events are related by a [partial order](#). This partial order has the physical meaning of the [causality relations](#) between spacetime events

End of quote

What we will assert is that the Rosen result, given in [2] may permit the introduction of the partial order in space-time which may allow for the introduction of quantum gravity

2.3. Formal development of the thermodynamics of space-time and its relations to DE

The key point of this mini chapter will be to summarize the derivation of the temperature[2]

$$T = \left(\rho_P / \sigma \right)^{1/4} \cdot \frac{\tilde{a} r^7}{\left(\tilde{a}^4 + r^4 \right)^2} \quad (6)$$

Whereas $\left(\rho_P / \sigma \right)^{1/4} = 1.574 \times 10^{32} \text{ K(kelvin)}$, and $\tilde{a} = 10^{-3} \text{ cm}$,

whereas

$$r_{initial} = (3 / 8\pi\rho_p)^{1/2} = 5.58 \times 10^{-34} \text{ cm}$$

$$\Rightarrow T_{initial} = 2.65 \times 10^{-180} \text{ K (kelvin)} \quad (6a)$$

$$r_{DE\text{-formation}} = \tilde{a} = 10^{-3} \text{ cm}$$

$$\Rightarrow T_{DE\text{-formation}} = 7.41 \times 10^{31} \text{ K (kelvin)} \quad (6b)$$

We will be deriving Eq.(6) in terms of $\rho = \rho(T), P = P(T)$ as a summary of what to expect in this treatment of nonsingular space-time. To do so we start off with [2] in pre matter and radiation periods with entropy S ,

$$dS(V, T) = \frac{1}{T} \cdot [d(\rho V) + PdV] \quad (7)$$

$$V = V(\text{volume}) = 2\pi^2 r^3 \quad (7a)$$

And an integrability condition on Eq. (6) leading to

$$\frac{dP}{dT} = \frac{1}{T} \cdot (\rho + P) \quad (7b)$$

Then the integral of Eq. (7) is given as

$$S = \frac{V}{T} \cdot (\rho + P) \quad (7c)$$

Also, we look at a given value of pressure as given in [2] for which

$$P = \frac{\rho}{3} \cdot \left(1 - \frac{4\rho}{\rho_P} \right) \quad (8)$$

Put Eq. (7d) into Eq. (7b) and then one will get after integrating Eq. (7b)

$$\rho \cdot \left(1 - \frac{\rho}{\rho_P} \right)^7 = \sigma T^4 \quad (8a)$$

Here, [2] treated σ as the Stephan-Boltzman constant, and so then if we add in the energy equation

$$\dot{\rho} + 3 \cdot (\dot{r} / r) \cdot (\rho + P) = 0 \quad (8b)$$

Then we put in Eq. (8) into Eq. Eq. (8b) we obtain

$$\rho = \bar{a}^4 \rho_P / \left(\bar{a}^4 + r^4 \right) \quad (8c)$$

We claim that Eq. (8c) put into Eq. (8a) we will then obtain Eq. (6) with the conditions as we specified . We assert that we obtain through Eq. (6), Eq. (6a) and Eq.(6b) when the temperature is in the vicinity Planck temperature, that then we can introduce conditions for which we have Eq. (4) implemented[3]. Where we have a value of Planck's constant is at the value given in Eq. (6b) which [2] is the prematter -radiation boundary, so then that we are initiating DE as a function as the onset of the radiation

era of cosmology, and when DE commences we have by Eq.(1) conditons for the onset of gravitational physics

2.4 Analysis of the interrelationship between terms in the inflaton, for cosmology, inhomogeniety and Temperature T, at the prematter-radition boundary

We will set forth the following relationship

$$\frac{H^2}{\dot{\phi}} \approx 10^{-5} \quad (9)$$

Whereas, we are using by [10] , page 481 of that reference

$$H = 1.66\sqrt{g_*} \cdot \frac{T_{temp}^2}{m_P} \quad (9a)$$

Whereas we have from [11] the following time derivative value of the inflaton leading to, if we use Eq. (9) and also Eq. (9a)

$$\begin{aligned}
a(t) &= a_{initial} t^\nu \\
\Rightarrow \phi &= \ln \left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t \right)^{\sqrt{\frac{\nu}{16\pi G}}} \\
\Rightarrow \dot{\phi} &= \sqrt{\frac{\nu}{4\pi G}} \cdot t^{-1} \\
\Rightarrow \frac{H^2}{\dot{\phi}} &\approx \sqrt{\frac{4\pi G}{\nu}} \cdot t \cdot T^4 \cdot \frac{(1.66)^2 \cdot g_*}{m_P^2} \approx 10^{-5} \tag{10}
\end{aligned}$$

If we make use of Planck units, for $t \sim$ Planck time, $G = 1$, and Planck mass set = 1, and Planck Temperature T set also to 1 then Eq.(10) says that the coefficient ν just before turnabout, i.e. where the acceleration of inflation stopped is still very large, but not infinite, whereas if we do not do such Planck units, the terms t times T , representing time, t , and the 4th power of temperature T , mean that if we have, indeed nearly Planck temperatures, for T , that the time element t would be very small and so verifying the largeness of coefficient ν just before we have a cessation of acceleration, initially. If instead of using Eq.(9) for H , we use instead from [11] the following value of H as given in [11] only

$$H^2 = V_0 \exp\left(-\sqrt{\frac{16\pi G}{\nu}}\phi\right) = V_0 \cdot \left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t\right)^{\frac{1}{2} \cdot \sqrt{\frac{\nu}{\pi G}} - 4 \sqrt{\frac{\pi G}{\nu}}} \quad (11)$$

We then will get

$$\frac{H^2}{\dot{\phi}} \approx \sqrt{\frac{4\pi G}{\nu}} \cdot V_0 \cdot \left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}}\right)^{\frac{1}{2} \cdot \sqrt{\frac{\nu}{\pi G}} - 4 \sqrt{\frac{\pi G}{\nu}}} t^{\frac{1}{2} \cdot \sqrt{\frac{\nu}{\pi G}} - 4 \sqrt{\frac{\pi G}{\nu}} + 1} \approx 10^{-5} \quad (11a)$$

If we wish to just focus upon a general value for this 10^{-5} value, semi independent of time we can set

$$\frac{1}{2} \cdot \sqrt{\frac{\nu}{\pi G}} - 4 \sqrt{\frac{\pi G}{\nu}} + 1 = 0 \quad (11b)$$

A particular solution if we look at Planck units for which $G=1$ is

$$\nu = 5 \text{ which would then put every precise conditions upon } V_0, \text{ i.e.}$$

in Planckian units with $G=1$ we would have in Planck units,

normalized to = 1

$$\sqrt{\frac{4\pi}{5}} \cdot V_0 \cdot \left(\sqrt{\frac{8\pi V_0}{5 \cdot (14)}}\right)^{\frac{1}{2} \cdot \sqrt{\frac{5}{\pi}} - 4 \sqrt{\frac{\pi}{5}}} \approx 10^{-5} \quad (11c)$$

Note this is a particular solution but it would serve to put in

approximate values for V_0 about the time we would have the

formation of DE , and the cosmological constant, at a time step value approximately 10^{-42} seconds, at the time we have the first case of when the velocity would be maximized in inflation at the boundary of pre-matter and radiation, to quote [2]

2.3 What does Eq. (5) tell us about the arrow of time, problem ? using [2] ?

Were this to be true and the near zero temperature as given by Eq. (6a) versus the near Planck temperature at Eq.(6b), then going to the entropy expression of Eq. (5), we do have in this situation matching the requirements given by 't Hooft, [5] for which we can state that the construction of Eq.(5) combined with 't Hooft's particular solutions for initial conditions to the arrow of time, may indeed give a consistent arrow of time solution

2.4. What about the matter of Causal relations and initial conditions, using Dowker's construction and discussion of Posets ?

The author in [12]. had this initial construction, i.e. and is replicated for the record with several given Changes. We first give an initial equations of [7], [13] and then afterwards relate it to the Dowker physics [8]results Here, the idea would be, to make the following equivalence, i.e. look at, [7] where we have what we call Initial entropy value for when we

identify the cosmological constant. The value of Eq.(12) is assumed to be in magnitude about 10^{90} or so, which is the value of entropy if we use the following sort of model

$$\left[\left[\frac{\Lambda_{Max} r^4}{8\pi G} \right] \cdot (4/3) \cdot \left[\frac{2\pi^2 g_*}{45} \right]^{1/3} \right]^{3/4} \sim S_{initial} \quad (12)$$

We furthermore, make the assumption of a minimum radius of [14,15] where the r in Eq.(12) is the same in Eq. (13) below, and in magnitude 10^{30} times larger than when Entropy was effectively zero.

$$R_{initial} \text{ (when } \Lambda \text{ forms)} = \text{radius is } 10^{30} \text{ Planck length} \quad (13)$$

This Eq. (12) will be put as the minimum value of r , where we have in this situation [16, 17] with M the amount of Space-time matter energy at the start of the radiation era, and $l \approx 10^{29} - 10^{30}$ start – radius as given in Eq. (6b), with the start radius when we have almost zero entropy. If so then we have at 10^{-3} centimeters

$$\#bits \sim \left[\frac{E}{\hbar} \cdot \frac{l}{c} \right]^{3/4} \approx \left[\frac{Mc^2}{\hbar} \cdot \frac{l}{c} \right]^{3/4} \quad (14)$$

Needless to say we would have entropy defined as Eq.(14) to the $4/3^{\text{rd}}$ power, as to have a linkage between Entropy, bits and also the grid points in a space-time

lattice which may give us quantum gravity. Afterwards we likely to keep fidelity with the results we have worked with prior to this section have an invariant cosmological constant and would be applying our inquiry as to the application of Eq.(12) as of about where the cosmological constant formed up in an identifiable manner. Meaning, after 10^{-42} seconds, and at a radius of 10^{-3} centimeters, in line with the mass M being the “equivalent matter energy” at the boundary between pre matter states, and radiation as given in [2]. Keep in mind that the Energy E as given in Eq. (14) would have a temperature dependence as given in Eq.(6b) with an input parameter of E which can go into Eq. (14)

$$E = \frac{k_B (\text{dim} - \text{space} - \text{time})}{2} \cdot T \quad (15)$$

The points where we have bits, as computationally given would be the grid points to the Poset argument as in [8]. Whereas we can give the following relationship as to specify the inter-relationship between E and time. We pick Entropy as represented by an energy term E driving the entropy as given by T temperature dependence. As given in Eq.(15), whereas we use the cube of the same Temperature T driving entropy in Eq. (12)

2.5 Coming up with a “modified HUP so as to obtain the grid points implied by Eq. (14)

Shalyt-Margolin and Tregubovich (2004, p.73)[18], Shalyt-Margolin (2005, p.62)[19][20] have this Relationship. Here Delta E is assumed to be consistent in a change in energy from almost zero to the Energy value givein in Eq. (15)

$$\Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_P^2 \frac{\Delta E}{\hbar} \Rightarrow (\Delta E)^2 - \frac{\hbar \Delta t}{\gamma t_P^2} (\Delta E) + \frac{\hbar^2}{\gamma t_P^2} = 0$$

$$\Rightarrow \Delta E = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 + \sqrt{1 - \frac{4\hbar^2}{\gamma t_P^2 \cdot \left(\frac{\hbar \Delta t}{2\gamma t_P^2}\right)^2}} \right) = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \sqrt{1 - \frac{16\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2}} \right) \quad (16)$$

For sufficiently small γ . The above could be represented by[20]

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \left(1 - \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \right) \right)$$

$$\Rightarrow \Delta E \approx \text{either } \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2}, \text{ or } \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(2 - \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \right) \quad (17)$$

This would lead to a minimal relationship between change in E and change in time as represented by Eq. (17), so that we could to first order, say be looking at something very close to the traditional Heisenberg uncertainty principle results of approximately

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \equiv \frac{4\hbar}{\Delta t} \quad (18)$$

Or

$$\Delta E \Delta t \approx 4\hbar \quad (19)$$

i.e. stepping through an iteration of delta t, per causal structure in early space-time would with refinements help construct the grid and causal structures alluded to in [8], in particular see her discussion of the causal set given in page 3 of [8] whereas the grid defined by [8] would reflect the computational ‘bits’ given in Eq. (14)

3. Good points and limitations as to the given analysis, and what needs to be filled in

Our analysis has given evidence that we can satisfy the “tHoof” idea of special initial conditions as to forming the arrow of time. This is important, since there is a basic symmetry in the GR equations of space-time, which means forming the arrow of time, will necessitate specialized initial condtions even if the general GR equations do NOT depend upon specialized initial conditions. In addition, in the face of virtually unanimous complaints on the part of reviewers, the author has avoided describing the parcicular origins of a non singular start to expansion of the universe. The Rosen and Israel model assumes this nonsingular start, as seen in [2], without trying to derive where it came from. The author states that [2] gives a thermodynamically consistent nonsingular univese model which satisfied a mathematically consistent origin to the arrow of time problem

Due to the quirkiness of the [2] model, Rosen and Israelit also called the start of this expansion as a Point of zero time. That is right. The start to expansion is called time value “zero”

The unusual nature of this designation allowed the author to then go to a minimum time step, Δt which may be measurable, if one obtains in data sets a boundary regime which delinates the start of the radiation regime in cosmology.

Now for the limitations

The author abandoned any attempt in this document to specify WHERE the nonsingular start to cosmological expansion came from. As a physics researcher, this is an appalling omission, and is only done due to the innate conservatism of the general research community. What is mandatory is that a derivational approach to the origins of this nonsingular start be somehow meshed into a research program of hopefully gravitational physics data sets. In addition, [21] in terms of holographic principle applications of an interrelationship between the mass of a graviton and information needs to be explored

In lieu of specifying the time of Δt approximately 10^{-42} seconds and a defined initial space-time the following was obtained in terms of probable GW signals, from this early universe configuration

3.1 How do we obtain relic high frequency Gravity waves?

With redshift about $z = 10^{25}$ we go work with the following approximation

$$\begin{aligned}
(1 + z_{initial-era}) &\equiv \frac{a_{today}}{a_{initial-era}} \approx \left(\frac{\omega_{Earth-orbit}}{\omega_{initial-era}} \right)^{-1} \\
\Rightarrow (1 + z_{initial-era}) \omega_{Earth-orbit} &\approx 10^{25} \omega_{Earth-orbit} \approx \omega_{initial-era}
\end{aligned} \tag{20}$$

We postulate that we specify an initial era frequency via dimensional analysis which is slightly modified by Maggiore for the speed of a graviton [22] whereas we use that we assume having the following relationship of

$$c(\text{light-speed}) \approx \omega_{initial-era} \cdot (\lambda_{initial-post-bubble} = \ell_{Planck}) \text{ and that dimensional comparison with initially } T_{universe} \approx T_{Plank-temperature} = 1.22 \times 10^{19} \text{ GeV} .$$

If so then the initial temperature would be extremely high leading to a change in temperature from Pre Planckian conditions to Planck era .Where we would be

$$\text{assuming } \omega_{initial-era} \approx \frac{c}{\ell_{planck}} \leq 1.8549 \times 10^{43} \text{ Hz} \text{ so then we then}$$

would be looking at having frequencies on Earth from gravitons of mass $m(\text{graviton})$ less than or equal to

$$\omega_{Earth-orbit} \leq 10^{-25} \omega_{initial-era} \tag{21}$$

This is what would necessitate new technological developments and likely space-borne systems to analyze. The final point being that the brilliant work done by Rosen in [24] needs to be explored as possibly being relevant to the origins of the nonsingular start to the cosmological expansion. The author views [23] as a worthy starting point to quantum mechanical analogues which may explain this datum, missing in the [2] nonsingular start to the present universe which should be explored, as to its relevance to quantum mechanics and near space-time singularities. Rosen's [23] model may provide a bridge between interior conditions which may exist in a nonsingular start to expansion of the universe, and what is happening in our present cosmos. That as doing generalizations of what Ng [17] proposed as far as infinite quantum statistics for a counting algorithm style approach to early universe entropy. We also view that what is presented in [24] as to a quantum vacuum will be decisively important to explain the transition from the preinflationary state, as implied by [2] and the rapid expansion as given in Eq. (6), Eq. (6a) and Eq.(6b)

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