

Testing modified gravity theories with marked statistics

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AA, K. Koyama, H. Winther, J.L. Cervantes-Cota, B. Li. (1911.06362)
O. Philcox, AA, E. Massara. (2010.05914)
DESI collaboration: S. Alam, AA, R. Bean, et al. (2011.05771)

Sixteenth Marcel Grossmann Meeting

- 1. Motivation
- 2. Marked Density Field
- 3. Marked Correlation Function
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Motivation



Power spectrum: $(2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') P(\mathbf{k}) = \langle \delta(\mathbf{k}) \delta(\mathbf{k}') \rangle$



Baojiu Li's ELEPHANT simulations

- 1. Modified gravities which aim to explain the accelerated expansion of the Universe often invoque *screening* mechanisms to recover GR in certain limits.
- 2. The "standard" statistics used in Cosmology give the same weight to all observed objects.



Point 2 means that regions where there are more tracers become overrepresented in "standard" statistics. In turn, because of point 1, this implies that we are probing regions where the screenings are more efficient.

Chameleon screening mechanism

One of the most popular screening mechanisms is the so-called chameleon

$$V(\phi) = V_{\text{vacuum}}(\phi) + \tilde{V}(\phi, \rho)$$



$$m_{eff}^2 = V''(\phi_0) \gg V''_{\text{vacuum}}(\phi_{v,0}) = m^2$$



Cosmic voids radial profile



[P. Zivicki et al, 1411.5694]

Marked statistics are tailored to probe cosmological regions where screenings are not efficient.

[astro-ph.CO] 16 Nov 2016 arXiv:1609.08632v2

A marked correlation function for constraining modified gravity models

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Abstract. Future large scale structure surveys will provide increasingly tight constraints on our cosmological model. These surveys will report results on the distance scale and growth rate of perturbations through measurements of Baryon Acoustic Oscillations and Redshift. Space Distortions. It is interesting to ask: what further malyses should become routine, so as to test as-yet-unknown models of cosmic acceleration? Models which aim to explain the accelerated expansion rate of the Universe by modifications to General Relativity offern invoke screening mechanisms which can imprint a non-standard density dependence on their predictions. This suggests density-dependent clustering as a 'generic' constraint. This paper argues that a density-marked correlation function provides a density-dependent statistic which is continely available to such survey analyses. We give one realization of this idea and study it using low order perturbation theory. We encourage groups developing modified gravity theories to see Whether such statistics provide discriminatory power for their models.

Keywords: cosmological parameters from LSS - power spectrum - galaxy clustering

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Marked Density Field

Marked Density Field δ_M

As usual, the density fluctuation is introduced by $\rho(x, t) = \bar{\rho}(t) [1 + \delta(x, t)]$ Define the mark

$$m[\delta; R] = \left(\frac{1+\delta^*}{1+\delta^*+\delta_R}\right)^p,$$

If δ^* , p > 0, the mark up-weights low density regions.

 δ_R is the overdensity field δ averaged over regions of size R. It is the smoothed density fluctuation

$$\delta_R(\boldsymbol{x},t) = \int d^3 x' W_R(\boldsymbol{x}-\boldsymbol{x}')\delta(\boldsymbol{x}',t).$$

Finally, weight the (non-smoothed) density field with the mark

$$1 + \delta_M(\boldsymbol{x}, t) = \frac{m(\delta; R)}{\bar{m}} \left[1 + \delta(\boldsymbol{x}, t) \right]$$





[E. Massara ++, 2001.11024]

Marked Correlation Function

The *marked correlation function*, \mathcal{M} , is the 2-point configuration space correlation of marked fields. It is the sum of pairs of objects separated by a distance r, weighted by the ratio of the mark function value to the mean mark m_i/\bar{m} at each point and divided by the number of such pairs n(r):

$$\mathcal{M}(r) = \sum_{\{i,j\} \mid r_{ij} = r} \frac{m_i m_j}{n(r)\bar{m}^2}.$$

For an analytical treatment it is convenient to rewrite the above equation as

$$\mathcal{M}(r) = \frac{1 + W(r)}{1 + \xi(r)}$$

with

$$1 + W(r) = \frac{1}{\bar{m}^2} \langle \left[1 + \delta_M(\boldsymbol{x}_2) \right] \left[1 + \delta_M(\boldsymbol{x}_1) \right] \rangle$$

with $r = |\mathbf{x}_2 - \mathbf{x}_1|$, and $\xi(r) = \langle \delta(\mathbf{x}_2) \delta(\mathbf{x}_1) \rangle$ the correlation function,

and the mean mark $\bar{m} = \langle m(\delta_R)(1+\delta) \rangle$.



[S. Alam, AA, R. Bean, et al, 2011.05771]

Analytic description of the marked correlation function: [AA, K. Koyama, H. Winther, J.L. Cervantes & B.Li (1911.06362)]

Lagrangian Perturbation Theory:

$$1 + W(r) = \int d^3q \frac{e^{-\frac{1}{2}(r-q)^T \mathbf{A}^{-1}(r-q)}}{(2\pi)^{3/2} |\mathbf{A}|^{1/2}} \int d^3Q \frac{e^{-\frac{1}{2}(\mathbf{R}-\mathbf{Q})^T \mathbf{C}^{-1}(\mathbf{R}-\mathbf{Q})}}{(2\pi)^{3/2} |\mathbf{C}|^{1/2}} \Big[\underbrace{1}_{\mathbf{Z}\mathbf{A}} + \mathcal{I}\Big],$$

with $\mathcal{I} = \mathcal{I}(R, Q, r, q)$.

The components of the matrices \mathbf{A} and \mathbf{C} are

$$A_{ij}(\boldsymbol{q},t) = 2 \int \frac{d^3k}{(2\pi)^3} \left(1 - e^{i\boldsymbol{k}\cdot\boldsymbol{q}}\right) \frac{k_i k_j}{k^4} P_L(k)$$
$$C_{ij}(\boldsymbol{q},t) = \sigma_{\Psi}^2 \delta_{ij} - \frac{1}{4} A_{ij}^L(\boldsymbol{q},t),$$

with the "velocity" variance $\sigma_{\Psi}^2 = \frac{1}{6\pi^2} \int_0^\infty dp \ P_L(p)$

Biasing $\delta_X = b_{LS}\delta$

- Tracers, as halos, galaxies, quasars,..., are biased objects of the underlying CDM field.
- Large scale bias (b_{LS}) depends mainly on the quantity $\nu_c(M) = \delta_c(M)/\sigma^2(M)$.
- The density threshold for collapse $\delta_c(M)$ becomes mass dependent in MG due to a violation of Birkhoff's theorem and $\delta_c^{\text{MG}}(M) < \delta_c^{\text{GR}}(M)$.
- Meanwhile, the variances of perturbations on balls enclosing a mass M have $\sigma_{MG}^2(M) > \sigma_{GR}^2(M)$.

Hence, one typically finds $b_{\rm LS}^{\rm MG} < b_{\rm LS}^{\rm GR}$



Halo correlation function $\xi_h(r)$



Marking and biasing processes are degenerated:



This effect of inversion of the trends for the mCFs for tracers and matter can be interpreted by considering the mean mark, $\bar{m} \approx b_1 B_1 \sigma_R^2$, which shows that for the unbiased case $(b_1 = 1) \bar{m}_{\text{matter}}^{\text{MG}} < \bar{m}_{\text{matter}}^{\text{GR}}$, because $\sigma_R^{\text{MG}} > \sigma_R^{\text{GR}}$ and B_1 is negative. However, if the differences in linear local bias are sufficiently large they yield

 $\bar{m}_{\mathrm{tracers}}^{\mathrm{MG}} > \bar{m}_{\mathrm{tracers}}^{\mathrm{GR}}$

Marked correlation functions are highly linear



Comparison of models



Marked correlation function for halos



Marked Power Spectrum

Using the Marked Power Spectrum to Detect the Signature of Neutrinos in Large-Scale Structure

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Cosmological neutrinos have their greatest influence in voids: These are the regions with the highest neutrino to dark matter density ratios. The marked power spectrum can be used to emphasize low-density regions over high-density regions and, therefore, is potentially much more sensitive than the power spectrum to the effects of neutrino masses. Using 22 000 N-body simulations from the Quijote suite, we quantify the information content in the marked power spectrum of the matter field and show that it outperforms the standard power spectrum by setting constraints improved by a factor larger than 2 on all cosmological parameters. The combination of marked and standard power spectra allows us to place a 4.3 σ constraint on the minimum sum of the neutrino masses with a volume equal to 1 (Gpc h^{-1})³ and without cosmic microwave background priors. Combinations of different marked power spectra yield a 6 σ constraint within the same conditions.

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Through a Fisher information matrix analysis, Massara++ (2001.11024) concluded that the marked power spectrum can put tight constraints on the sum of the neutrino masses:

Power spectrum:	$\sigma(M_{\nu}) = 0.8 \mathrm{eV}$
Marked power spectrum:	$\sigma(M_{\nu}) = 0.017 \mathrm{eV}$

The marked PS is $47 \times$ more constrictive than the "standard" power spectrum!

(Warning: This analysis was done for CDM in configuration space.)

PT for the marked power spectrum

 $(2\pi)^2 \delta_{\rm D}(\boldsymbol{k} + \boldsymbol{k}') \mathcal{M}(\boldsymbol{k}) = \langle \delta_M(\boldsymbol{k}) \delta_M(\boldsymbol{k}') \rangle$

 $\mathcal{M}(k,\mu) = \left[C_0 - C_1 W_R(k)\right] \left[(\tilde{a}_0 + \tilde{a}_2 \mu^2) P_L(k) + \tilde{b}_0 \right] + 1 \text{-loop}$



O. Philcox, AA & E. Massara (arxiv:2010.05914)