String-like configurations in hybrid metric-Palatini gravity

XVIth Marcel Grossmann Meeting

Tiberiu Harko, Francisco Lobo, **Hilberto Silva**
(arXiv:2003.09751)

Instituto de Astrofísica e Ciências do Espaço

July 6, 2021

Overview

1. Hybrid metric-Palatini
2. Cosmic Strings
3. Solutions for different scalar field potentials
4. Conclusions

Addressing the imbalance - Motivation for Modified Gravity

- Adding a previously unaccounted source. (e.g Λ,...)

- Changing the governing equations.(e.g Gauss-Bonnet gravity, $f(R)$ gravity,...)

Hybrid metric-Platini

Both metric (arXiv:astro-ph/0306438) and Palatini(arXiv:1101.3864) approach are successful in reproducing acceleration rate of the Universe, but are flawed, particularly changing the solar system dynamics. So, a new approach was proposed(arXiv:1508.04641), the hybrid metric-Palatini:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4 x \sqrt{-g}\, [R + f(\mathcal{R})] + S_m \ , \tag{1}$$

$$\mathcal{R}_{\mu\nu} \equiv \hat{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \hat{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \hat{\Gamma}^{\alpha}_{\alpha\lambda}\hat{\Gamma}^{\lambda}_{\mu\nu} - \hat{\Gamma}^{\alpha}_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\alpha\nu} \ . \tag{2}$$

Dynamic equations(in scalar-tensor representation)

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left[(1+\phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m \ . \tag{3}$$

Performing the variation of the action with respect to the metric and the scalar field ϕ yields the field equations

$$\begin{aligned}(1+\phi)G_{\mu\nu} &= \kappa^2 T_{\mu\nu} + \nabla_\mu\nabla_\nu\phi - \nabla_\alpha\nabla^\alpha\phi g_{\mu\nu} \\ &- \frac{3}{2\phi}\nabla_\mu\phi\nabla_\nu\phi + \frac{3}{4\phi}\nabla_\lambda\phi\nabla^\lambda\phi g_{\mu\nu} - \frac{1}{2}Vg_{\mu\nu}\,,\end{aligned} \tag{4}$$

and

$$-\nabla_\mu\nabla^\mu\phi + \frac{1}{2\phi}\partial_\mu\phi\partial^\mu\phi + \frac{\phi}{3}\left[2V - (1+\phi)V_{,\phi}\right] = \frac{\phi\kappa^2}{3}T\,, \tag{5}$$

Cosmic strings

- Topological defects formed at phase transitions after SSB.
- Featured on several Grand Unified scenarios.
- Possible impact on CMB anisotropies, small scale structure formation, Gamma Ray Bursts, stochastic background GW spectra.

Straight infinite Cosmic String

$$T_t^t = T_z^z = -\sigma(r)\,, \tag{6}$$

where σ is the string tension. We consider a general cylindrically symmetric static metric

$$ds^2 = -e^{2(K-U)}dt^2 + e^{2(K-U)}dr^2 + e^{-2U}W^2 d\theta^2 + e^{2U}dz^2, \tag{7}$$

where t, r, θ and z denote the time, radial, angular and axial cylindrical coordinates, respectively, and K, U and W are functions of r alone.

Full set of equations

$\nabla_\mu T^\mu_\nu = 0 \implies K' = 0;$ Boost invariance (t and z) $\implies$ U=0; r=$\beta\xi$

$$\xi + C_0 = \int \frac{\phi^{-3/4} d\phi}{\sqrt{\left[C - \beta^2 \int \phi^{-3/2} V(\phi) d\phi\right]}}, \tag{8}$$

$$W(\phi) = W_0 \phi^{-3/4} \sqrt{C - \beta^2 \int \phi^{-3/2} V(\phi) d\phi}, \tag{9}$$

$$\kappa^2 \sigma(\phi) = \frac{1}{4\phi} \left\{ \left[2\phi(\phi+1) V'(\phi) + 3\sqrt{\phi} \int \frac{V(\phi)}{\phi^{3/2}} d\phi \; -2(2\phi+3) V(\phi) \right] - 3\left(C/\beta^2\right) \sqrt{\phi} \right\} \tag{10}$$

An important physical parameter characterizing the cosmic string properties is the mass per unit length of the string, which is defined as

$$m(r) = \int_0^{2\pi} d\theta \int_0^{R_s} \sigma(r) W(r) dr = 2\pi \int_0^{R_s} \sigma(r) W(r) dr, \tag{11}$$

where R_s is the string radius.

$$m(\xi_s) = 2\pi\beta \int_0^{\xi_s} \sigma(\xi) W(\xi) d\xi, \tag{12}$$

where $\xi_s = R_s/\beta$.

V=V$_0$

$\beta^2 V_0 = 1$

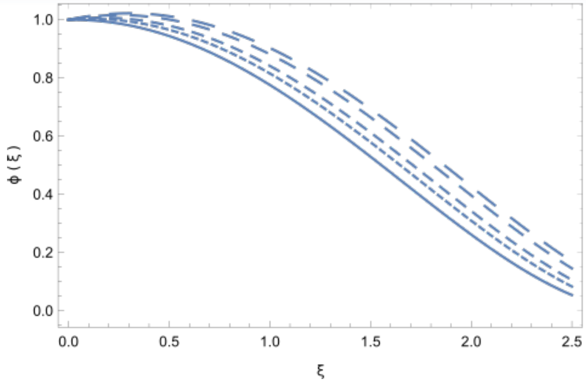

Figure: Variation of the scalar field in the presence of a constant potential for $\phi(0) = \phi_0 = 1$, and for different values of ϕ_0'.

V=V$_0$

$\beta^2 V_0 = 1$

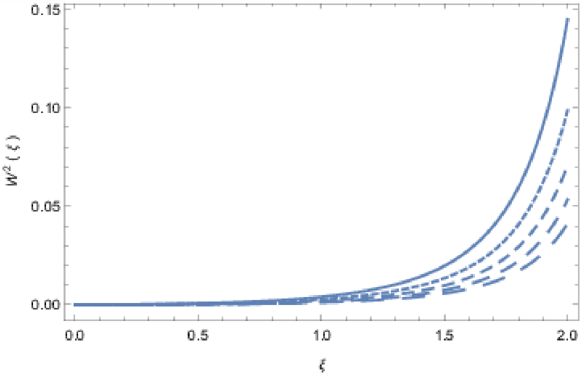

Figure: Variation of the metric tensor component $W^2(\xi)$in the presence of a constant potential for $\phi\left(10^{-7}\right) = 1,\ W\left(10^{-7}\right) = 10^{-3}$, and for different values of ϕ_0'.

V=V$_0$

$\beta^2 V_0 = 1$

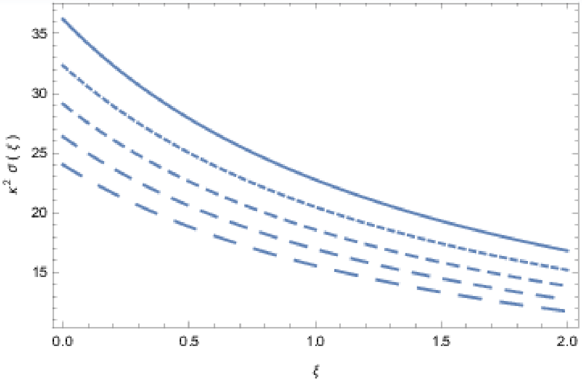

Figure: Variation of the energy density $\kappa^2\sigma(\xi)$ in the presence of a constant potential for $\phi(0) = 1$, and for different values of ϕ_0'.

$$V = V_0 e^{-\lambda\phi}$$

$\phi = \Phi/\lambda; \quad \beta = \sqrt{2/V_0\lambda}$

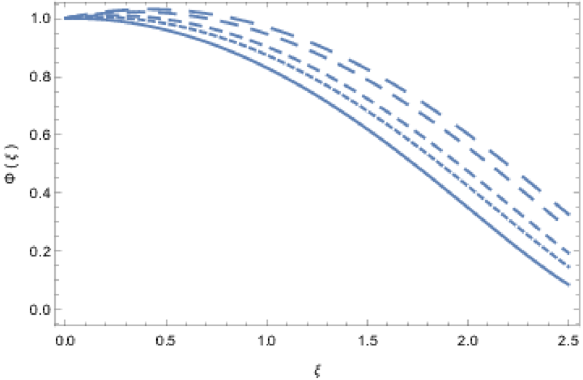

Figure: Variation of the scalar field in the presence of an exponential potential $V(\Phi) = e^{-\Phi}$ for $\Phi(0) = \Phi_0 = 1$, and for different values of Φ_0'.

$V = V_0 e^{-\lambda\phi}$

$\phi = \Phi/\lambda; \quad \beta = \sqrt{2/V_0\lambda}$

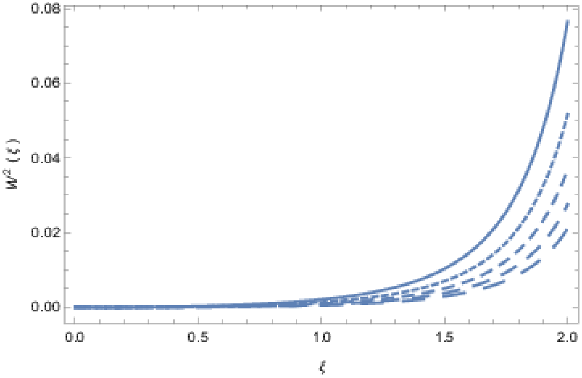

Figure: Variation of the metric tensor component $W^2(\xi)$ in the presence of an exponential potential $V(\Phi) = e^{-\Phi}$ for $\Phi(0) = 1$, $W(0) = 10^{-3}$, and for different values of Φ_0'.

$V = V_0 e^{-\lambda\phi}$

$\phi = \Phi/\lambda; \quad \beta = \sqrt{2/V_0\lambda}$

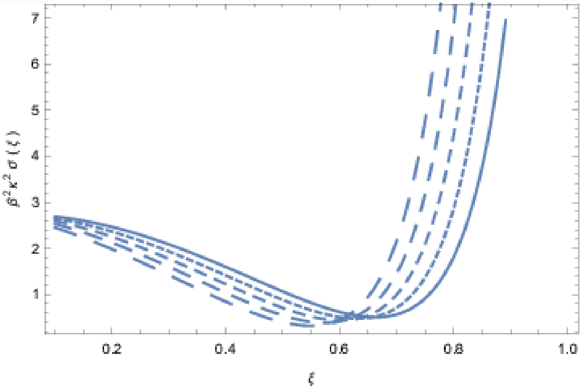

Figure: Variation of the energy density $\kappa^2\sigma(\xi)$ in the presence of an exponential potential $V(\Phi) = e^{-\Phi}$ for $\Phi(0) = 1$, and for different values of Φ_0'.

$$V(\phi) = \pm\frac{\bar{\mu}^2}{2}\phi^2 + \frac{\nu}{4}\phi^4$$

$$r = \sqrt{2}\bar{\mu}\xi; \qquad \phi = \frac{\Phi}{(\nu\bar{\mu})^{\frac{1}{3}}}$$

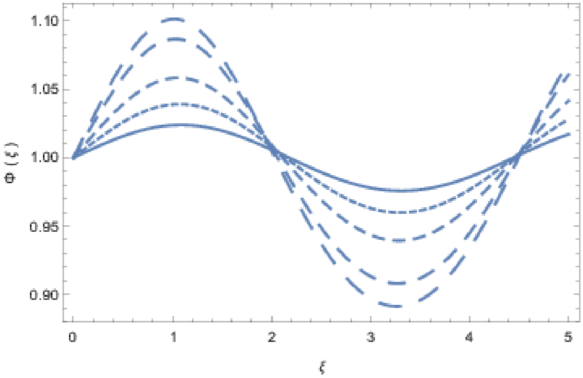

Figure: Variation of the scalar field in the presence of a Higgs-type potential $V(\Phi) = -\Phi^2 + \Phi^4$ for $\Phi(0) = \Phi_0 = 1$, and for different values of Φ_0'.

$$V(\phi) = \pm\frac{\bar{\mu}^2}{2}\phi^2 + \frac{\nu}{4}\phi^4$$

$r = \sqrt{2}\bar{\mu}\xi; \qquad \phi = \frac{\Phi}{(\nu\bar{\mu})^{\frac{1}{3}}}$

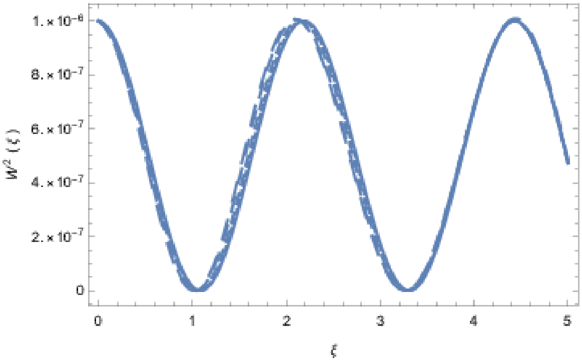

Figure: Variation of the metric tensor component $W^2(\xi)$ in the presence of a Higgs type potential $V(\Phi) = -\Phi^2 + \Phi^4$ for $\Phi\,(0) = 1$, $W\,(0) = 10^{-3}$, and for different values of Φ_0'.

$$V(\phi) = \pm\frac{\bar{\mu}^2}{2}\phi^2 + \frac{\nu}{4}\phi^4$$

$r = \sqrt{2}\bar{\mu}\xi; \qquad \phi = \frac{\Phi}{(\nu\bar{\mu})^{\frac{1}{3}}}$

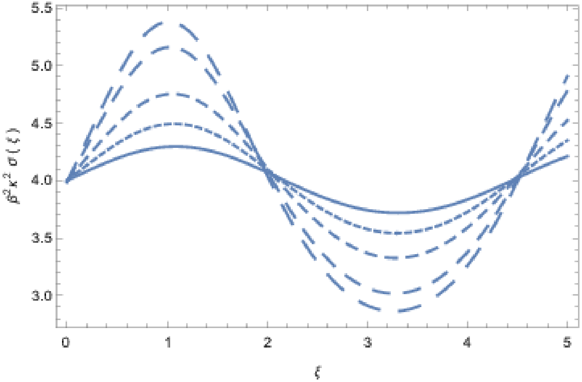

Figure: Variation of the string tension $\kappa^2\sigma(\xi)$ in the presence of a Higgs type potential $V(\Phi) = -\Phi^2 + \Phi^4$ for $\Phi(0) = 1$, $W(0) = 10^{-3}$, and for different values of Φ_0'.

Conclusions and Future Prospects

- Vilenkin-type strings are inconsistent in Brans-Dicke gravity, but not in hybrid metric-Palatini.
- In all the considered solutions, one can't define an axis, since $W^2(0) \neq 0$
- Behaviour strongly dependent on $\phi(0)$ and $\phi'(0)$
- With a Higgs potential, strings tension does not vanish for any value of the radial coordinate.
- $V = 0$ and $V(\phi) = V_0\phi^{3/4}$ have analitical solutions (please, see paper for more on this)
- More general type of string configurations must be studied in the hybrid metric-Palatini gravity (e.g Superconducting, loops, etc.)