Constraining neutrino mass using three-point mean relative velocity statistics

(Based on Kuruvilla & Aghanim, Submitted to A&A, 2020; arXiv:2102.06709)

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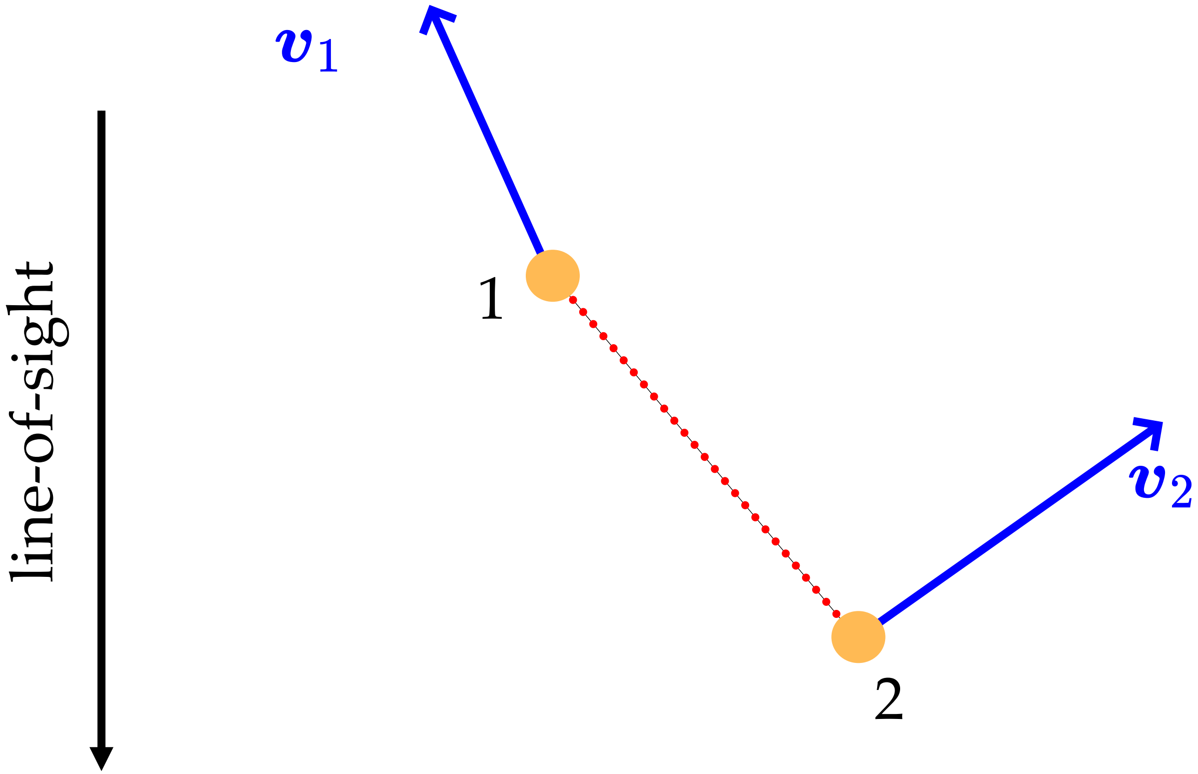
Current constraints on summed neutrino mass

- The Karlsruhe Tritium Neutrino (KATRIN) experiment has reported the first direct detection of sub-eV neutrino mass, with an upper limit on the 'effective neutrino mass' of 0.8 eV. (Aker et al. 2021)
- By combining cosmic microwave background, baryonic acoustic oscillation, and redshift-space galaxy clustering to obtain an upper limit of 0.102 eV. (eBOSS Collaboration 2021)
- Considering extensions of the standard cosmological model, the upper limit becomes less stringent. (e.g. Vagnozzi et al. 2018; Choudhury & Hannestad 2020)

Pairwise velocity

$$w_{\rm r}(r_{12}) = (\boldsymbol{v}_2 - \boldsymbol{v}_1) \cdot \hat{\boldsymbol{r}}_{12}$$



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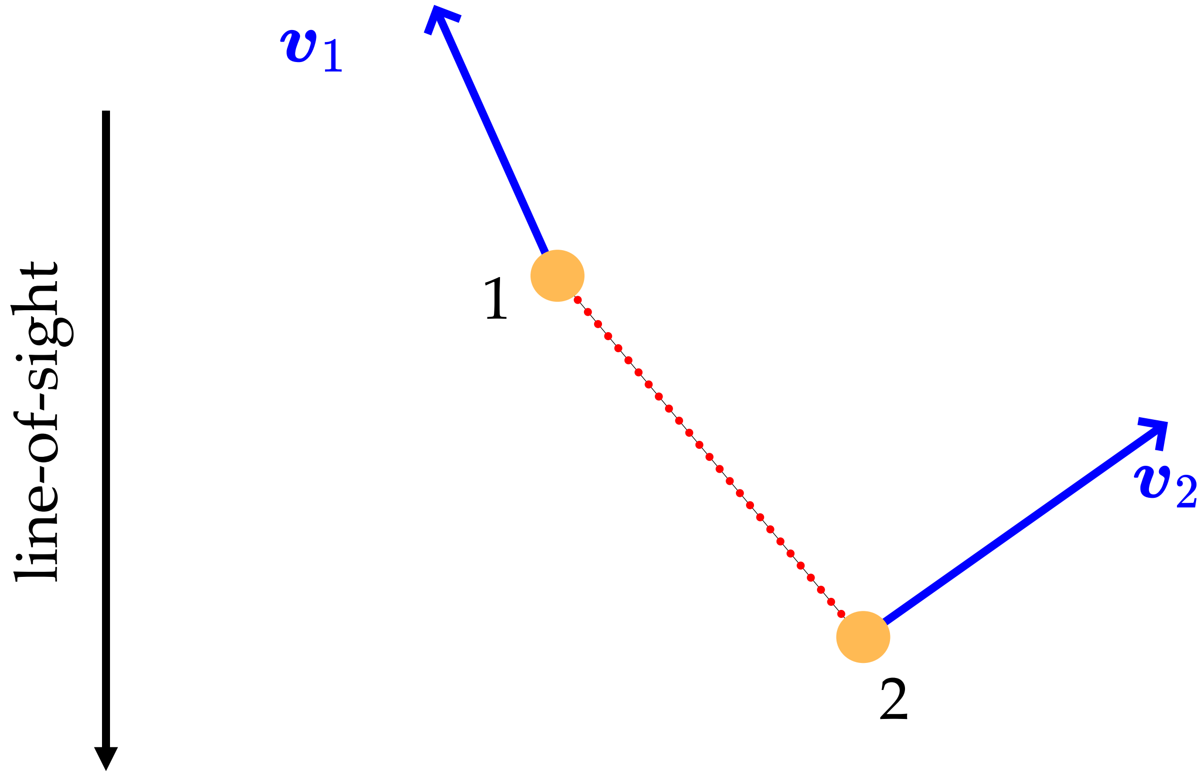
$$\langle \boldsymbol{w}_{12} | \boldsymbol{r}_{12} \rangle_{\rm p} = \frac{\langle (1+\delta_1)(1+\delta_2)(\boldsymbol{v}_2 - \boldsymbol{v}_1) \rangle}{\langle (1+\delta_1)(1+\delta_2) \rangle}$$



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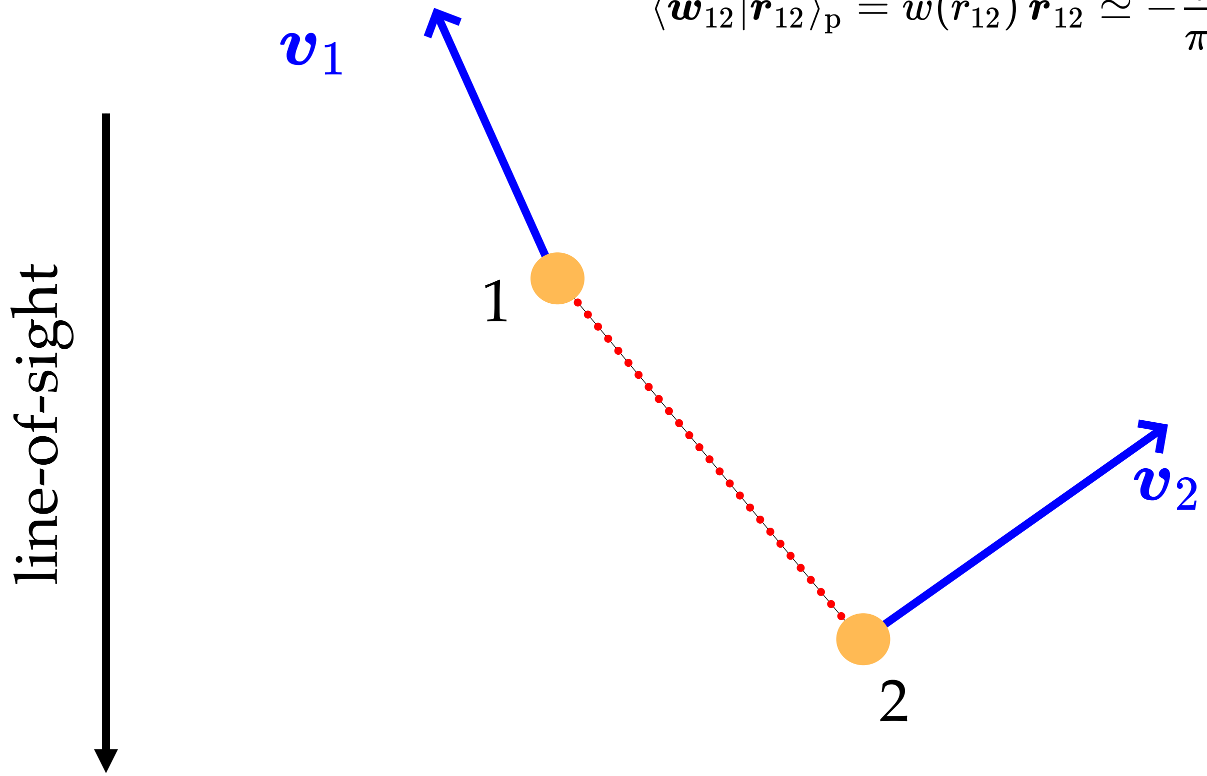
$$\langle \boldsymbol{w}_{12} | \boldsymbol{r}_{12} \rangle_{\mathrm{p}} = \bar{w}(r_{12})\, \hat{\boldsymbol{r}}_{12} \simeq -\frac{f}{\pi^2}\, \hat{\boldsymbol{r}}_{12} \int_0^\infty k\, j_1(kr_{12})\, P(k)\, \mathrm{d}k$$



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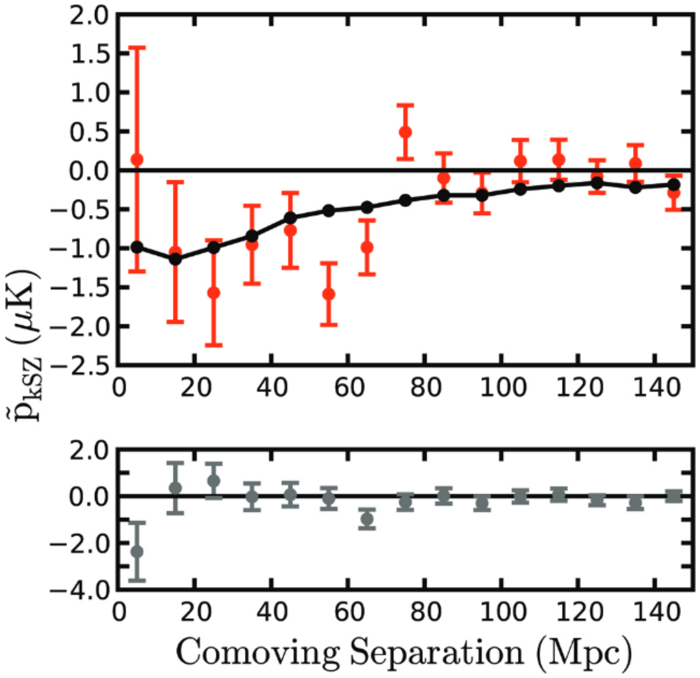
- The first significant detection of the kinetic Sunyaev-Zeldovich (kSZ) effect was achieved through the mean pairwise mean velocity. (Hand et al. 12)



(Hand et al. 12)

$$\frac{\Delta T^{\mathrm{kSZ}}(r_{12})}{T_{\mathrm{cmb}}} \simeq -\tau \frac{\bar{w}(r_{12})}{c}$$

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- The first significant detection of the kinetic Sunyaev-Zeldovich (kSZ) effect was achieved through the mean pairwise mean velocity. (Hand et al. 12)

- The pairwise velocity measurement from kSZ experiments has been shown to be a novel probe to constrain the summed neutrino mass. (Mueller et al. 15)

- The impact of massive neutrinos and its interplay with baryonic physics at non-linear scales.

(JK, Aghanim & McCarthy, A&A, 2020)

Can we generalise the relative velocity statistics to three-point?

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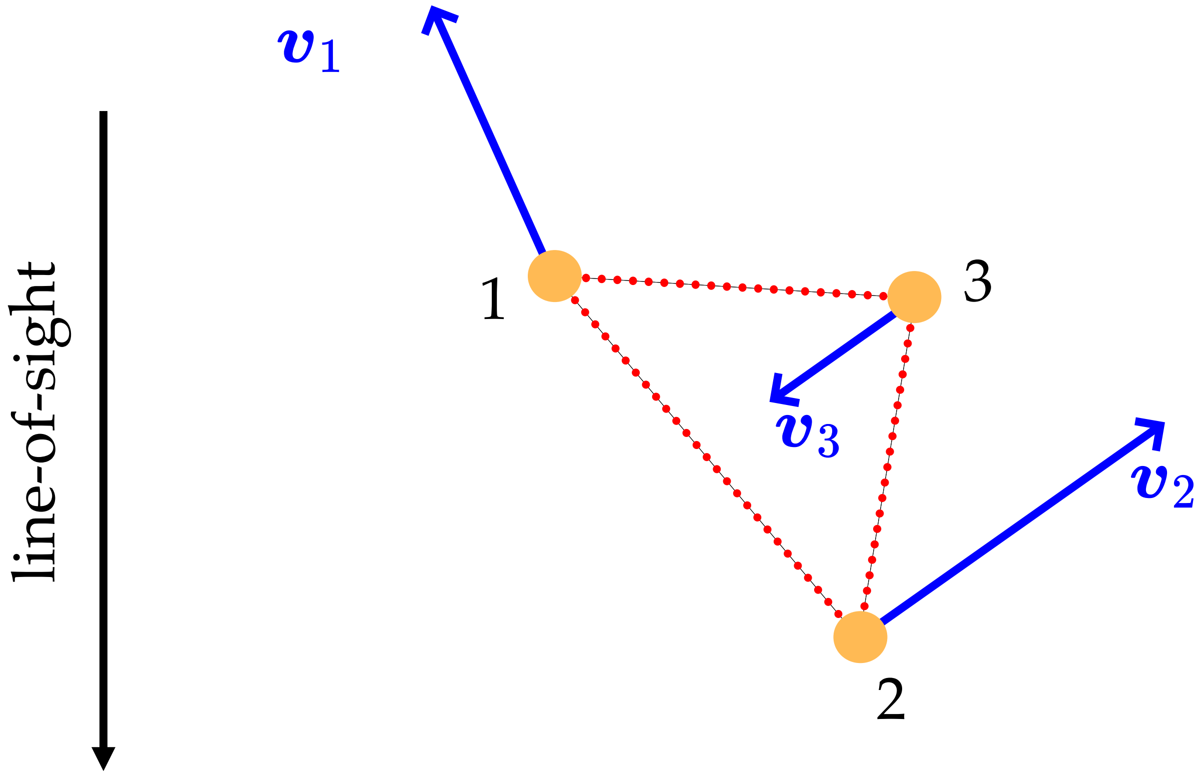
Can we generalise the relative velocity statistics to three-point?

(JK & Porciani, JCAP, arXiv:2005.05331, 2020)

Three-point relative velocity



$$R_{12}(r_{12}, r_{23}, r_{31}) = (\boldsymbol{v}_2 - \boldsymbol{v}_1) \cdot \hat{\boldsymbol{r}}_{12}$$

$$R_{23}(r_{12}, r_{23}, r_{31}) = (\boldsymbol{v}_3 - \boldsymbol{v}_2) \cdot \hat{\boldsymbol{r}}_{23}$$

Can we use the relative velocity statistics to constrain cosmology?

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Halos - Mean relative velocity between pairs in a triplet

- Fisher matrix formalism

Halos - Mean relative velocity between pairs in a triplet

$$L_{\text{box}} = 1\,h^{-1}\text{Gpc} \;;\; N_{\text{p}} = 512^3$$

Qujiote suite

Fiducial	0.0	0.3175	0.049	0.6711	0.9624	0.834	0.834	2LPT	15,000
Fiducial ZA	0.0	0.3175	0.049	0.6711	0.9624	0.834	0.834	Zel'dovich	500
M_ν^+	$\underline{0.1}$	0.3175	0.049	0.6711	0.9624	0.834	0.834	Zel'dovich	500
M_ν^{++}	$\underline{0.2}$	0.3175	0.049	0.6711	0.9624	0.834	0.834	Zel'dovich	500
M_ν^{+++}	$\underline{0.4}$	0.3175	0.049	0.6711	0.9624	0.834	0.834	Zel'dovich	500
Ω_m^+	0.0	$\underline{0.3275}$	0.049	0.6711	0.9624	0.834	0.834	2LPT	500
Ω_m^-	0.0	$\underline{0.3075}$	0.049	0.6711	0.9624	0.834	0.834	2LPT	500
Ω_b^+	0.0	0.3175	$\underline{0.051}$	0.6711	0.9624	0.834	0.834	2LPT	500
Ω_b^-	0.0	0.3175	$\underline{0.047}$	0.6711	0.9624	0.834	0.834	2LPT	500
h^+	0.0	0.3175	0.049	$\underline{0.6911}$	0.9624	0.834	0.834	2LPT	500
h^-	0.0	0.3175	0.049	$\underline{0.6511}$	0.9624	0.834	0.834	2LPT	500
n_s^+	0.0	0.3175	0.049	0.6711	$\underline{0.9824}$	0.834	0.834	2LPT	500
n_s^-	0.0	0.3175	0.049	0.6711	$\underline{0.9424}$	0.834	0.834	2LPT	500
σ_8^+	0.0	0.3175	0.049	0.6711	0.9624	$\underline{0.849}$	$\underline{0.849}$	2LPT	500
σ_8^-	0.0	0.3175	0.049	0.6711	0.9624	$\underline{0.819}$	$\underline{0.819}$	2LPT	500

(Hahn et al. 20)

$$\frac{\partial \boldsymbol{S}}{\partial \theta} \simeq \frac{\boldsymbol{S}(\theta+\mathrm{d}\theta)-\boldsymbol{S}(\theta-\mathrm{d}\theta)}{2\;\mathrm{d}\theta} \qquad \frac{\partial \boldsymbol{S}}{\partial M_\nu} \simeq \frac{-\boldsymbol{S}(M_\nu{=}0.4)+4\boldsymbol{S}(M_\nu{=}0.2)-\boldsymbol{S}(M_\nu{=}0)}{0.4}$$

Halos - Mean relative velocity between pairs in a triplet

- Compute the covariance matrix for the relative velocity statistics using 15,000 simulations from the Quijote suite.
- Derivatives also computed directly using the Quijote suite.

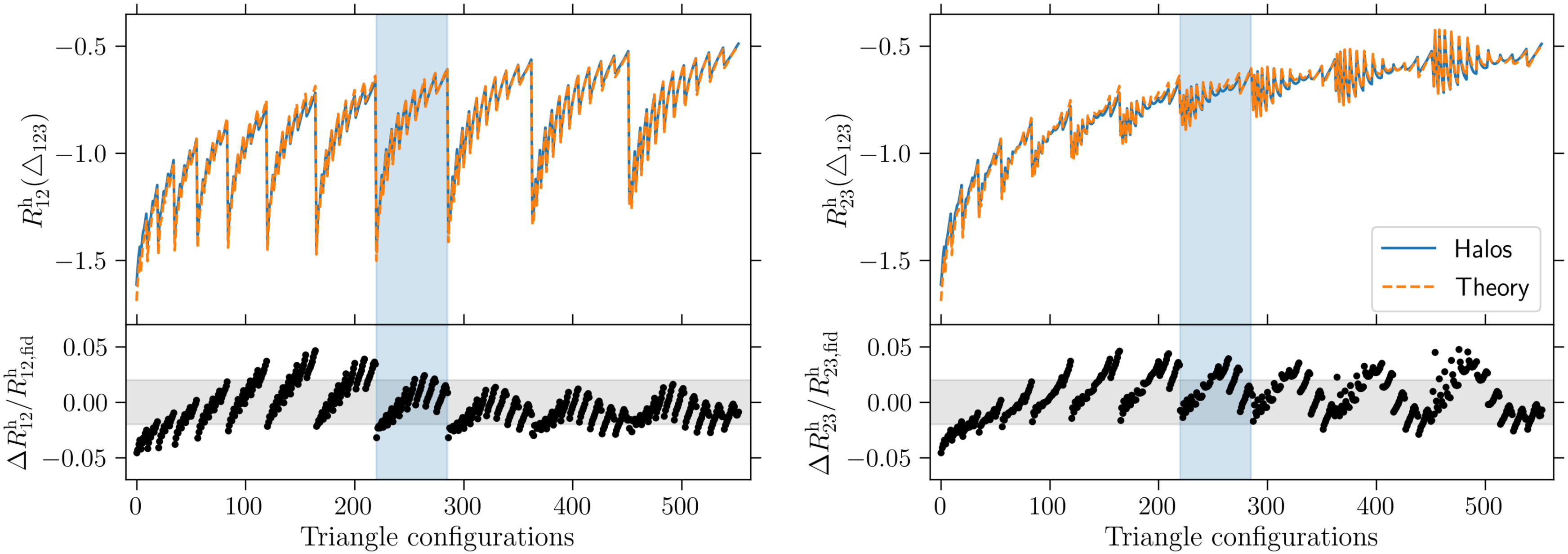
- The Quijote simulations is a suite of 44,100 full N-body simulations. (Villaescusa-Navarro et al. 20)
- This work runs parallel to the quantification of the information content from the halo redshift-space bispectrum using the Quijote suite. (Hahn et al. 20)

Halos - Mean relative velocity between pairs in a triplet

Triangular configurations: $50 \leq r_{31} \leq r_{23} \leq r_{12} \leq 120\ h^{-1}\mathrm{Mpc}$

Minimum halo mass: $M_\mathrm{h} > 5 \times 10^{13}\ h^{-1} M_\odot$

15,000 simulations of the Quijote suite of simulations



$$R^\mathrm{h}_{ij}(\triangle_{123}, M_\mathrm{h}) = b(M_\mathrm{h})\, R_{ij}(\triangle_{123})$$

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Halos - Mean relative velocity between pairs in a triplet

$$\langle \boldsymbol{w}_{12} | \triangle_{123} \rangle_{\rm t} = \frac{\langle (1+\delta_1)(1+\delta_2)(1+\delta_3)(\boldsymbol{v}_2 - \boldsymbol{v}_1) \rangle}{\langle (1+\delta_1)(1+\delta_2)(1+\delta_3) \rangle}$$

$$\simeq \langle \delta_1 \boldsymbol{v}_2 \rangle - \langle \delta_2 \boldsymbol{v}_1 \rangle + \langle \delta_3 \boldsymbol{v}_2 \rangle - \langle \delta_3 \boldsymbol{v}_1 \rangle$$

$$= \bar{w}(r_{12})\, \hat{\boldsymbol{r}}_{12} - \frac{1}{2} \left[\bar{w}(r_{23})\, \hat{\boldsymbol{r}}_{23} + \bar{w}(r_{31})\, \hat{\boldsymbol{r}}_{31} \right]$$

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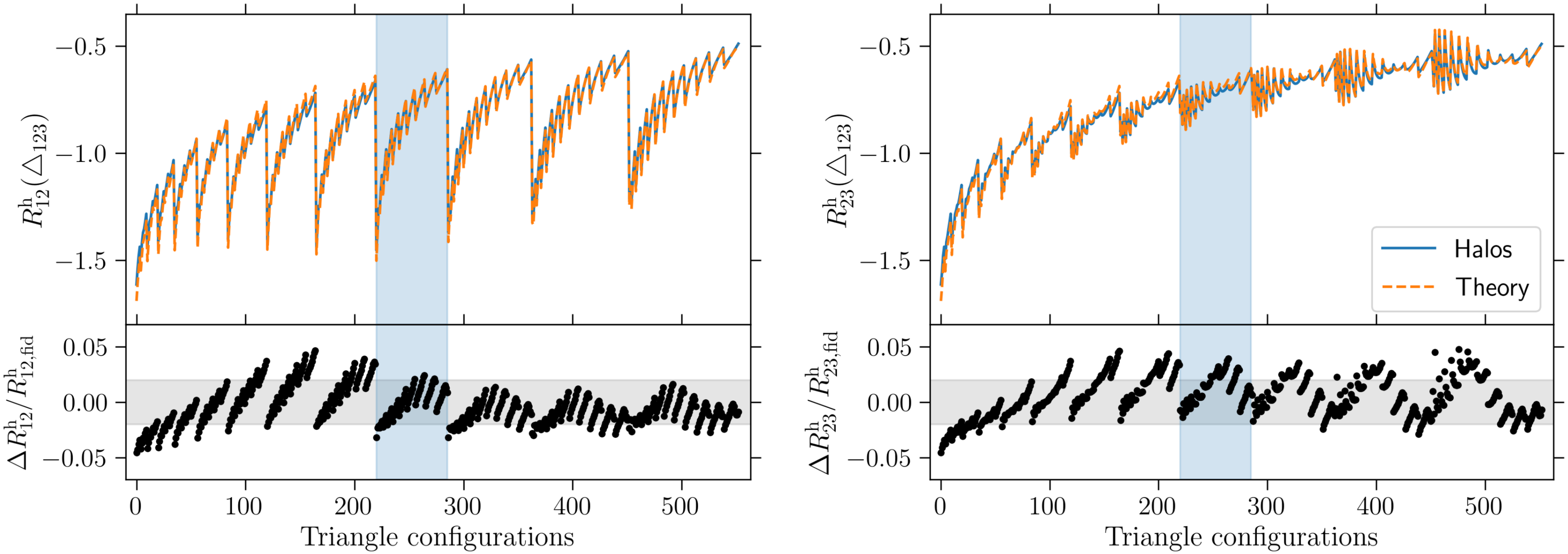

$$R_{12}(\triangle_{123}) = \bar{w}(r_{12}) - \frac{1}{2}\left[\bar{w}(r_{23})\;\cos\chi - \bar{w}(r_{31})\;\frac{r_{12}+r_{23}\cos\chi}{\sqrt{r_{12}^2+r_{23}^2+2r_{12}r_{23}\cos\chi}}\right]$$

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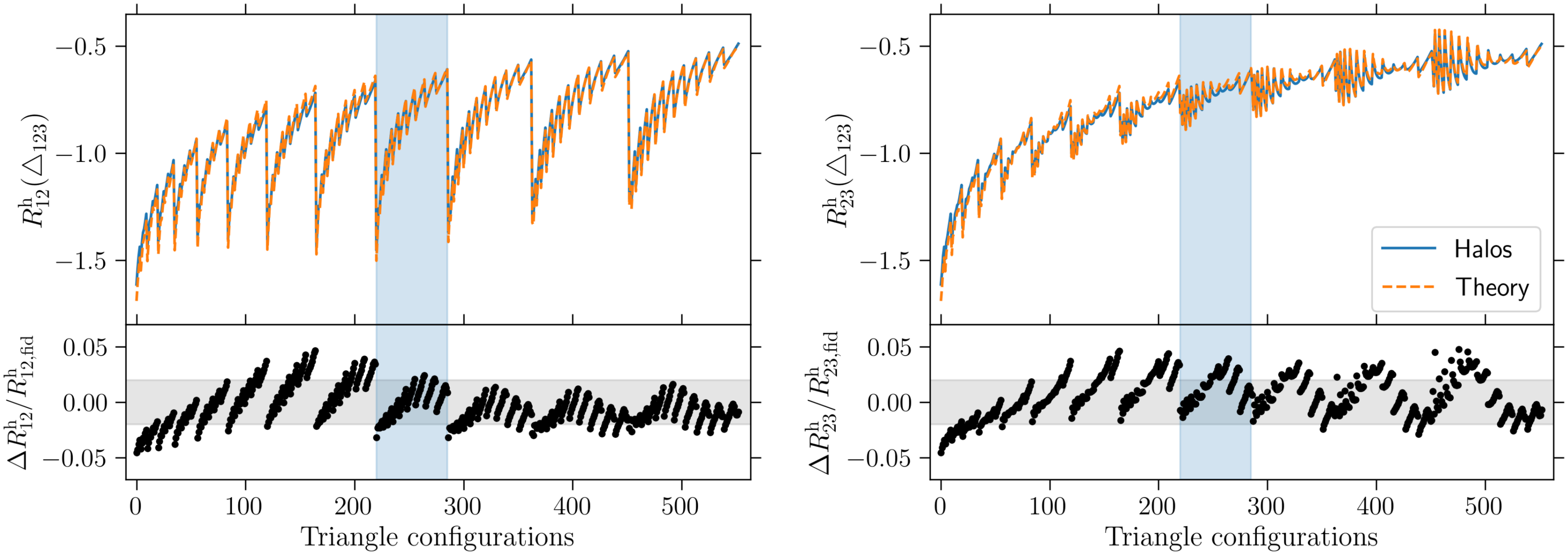

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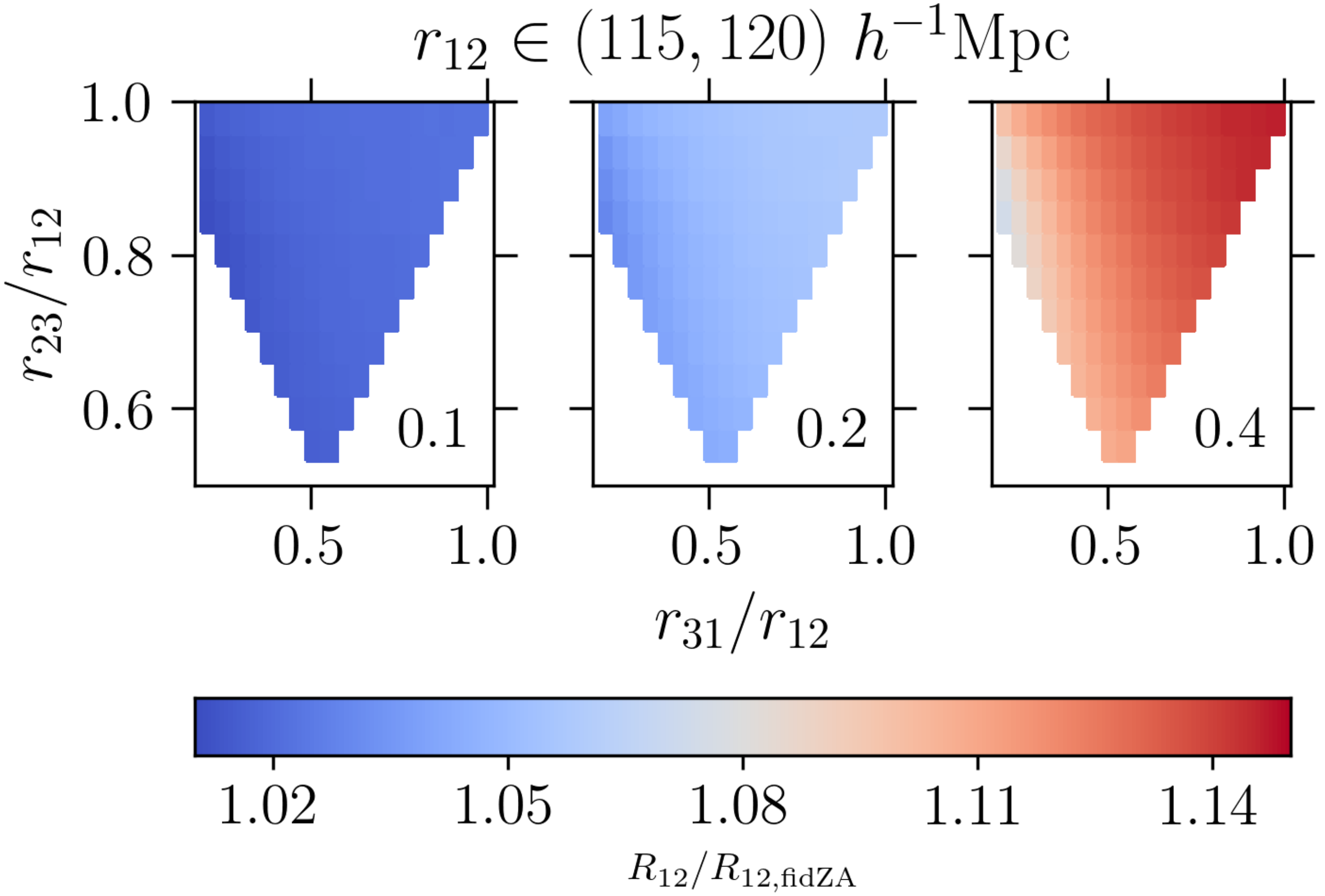
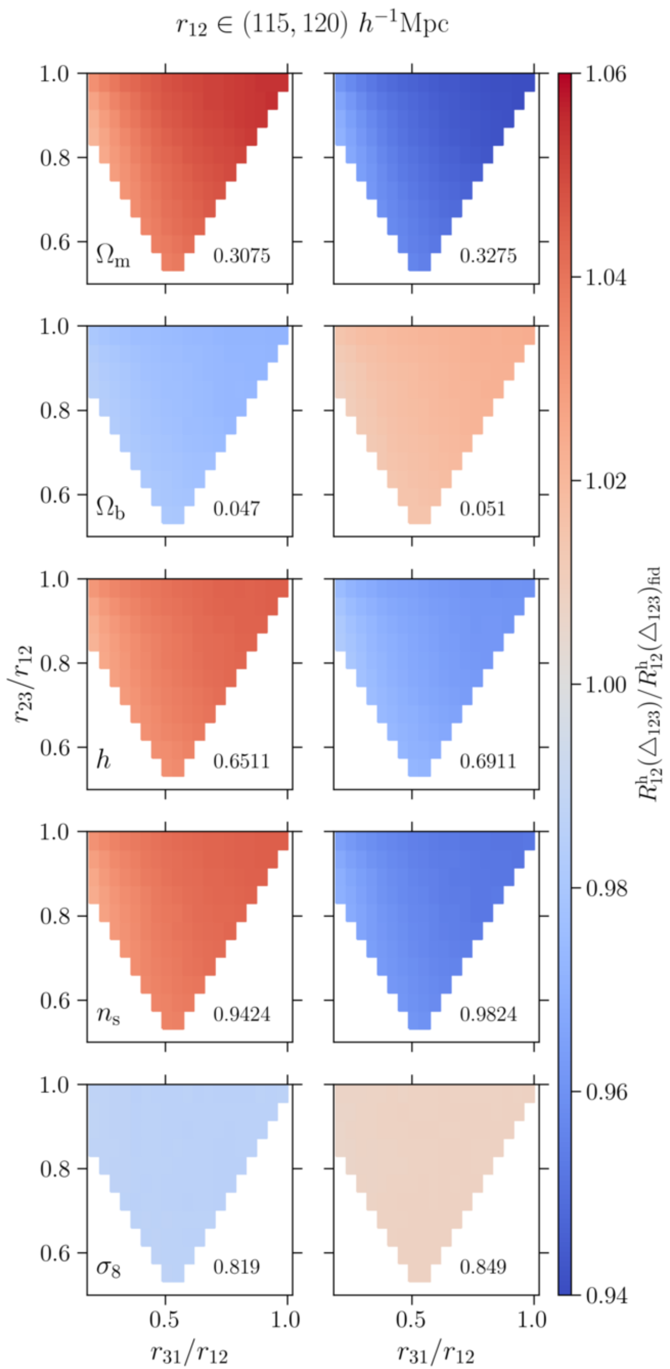
Effect of massive neutrinos

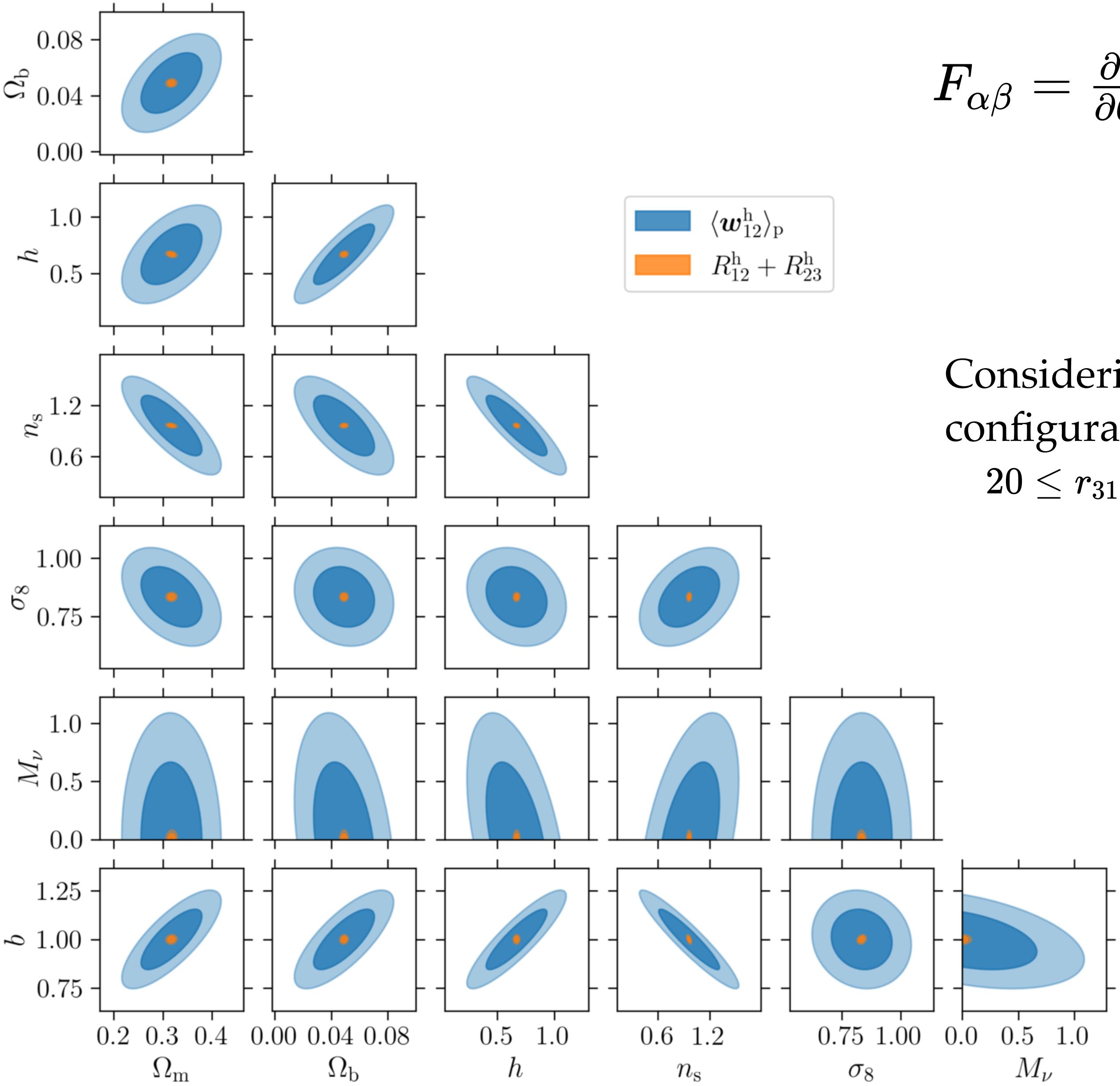


(JK & Aghanim 21)

Effect of cosmological parameters



$$F_{\alpha\beta} = \frac{\partial \boldsymbol{S}}{\partial \theta_\alpha} \cdot \hat{\mathbf{C}}^{-1} \cdot \frac{\partial \boldsymbol{S}^{\mathsf{T}}}{\partial \theta_\beta}$$



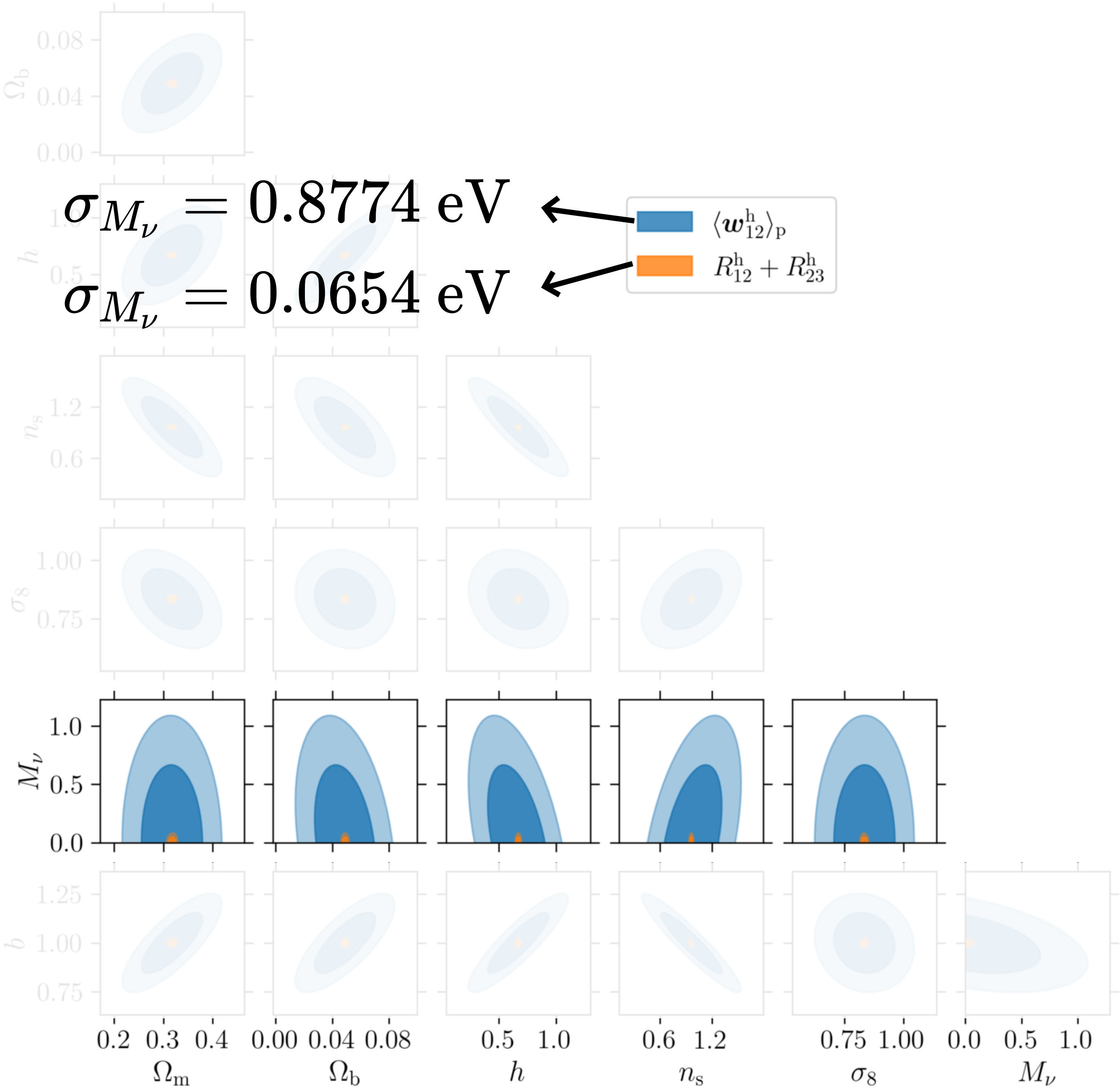
$$F_{\alpha\beta} = \frac{\partial \boldsymbol{S}}{\partial \theta_\alpha} \cdot \hat{\mathbf{C}}^{-1} \cdot \frac{\partial \boldsymbol{S}^{\mathsf{T}}}{\partial \theta_\beta}$$

Considering all triangular configurations:

$$20 \leq r_{31} \leq r_{23} \leq r_{12} \leq 120$$

$$\sigma_{M_\nu} = 0.8774 \text{ eV}$$

$$\sigma_{M_\nu} = 0.0654 \text{ eV}$$



But is this really competitive with clustering statistics?

Summary statistics	Matter density	Baryon density	Hubble parameter (h)	Spectral index	Sigma 8	Summed neutrino mass (eV)
mean relative velocity	0.0091	0.0024	0.0226	0.0221	0.0156	0.0655
power spectrum multipoles	**2.6**	**4.8**	**4.9**	**5.7**	**2.3**	**4.5**
bispectrum monopole	**1.2**	**1.7**	**1.7**	**1.5**	**0.9**	**0.8**

(Hahn et al. 20)

* Numbers within the orange box denotes the factor of improvement of 1 sigma constraint from three-point relative velocity with respect to the corresponding clustering statistics.

Conclusions

- Leading order perturbation theory prediction for the mean three-point relative velocity works quite well on quasi-linear scales and above when compared against the direct measurement from simulation.

- Cosmological constraints from the mean three-point relative velocity statistics are competitive with those obtained from the bispectrum, while having sizeable improvements with respect to the power spectrum.

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Thank you for listening!