The Crab Pulsar’s radio emission is unusual, consisting predominantly of giant pulses, with durations of about a micro-second but structure down to the nano-second level, and brightness temperatures of up to $10^{37}$ K. It is unclear how giant pulses are produced, but they likely originate near the pulsar’s light cylinder, where corotating plasma approaches the speed of light. We report observations in the 400-800 MHz frequency band, where the pulses are broadened by scattering in the surrounding Crab nebula. We find that some pulse frequency spectra show strong bands, which vary during the scattering tail, in one case showing a smooth upward drift. While the banding may simply reflect interference between nano-second scale pulse components, the variation is surprising, as in the scattering tail the only difference is that the source is observed via slightly longer paths, bent by about an arcsecond in the nebula. The corresponding small change in viewing angle could nevertheless reproduce the observed drift by a change in Doppler shift, if the plasma that emitted the giant pulses moved highly relativistically, with a Lorentz factor $\gamma \sim 10^4$ (and without much spread in $\gamma$). If so, this would support models that appeal to highly relativistic plasma to transform ambient magnetic structures to coherent GHz radio emission, be it for giant pulses or for potentially related sources, such as fast radio bursts.