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Most Reliable Strong and Electroweak Evolution

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Abstract:

Both QCD and EW eras play essential roles in laying seeds for nucleosynthesis and even dictating the cosmological large-scale structure. Taking advantage of recent developments in ultrarelativistic nuclear experiments and nonperturbativ and perturbative lattice simulations, various thermodynamic quantities including pressure, energy density, bulk viscosity, relaxation time, and temperature have been calculated up to the TeV-scale, in which the possible influence of finite bulk viscosity is characterized for the first time and the analytical dependence of Hubble parameter on the scale factor is also introduced.

Main Problem and Questions

- **Einstein Field Equations combine *classical* space (geometry) with *quantized* energy-momentum tensor**

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **In physical GR cosmology, the Friedman equations govern the expansion of *classical* space in homogeneous and isotropic background of the universe.**
- **For Friedmann–Lemaître–Robertson–Walker metric**

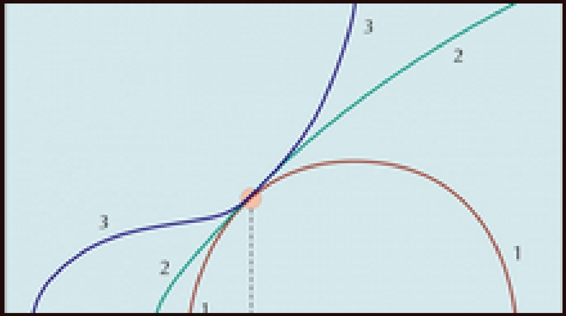
$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right],$$

and a perfect fluid with a mass density ρ and pressure p. two independent solutions

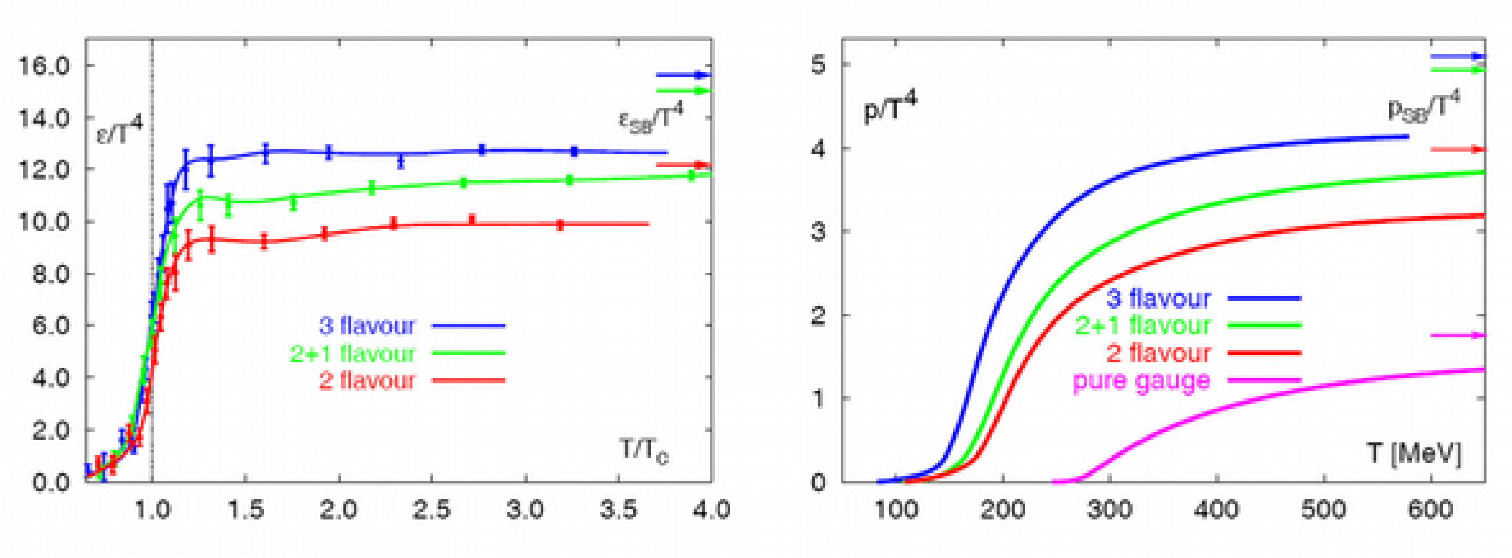
$$\begin{aligned} H(t)^2 &= \frac{8\pi}{3}\rho(t) - \frac{k}{a(t)^2} + \frac{\Lambda}{3}, \\ \dot{H}(t) + H(t)^2 &= -\frac{4\pi}{3}\left[\rho(t) + 3p_{\mathtt{eff}(t)}\right] + \frac{\Lambda}{3}, \end{aligned}$$

Main Problem and Questions

- The Friedmann equations can be solved exactly for any cosmic background having reliable equation of state (EoS).



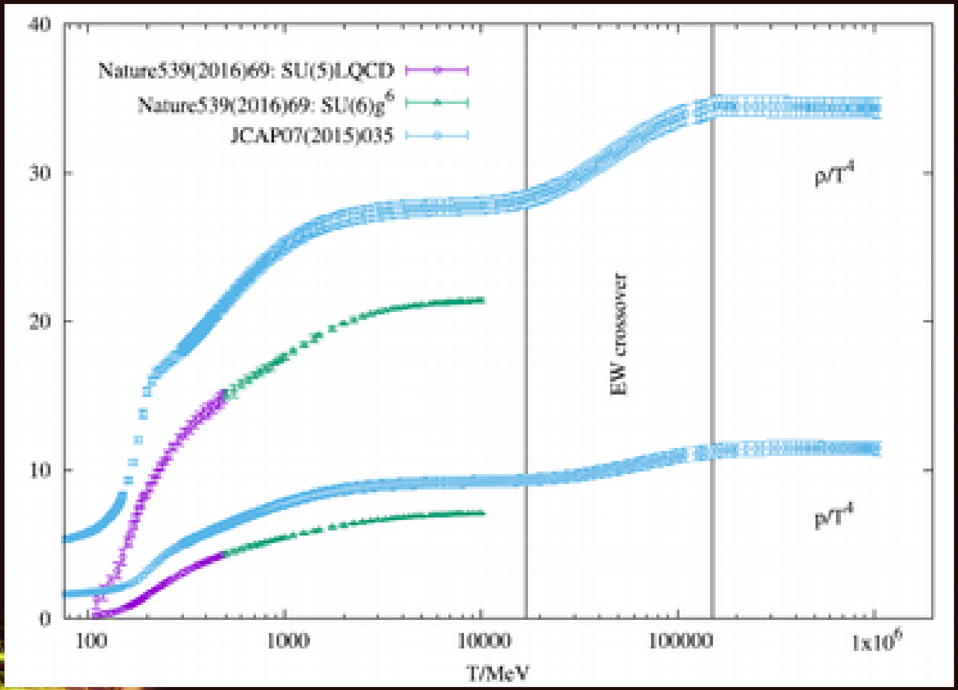
- Despite hypothetical assumption, we can now rely on recent relativistic experiments and perturbation and non-perturbation calculations for barotropic EoS.

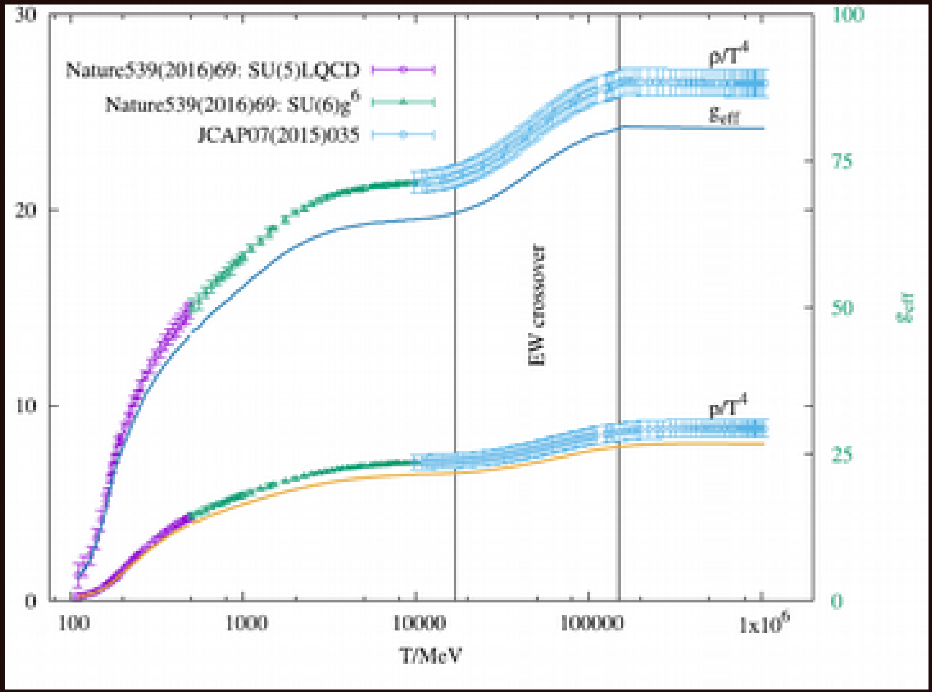


(Non)Perturbatice Simulations

By combining recent non-perturbative and perturbative calculations with other dof, such as photons, neutrinos, leptons, electroweak particles, and Higgs bosons, various thermodynamic quantities for baryon-free cosmic matter have been calculated up to the TeV-scale.

Tawfik, A.N.; Mishustin, I. Equation of State for Cosmological Matter at and beyond QCD and Electroweak Eras. *J. Phys.* 2019, *G46*, 125201, [arXiv:hep-ph/1903.00063]. doi:10.1088/1361-6471/ab46d4.





Equations of State: Pressure

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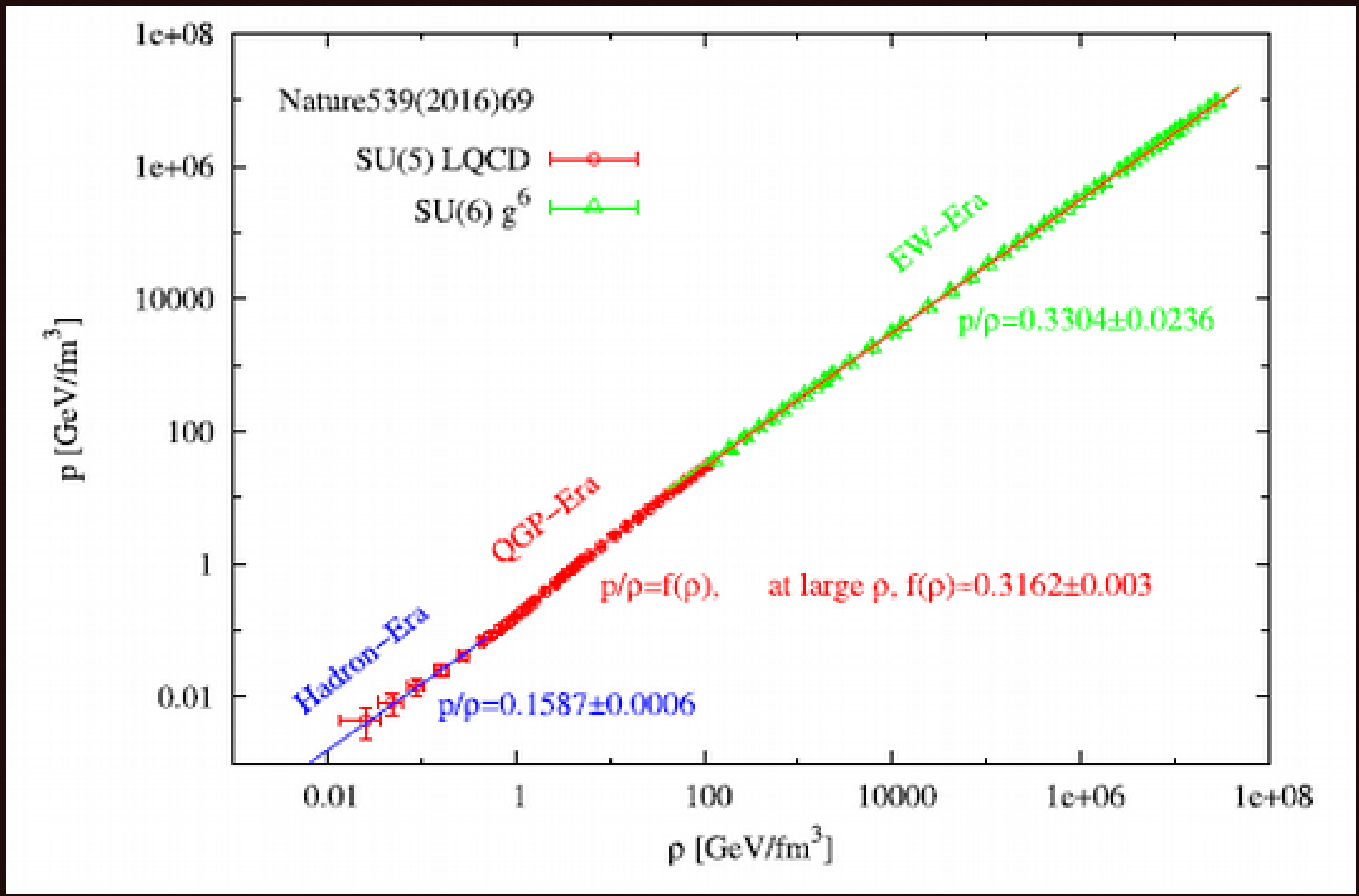

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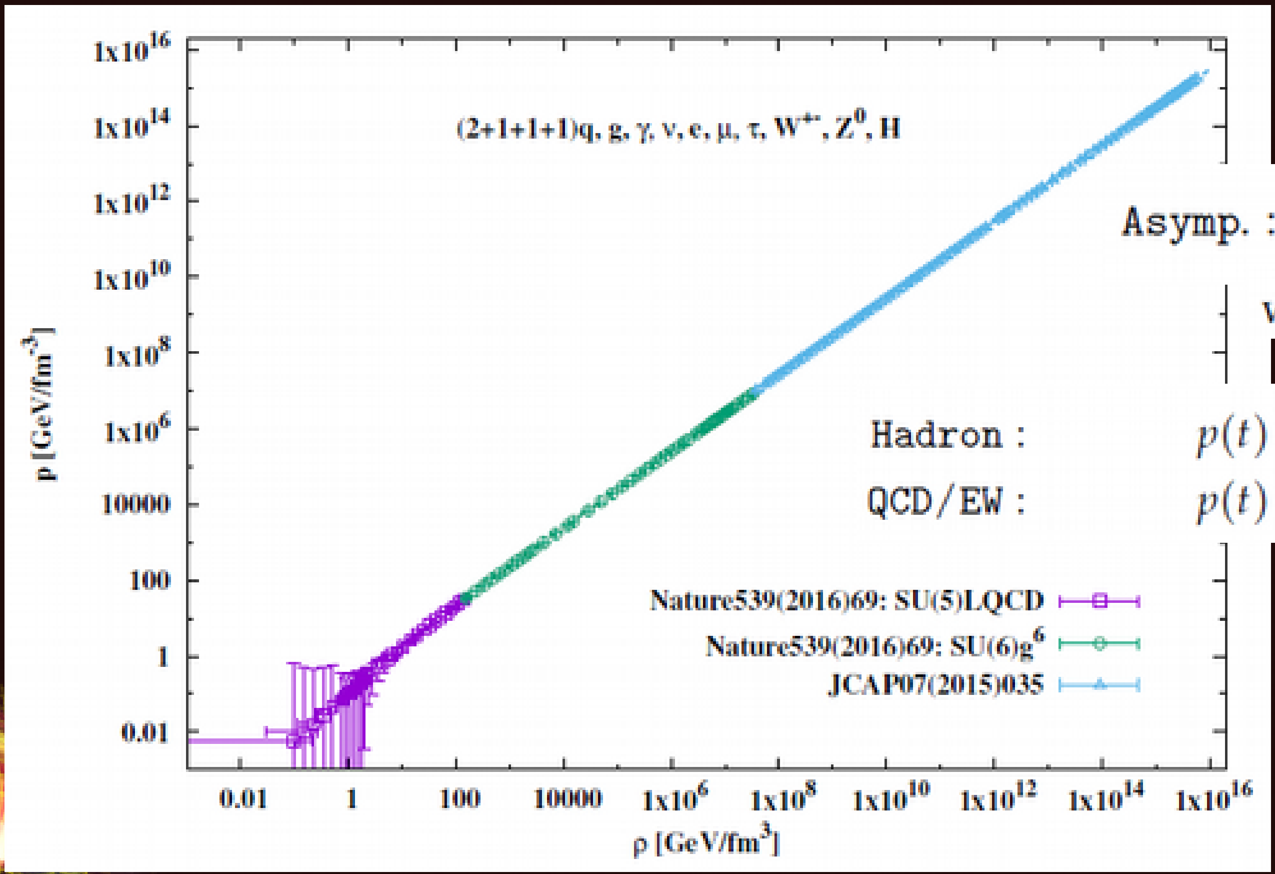


$$\mathtt{Asymp.:} \quad p(t) = \gamma_3 \rho(t),$$

where $\gamma_3 = 0.3304 \pm 0.0236.$

$$\mathtt{Hadron:} \quad p(t) = \alpha_1 + \beta_1 \rho(t),$$

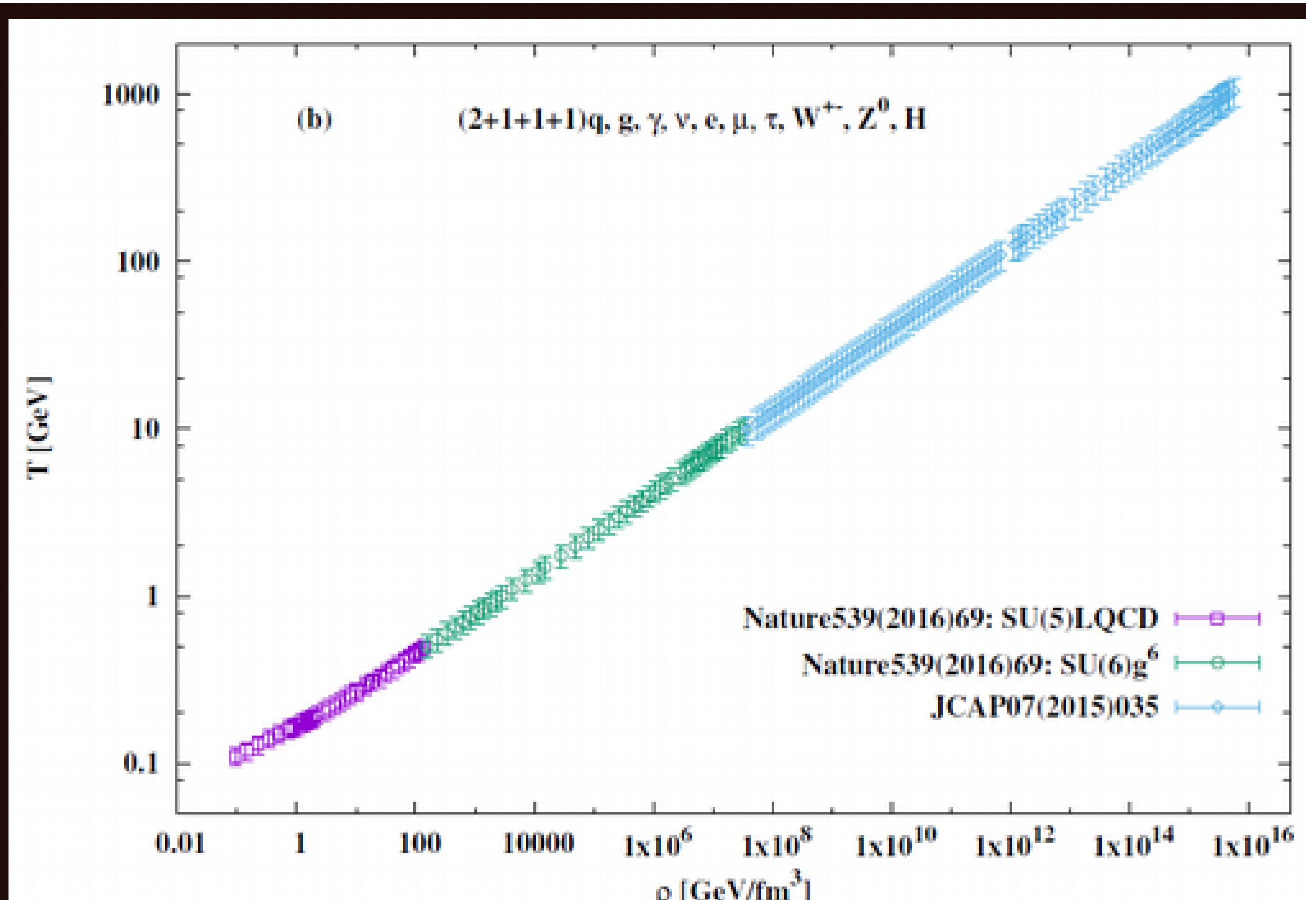
$$\mathtt{QCD/EW:} \quad p(t) = \alpha_2 + \beta_2 \rho(t) + \gamma_2 \rho(t)^{\delta_2},$$

Equations of State: Temperature

The dependence of temperature T on energy density; arotropic EoS, is found almost linear.

$$T(t) = \alpha_4 + \beta_4 \rho(t)^{\gamma_4},$$

where $\alpha_4 = 0.048 \pm 0.001$, $\beta_4 = 0.13 \pm 2\times10^{-4}$, and $\gamma_4 = 0.25 \pm 8\times10^{-5}$



Viscous Properties

The energy-momentum tensor of the bulk viscous cosmological fluid filling the very early Universe can be expressed as

$$T_{\mu\nu} = (\rho + p + \Pi)\, u_\mu u^\nu - (p + \Pi) g_{\mu\nu},$$

where ρ is the energy density, p is the thermodynamic pressure, Π is the bulk viscous pressure, and u_μ is the four velocity satisfying the normalization condition $u_\mu u^\mu = 1$. The bulk pressure Π can formally be included in the thermodynamic pressure $p_{\texttt{eff}} = p + \Pi$. We shall discuss on how to evaluate Π, concretely the bulk viscous pressure, in framework of Eckart (first-order), section 4.2, and Israel-Stewart (second-order) theories, section 4.3, for relativistic viscous cosmic fluid.

For number density n, specific entropy s, finite temperature T, bulk viscosity coefficient ζ, and relaxation time τ, the particle and entropy fluxes are to be related to each other as $N^i = \nu^i$ and $S^i = sN^i - (\tau\Pi^2/2\zeta T)u^i$, respectively. It should be emphasized that the evolution of the cosmological fluid is subject to the dynamical laws of particle number conservation $N^i_{;i} = 0$ and Gibbs' equation $Td\rho = d(\rho/n) + pd(1/n)$ [18]. In what follows, we assume that the energy-momentum tensor of the cosmological fluid is locally conserved, i.e. $T^k_{i;k} = 0$, where ; denotes the covariant derivative with respect to the line metric.

Bulk Viscosity

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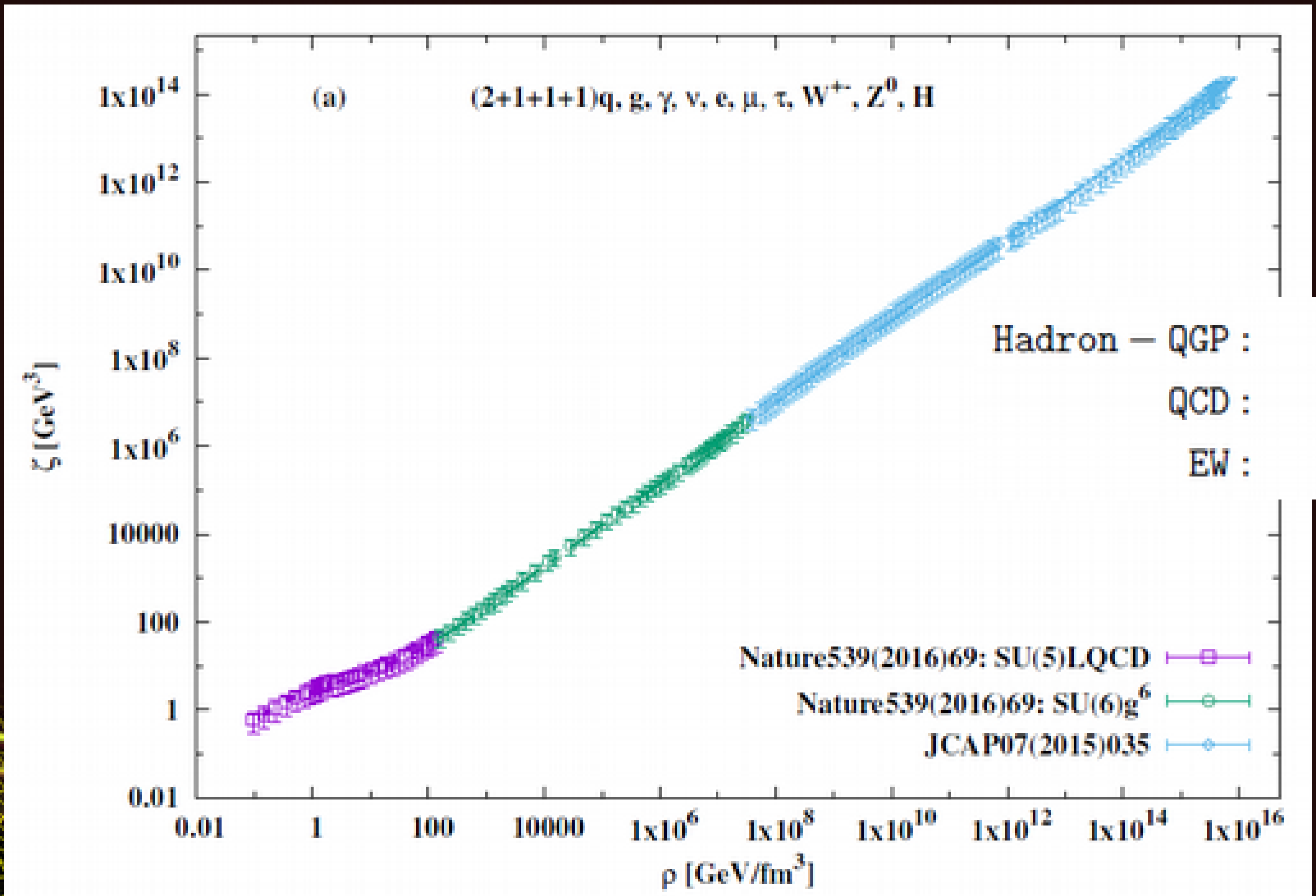


$$\begin{aligned}
\text{Hadron} - \text{QGP}: \quad & \zeta(t) = d_1 + d_2\rho(t) + d_3\rho(t)^{d_4},\\
\text{QCD}: \quad & \zeta(t) = e_1 + e_2\rho(t)^{e_3},\\
\text{EW}: \quad & \zeta(t) = f_1 + f_2\rho(t)^{f_3}.
\end{aligned}$$

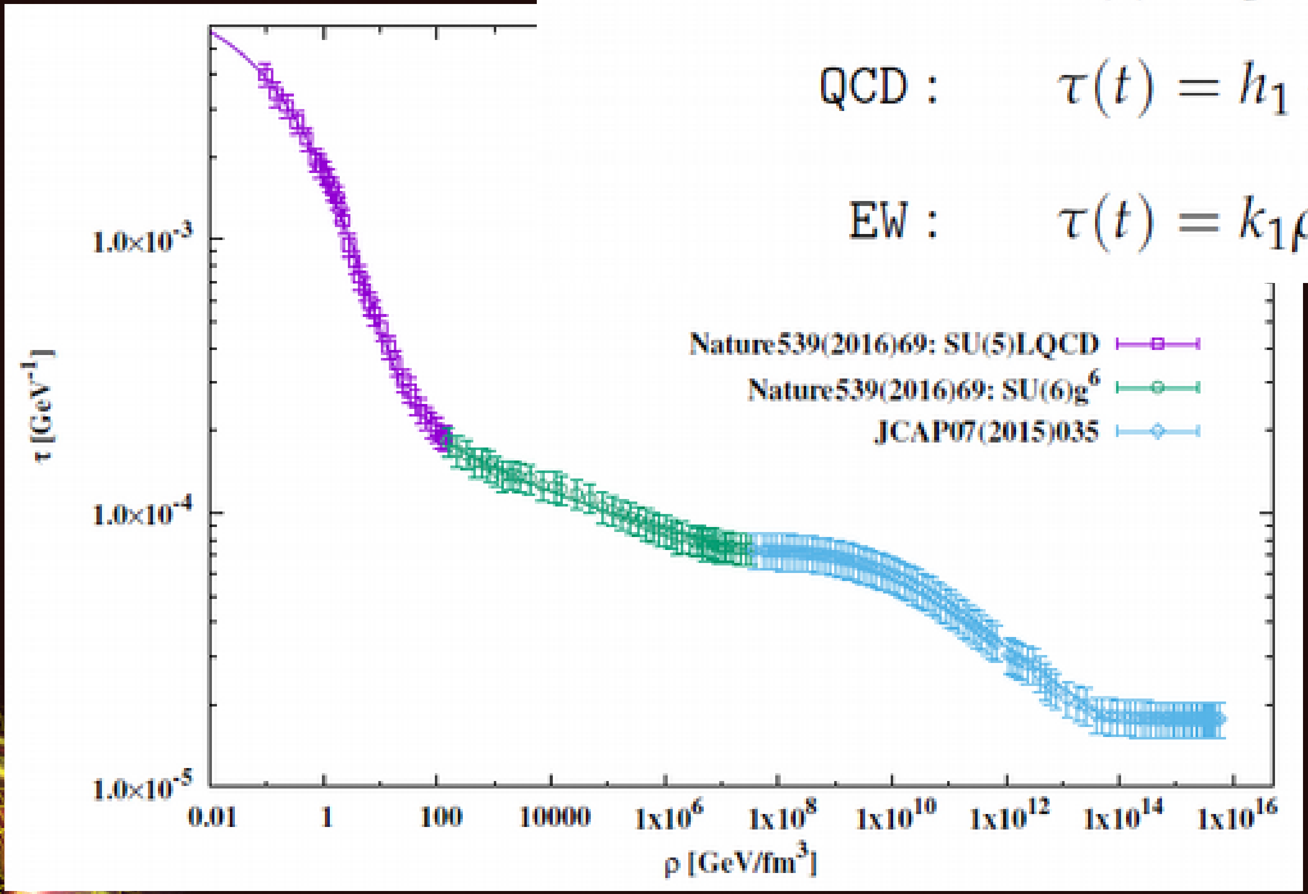
Relaxation Time

For the relaxation time $\tau(t) = \frac{\zeta(t)}{\rho(t)}.$

$$\text{Hadron}-\text{QGP}: \quad \tau(t) = g_1 + g_2 \exp(-g_3 \rho(t)^{g_4}),$$

$$\text{QCD}: \quad \tau(t) = h_1 + \frac{h_2}{h_3 + \log(h_4 \rho(t))},$$

$$\text{EW}: \quad \tau(t) = k_1 \rho(t)^{k_2} \log(k_3 \rho(t)),$$



Early Universe Evolution: Hadron

$$H(t)^2 = \frac{8\pi}{3}\rho(t) - \frac{k}{a(t)^2} + \frac{\Lambda}{3},$$

$$\dot{H}(t) + H(t)^2 = -\frac{4\pi}{3}\left[\rho(t) + 3p_{\mathtt{eff}(t)}\right] + \frac{\Lambda}{3},$$

At vanishing bulk viscosity, $\Pi(t) = 0$, the effective pressure $p_{\mathtt{eff}}(t) = p(t) + \Pi(t)$ can be simplified as the thermodynamic pressure $p(t)$. Then, the Friedmann equation (6) can be rewritten as

$$\ddot{a}(t)\,a(t) - \dot{a}(t)^2 + 4\pi[\rho(t) + p(t)]a(t)^2 - k = 0, \tag{11}$$

which can be solved if combined with set of closed equations, such as Eq. (4) and suitable EoS.

Hadron Era

$$\ddot{a}(t)\,a(t) + C_1\,\dot{a}(t)^2 + C_2\,a(t)^2 + C_1\,k = 0,$$

$$a(t) = c_2 \cosh\left[\sqrt{C_2(1-C_1)}(t+c_1)\right]^{1/(1-C_1)},$$

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{C_2}{1+C_1}}\,\tanh\left[\sqrt{C_2(1+C_1)}(t+c_1)\right]$$

Early Universe Evolution: QCD-EW

$$H(t)^2 = \frac{8\pi}{3}\rho(t) - \frac{k}{a(t)^2} + \frac{\Lambda}{3},$$

$$\dot{H}(t) + H(t)^2 = -\frac{4\pi}{3}\left[\rho(t) + 3p_{\mathtt{eff}(t)}\right] + \frac{\Lambda}{3},$$

QCD-EW

$$\ddot{a}(t)\,a(t) + C_1\,\dot{a}(t)^2 + C_2\,a(t)^2 + C_1\,k + 4\pi\gamma_2 a(t)^2\rho^{\delta_2} = 0,$$

$$\begin{aligned} u(t) &= \dot{a}(t)^2 \\ &= \frac{1}{C_3}\Big\{3k[C_2 + C_3 - 3(1+C_1)C_4] - 3C_3C_4a(t)^2 + c_1 a(t)^{-2C_1} e^{\frac{C_3}{3k}a(t)^2} \\ &\quad -C_1k[-3C_2 - 4C_3 + 9(1+C_1)C_4]e^{\frac{C_3}{3k}a(t)^2} Ein_{1-C_1}\left(\frac{C_3}{3k}a(t)^2\right)\Big\}, \end{aligned}$$

$$\begin{aligned} H(t) &= \frac{1}{C_3^{1/2}a(t)}\Big\{3k[C_2 + C_3 - 3(1+C_1)C_4] - 3C_3C_4a(t)^2 + c_4 a(t)^{-2C_1} e^{\frac{C_3}{3k}a(t)^2} \\ &\quad -C_1k[-3C_2 - 4C_3 + 9(1+C_1)C_4]e^{\frac{C_3}{3k}a(t)^2} Ein_{1-C_1}\left(\frac{C_3}{3k}a(t)^2\right)\Big\}^{1/2}. \end{aligned}$$

Early Universe Evolution: Asymp.

$$
\begin{aligned}
H(t)^2 &= \frac{8\pi}{3}\rho(t) - \frac{k}{a(t)^2} + \frac{\Lambda}{3}, \\
\dot{H}(t) + H(t)^2 &= -\frac{4\pi}{3}\left[\rho(t) + 3p_{\mathtt{eff}(t)}\right] + \frac{\Lambda}{3},
\end{aligned}
$$

Asymptotic Limit

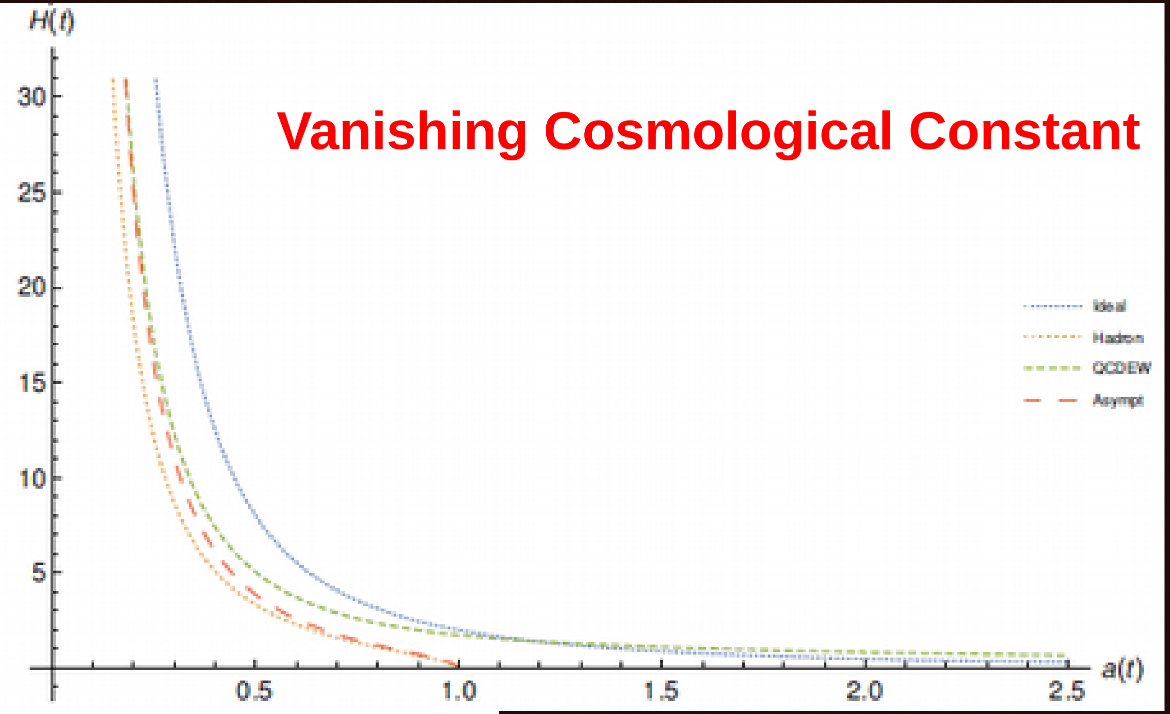
$$
\ddot{a}(t)\,a(t) + C_1\,\dot{a}(t)^2 + C_2\,a(t)^2 + C_1\,k = 0,
$$

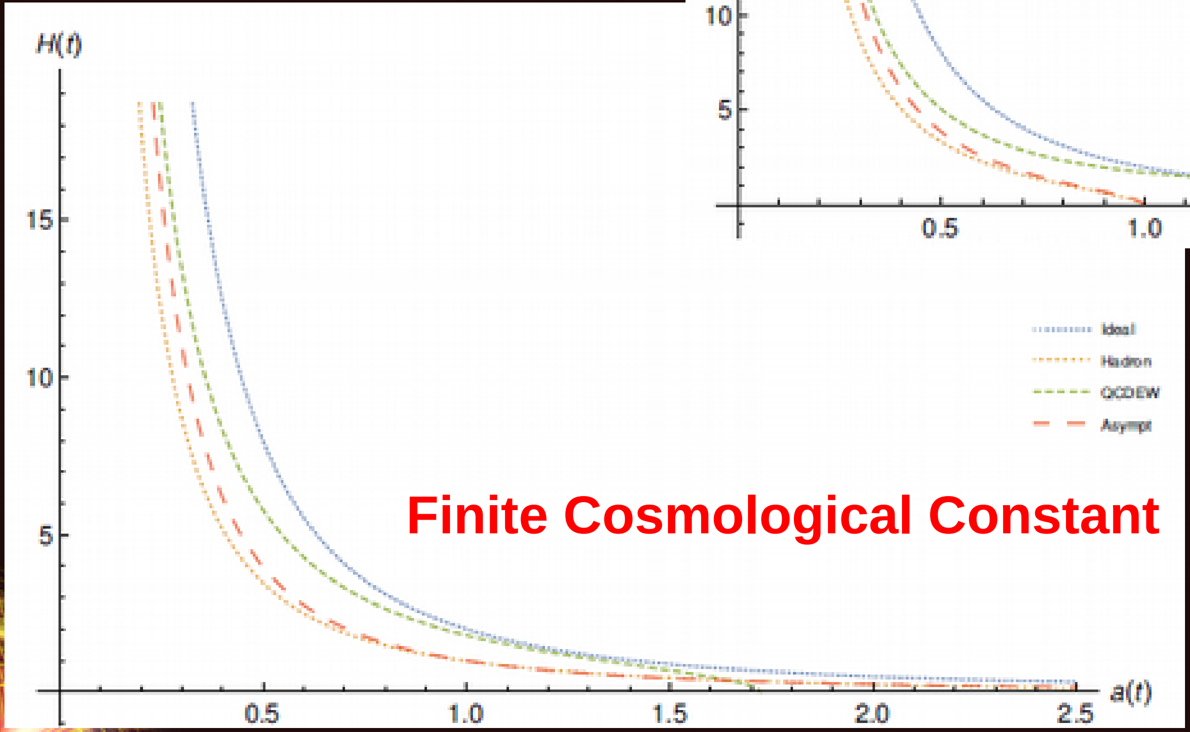
$$
\begin{aligned}
u(t) &= \dot{a}(t)^2 \\
&= c_4 a(t)^{-2C_1} - \left[\frac{C_2}{C_1+1} + k\right] a(t)^2,
\end{aligned}
$$

for which the Hubble parameter can be given as

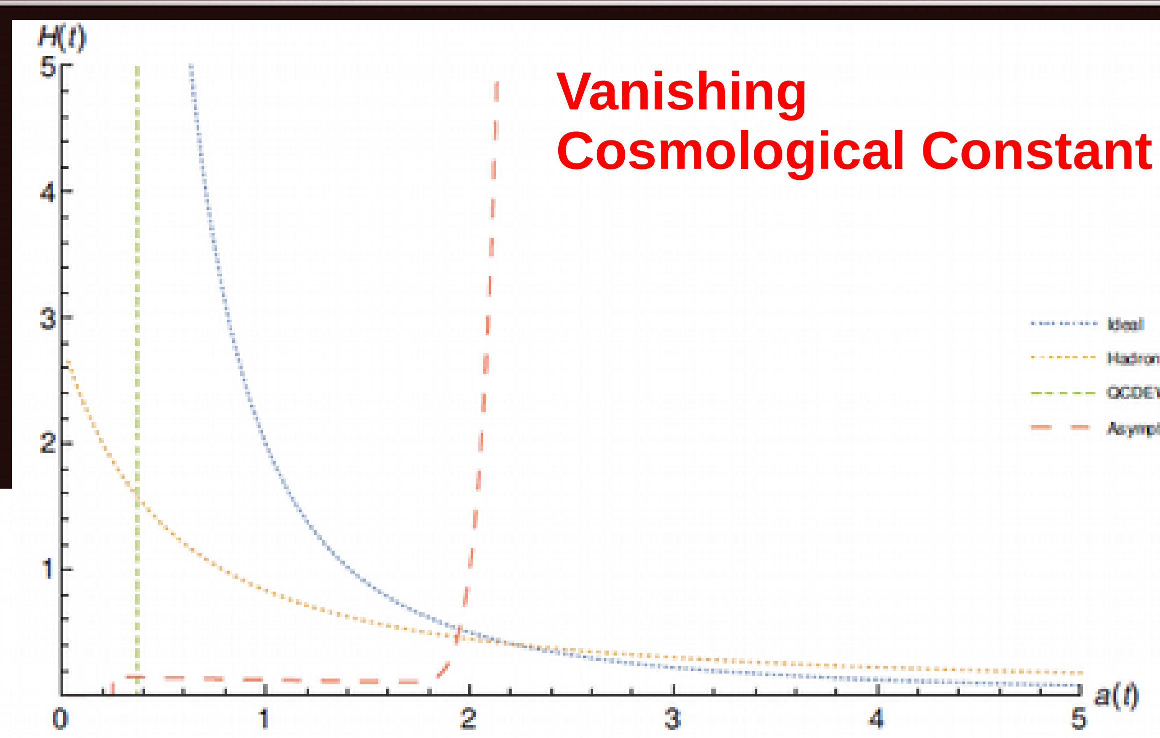
$$
H(t) = \left\{c_4 a(t)^{-2C_1} - \left[\frac{C_2}{C_1+1} + k\right] a(t)^2\right\}^{1/2} \frac{1}{a(t)}.
$$

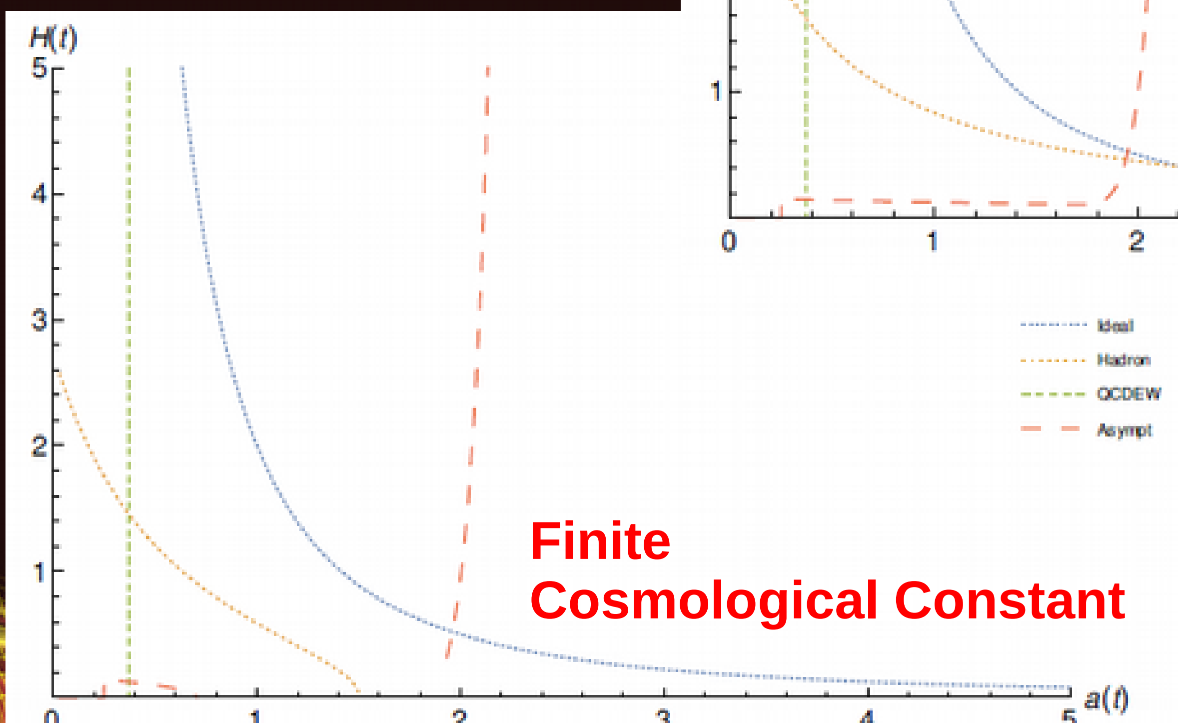
Results: Non-viscous fluid





Results: iscous fluid





Conclusions

The analytical solutions for relaible EoS, in which as much as possible contributions from both standard model for elementary particles and standard model for cosmology are taken into consideration, are sophisticated.

For Eckart theory, the only possible solutions relates the Hubble parameter with the scale factor, but none of them could be directly given in terms of the cosmic time.

For Israel-Stewart theory, the resulting differential equations are found higher-ordered nonlinear nonhomogeneous so that no analytical solution could be proposed, so far.