NEW PARTIAL RESUMMATION OF THE QED EFFECTIVE ACTION

Silvia Pla Garcia (silvia.pla@uv.es)

In collaboration with José Navarro-Salas

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Departamento de Fisica Teorica and IFIC, Centro Mixto Universidad de Valencia-CSIC.
Facultad de Fisica, Universidad de Valencia, Burjassot-46100, Valencia, Spain.

My goal today: I am going to explain a conjecture that states that the proper-time series expansion of the one-loop effective Lagrangian of QED can be partially summed in all terms containing the field-strength invariants $\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ and $\mathcal{G} = \frac{1}{4}\tilde{F}_{\mu\nu}F^{\mu\nu}$. In this talk I will focus on scalar QED.

Overview:

- One loop effective Lagrangian for QED.
  - Heisenberg-Euler Lagrangian.
  - The proper-time series expansion.
- One loop effective Lagrangian in presence of gravity.
  - R-summed form of the proper-time expansion.
- The conjecture.
- Main consequences.
- Summary and conclusions.

Heisenberg-Euler Lagrangian

- The Heisenberg-Euler Lagrangian (W. Heisenberg and H. Euler, 1936) describes the nonlinearities of quantum electrodynamics (QED) when the fermionic degrees of freedom of matter are integrated out and the strength of the electromagnetic background is kept constant. It can also be constructed for scalar QED (V. Weisskopf, 1936)

$$\mathcal{L}^{(1)}_{scalar} = -\frac{1}{(4\pi)^2}\int_0^\infty \frac{ds}{s^3}\, e^{-im^2 s}\left[\det\left(\frac{esF}{\sinh(esF)}\right)\right]^{1/2}, \tag{1}$$

where $F \equiv F^\mu{}_\nu$ and s is known as the “proper-time” parameter.

- The intrinsic nonlinearities of the quantum corrections have important implications: light-by-light scattering, vacuum polarization, pair creation from vacuum, etc...

- For arbitrary background configurations the form of the effective action is, in general, unknown.

One loop effective Lagrangian

- However, it is possible to build a general asymptotic expansion for the one-loop effective action in the number of external fields and the number of derivatives (D. Fliegner, P. Haberl, M.G. Schmidt and C. Schubert, 1998),

$$\mathcal{L}^{(1)}_{scalar} = \int_0^\infty \frac{ds}{s} e^{-im^2 s} g(x; is) \ , \tag{2}$$

with

$$g(x; is) = \frac{1}{(4\pi is)^2} \sum_{n=0}^{\infty} \frac{(-is)^n}{n!} O_n(x) \longrightarrow \left\{ \begin{array}{l} O_0 = 1, \quad O_1 = 0, \\ O_2 = -\frac{e^2}{6} F_{\kappa\lambda} F^{\kappa\lambda} \\ O_3 = -\frac{e^2}{20} \partial_\mu F_{\kappa\lambda} \partial^\mu F^{\kappa\lambda} \end{array} \right\} \tag{3}$$

- The coefficients O_n have been obtained up to 12th adiabatic order ($n = 6$), and their length grows substantially with n. O_6 has 41 terms.

One loop effective Lagrangian for gravitational backgrounds

- The role of the classical electromagnetic background can be replaced by a **gravitational field**, which is naturally coupled to quantized matter fields (that is, we can compute quantum corrections to the Einstein-Hilbert Lagrangian induced by a quantum scalar field).
- In general, the form of the effective action is not known, but it is also possible to make an asymptotic (adiabatic) expansion, as in the previous case (B. S. DeWitt, 1965), i.e.,

$$g(x; is) = \frac{1}{(4\pi is)^2} \sum_{n=0}^{\infty} (is)^n a_n(x) \longrightarrow$$

DeWitt coefficients. They are local, covariant, and gauge-invariant quantities of mass dimension 2n

The one loop effective action is known for the Static Einstein Universe,

$$g(x; is) \to \frac{e^{-i(\xi - \frac{1}{6})Rs}}{(4\pi is)^2}.$$

One can propose a new **adiabatic expansion** (L. Parker and D. J. Toms, 1985), defined by

$$\boxed{g(x; is) = e^{-i(\xi - \frac{1}{6})R(x)s}\, \bar{g}(x; is)}$$

- The new adiabatic expansion

$$e^{-i(\xi-\frac{1}{6})R(x)s}\ \bar{g}(x;is)=\frac{e^{-i(\xi-\frac{1}{6})R(x)s}}{(4\pi is)^2}\sum_{n=0}^{\infty}(is)^n\ \bar{a}_n(x) \tag{4}$$

has an important advantage with respect to the previous one: it does NOT contain any term that vanish when $R(x)$ is replaced by zero.

- The (non-perturbative) exponential factor captures the exact dependence on the Ricci scalar in the generic Schwinger-DeWitt adiabatic expansion.
- Important physical consequences: effective dynamics of the Universe, curvature dependence in the running of the gauge coupling constants...

Key idea: From a solvable case, we have extracted a non perturbative factor in the general adiabatic expansion that completely captures its dependence with $R(x)$. *Is it possible to do the same for the QED Lagrangian?*

Based on the gravitational case, we propose the following conjecture for the electromagnetic case:

The proper-time asymptotic expansion of the QED effective Lagrangian admits an exact resummation *in all terms involving the field-strength invariants* $\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ *and* $\mathcal{G} = \frac{1}{4} \tilde{F}_{\mu\nu} F^{\mu\nu}$*. The form of the factor is just the* Heisenberg-Euler Lagrangian *for QED, where the electric and magnetic fields depend arbitrarily on space-time coordinates.*

$$\mathcal{L}^{(1)}_{scalar} = \int_0^\infty \frac{ds}{s} e^{-im^2 s} g(x; is) \tag{5}$$

$$\downarrow$$

$$\mathcal{L}^{(1)}_{scalar} = \int_0^\infty \frac{ds}{s} e^{-im^2 s} \left[\det \left(\frac{esF(x)}{\sinh(esF(x))} \right) \right]^{1/2} \bar{g}(x; is) \tag{6}$$

Where $\bar{g}(x; is)$ does not have terms that vanish when $\mathcal{F}$ and $\mathcal{G}$ are replaced by zero.

- The new asymptotic expansion is given by

$$\bar{g}(x; is) = \frac{1}{(4\pi is)^2} \sum_{n=0}^{\infty} \frac{(-is)^n}{n!} \bar{O}_n(x). \tag{7}$$

- The adiabatic expansions are related by

$$g(x; is) = \left[\det \left(\frac{esF(x)}{\sinh(esF(x))} \right) \right]^{1/2} \bar{g}(x; is). \tag{8}$$

- If the conjecture works, the new coefficients $\bar{O}_n$ (obtained from O_n and the EH determinant) **will not depend on the EM invariants.**

$$O_0 + \frac{O_2}{2}(-is)^2 + \frac{O_3}{3!}(-is)^3 + \ldots = (EH_0 + EH_2(-is)^2 + \ldots) \cdot (\bar{O}_0 + \frac{\bar{O}_2}{2}(-is)^2 + \frac{\bar{O}_3}{3!}(-is)^3 + \ldots)$$

- We have checked that it is true up to $n = 6$ (the last available coefficient).

Original expansion $g(x;is)$, $O_0=1$,

$$
\begin{aligned}
O_1 &= 0,\quad O_2 = \boldsymbol{-\frac{e^2}{6}F_{\kappa\lambda}F^{\kappa\lambda}},\quad O_3 = -\frac{e^2}{20}\partial_\mu F_{\kappa\lambda}\partial^\mu F^{\kappa\lambda},\\
O_4 &= \boldsymbol{\frac{e^4}{15}F_\kappa{}^\mu F^{\kappa\lambda}F_\lambda{}^\nu F_{\mu\nu} + \frac{e^4}{12}F_{\kappa\lambda}F^{\kappa\lambda}F_{\mu\nu}F^{\mu\nu}}\\
&\quad -\frac{e^2}{70}\partial_\nu\partial_\mu F_{\kappa\lambda}\partial^\nu\partial^\mu F^{\kappa\lambda},\\
O_5 &= \frac{2e^4}{7}F^{\kappa\lambda}F^{\mu\nu}\partial_\lambda F_{\nu\rho}\partial_\mu F_\kappa{}^\rho - \frac{4e^4}{63}F_\kappa{}^\mu F^{\kappa\lambda}\partial_\lambda F^{\nu\rho}\partial_\mu F_{\nu\rho}\\
&\quad -\frac{e^4}{9}F_\kappa{}^\mu F^{\kappa\lambda}F^{\nu\rho}\partial_\mu\partial_\lambda F_{\nu\rho} - \frac{16e^4}{63}F^{\kappa\lambda}F^{\mu\nu}\partial_\mu F_\kappa{}^\rho\partial_\nu F_{\lambda\rho}\\
&\quad +\frac{5e^4}{18}F^{\kappa\lambda}F^{\mu\nu}\partial_\rho F_{\mu\nu}\partial^\rho F_{\kappa\lambda} + \frac{34e^4}{189}F^{\kappa\lambda}F^{\mu\nu}\partial_\nu F_{\lambda\rho}\partial^\rho F_{\kappa\mu}\\
&\quad +\frac{25e^4}{189}F^{\kappa\lambda}F^{\mu\nu}\partial_\rho F_{\lambda\nu}\partial^\rho F_{\kappa\mu} + \frac{4e^4}{21}F_\kappa{}^\mu F^{\kappa\lambda}\partial_\rho F_{\mu\nu}\partial^\rho F_\lambda{}^\nu\\
&\quad +\boldsymbol{\frac{e^4}{12}F_{\kappa\lambda}F^{\kappa\lambda}\partial_\rho F_{\mu\nu}\partial^\rho F^{\mu\nu}} - \frac{e^2}{252}\partial_\rho\partial_\nu\partial_\mu F_{\kappa\lambda}\partial^\rho\partial^\nu\partial^\mu F^{\kappa\lambda}.
\end{aligned}
$$

$(\mathcal{F},\mathcal{G})-$summed expansion $\bar{g}(x;is)$, $\bar{O}_0=1$,

$$\begin{aligned}
\bar{O}_1 &= 0,\ \ \bar{O}_2=0,\ \ \bar{O}_3=-\tfrac{e^2}{20}\partial_\mu F_{\kappa\lambda}\partial^\mu F^{\kappa\lambda},\\
\bar{O}_4 &= -\tfrac{e^2}{70}\partial_\nu\partial_\mu F_{\kappa\lambda}\partial^\nu\partial^\mu F^{\kappa\lambda},\\
\bar{O}_5 &= \tfrac{2e^4}{7}F^{\kappa\lambda}F^{\mu\nu}\partial_\lambda F_{\nu\rho}\partial_\mu F_\kappa{}^\rho-\tfrac{4e^4}{63}F_\kappa{}^\mu F^{\kappa\lambda}\partial_\lambda F^{\nu\rho}\partial_\mu F_{\nu\rho}\\
&-\tfrac{e^4}{9}F_\kappa{}^\mu F^{\kappa\lambda}F^{\nu\rho}\partial_\mu\partial_\lambda F_{\nu\rho}-\tfrac{16e^4}{63}F^{\kappa\lambda}F^{\mu\nu}\partial_\mu F_\kappa{}^\rho\partial_\nu F_{\lambda\rho}\\
&+\tfrac{5e^4}{18}F^{\kappa\lambda}F^{\mu\nu}\partial_\rho F_{\mu\nu}\partial^\rho F_{\kappa\lambda}+\tfrac{34e^4}{189}F^{\kappa\lambda}F^{\mu\nu}\partial_\nu F_{\lambda\rho}\partial^\rho F_{\kappa\mu}\\
&+\tfrac{25e^4}{189}F^{\kappa\lambda}F^{\mu\nu}\partial_\rho F_{\lambda\nu}\partial^\rho F_{\kappa\mu}+\tfrac{4e^4}{21}F_\kappa{}^\mu F^{\kappa\lambda}\partial_\rho F_{\mu\nu}\partial^\rho F_\lambda{}^\nu\\
&-\tfrac{e^2}{252}\partial_\rho\partial_\nu\partial_\mu F_{\kappa\lambda}\partial^\rho\partial^\nu\partial^\mu F^{\kappa\lambda}.
\end{aligned}$$

From the factorization, it is possible to find some exactly solvable electromagnetic backgrounds.

- For electric and magnetic fields pointing in the $\hat{z}$ direction that depend arbitrarily on the light-cone coordinate $x^+ = (t + z)$, we find that $\bar{O}_0 = 1$ and $\bar{O}_{n>0} = 0$. Therefore, the exact form of the unrenormalized effective Lagrangian should be (In agreement with T. Tomaras, N. Tsamis, and R. Woodard, 2000; H. Fried and R. Woodward, 2002; A. Ilderton, 2014)

$$\mathcal{L}^{(1)}_{scalar} = -\frac{1}{16\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-im^2 s} \frac{e^2 s^2 E(x^+) B(x^+)}{\sinh esE(x^+) \sin esB(x^+)}. \qquad (9)$$

- For a single plane wave electromagnetic field, $\mathcal{F}(x) = 0$ and $\mathcal{G}(x) = 0$, therefore, the one loop effective action vanishes $\longrightarrow$ no quantum corrections.

Our results seem to be consistent in presence of gravity:

- In this case, it is possible to perform a double factorization (the exponential $R(x)$ factorization is ensured; I. Jack and L. Parker, 1985).

$$g(x, is) = e^{-is(\xi - \frac{1}{6})R(x)} \left[\det \left(\frac{esF(x)}{\sinh(esF(x))} \right) \right]^{1/2} \tilde{g}(x; is) \tag{10}$$

The conjecture allows us to make some general predictions regarding the Schwinger's formula for the pair production rate.

- The factorization found suggests that the poles of the imaginary part of the one-loop effective Lagrangian are located at the same points as in the constant electric field case $\tau_n = n\pi/|eE(x)|$.

- The one-loop QED effective-Lagrangian is, in general, unknown. However, it is possible to obtain an adiabatic expansion of it $g(x; is)$.
- We have proposed an alternative adiabatic expansion $\bar{g}(x; is)$, which encapsulates in a global pre-factor all its dependence on the electromagnetic invariants $\mathcal{F}(x)$ and $\mathcal{G}(x)$.
- The form of the (non-perturbative) factor involved in this partial resummation is just the Heisenberg-Euler Lagrangian for QED, where the electric and magnetic fields depend arbitrarily on spacetime coordinates.
- The new expansion does not contain terms that vanish when $\mathcal{F}(x)$ and $\mathcal{G}(x)$ are replaced by zero.
- This new factorization allow us to obtain some exact solutions.
- The factorization seems to be **consistent** in presence of gravity.

THANKS FOR YOUR ATTENTION